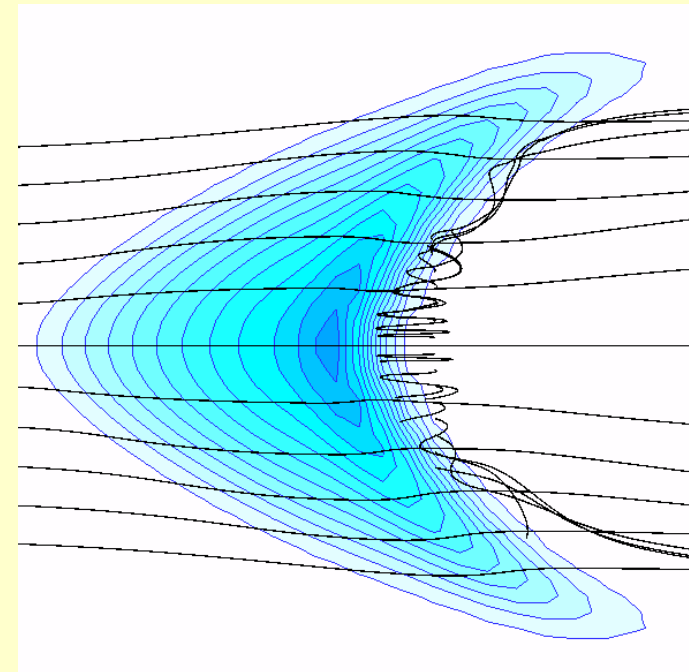
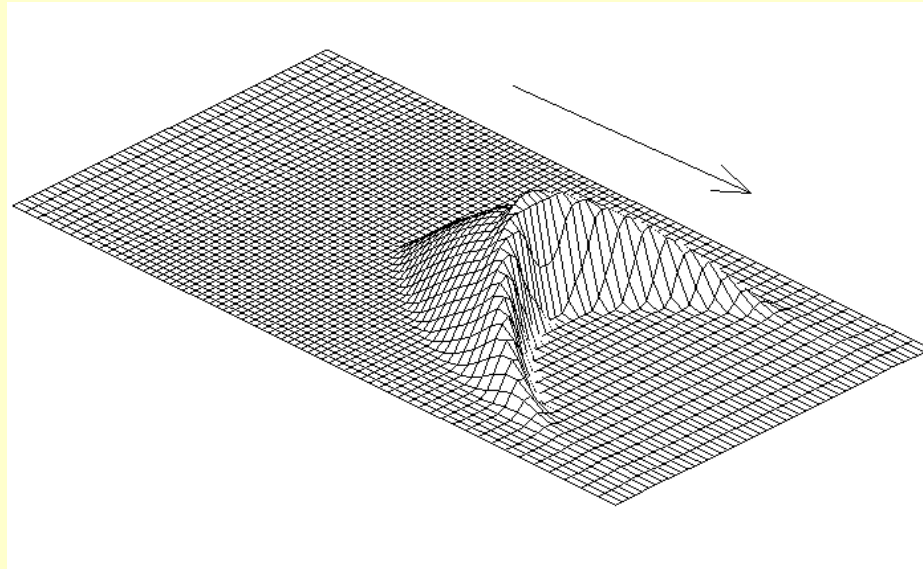
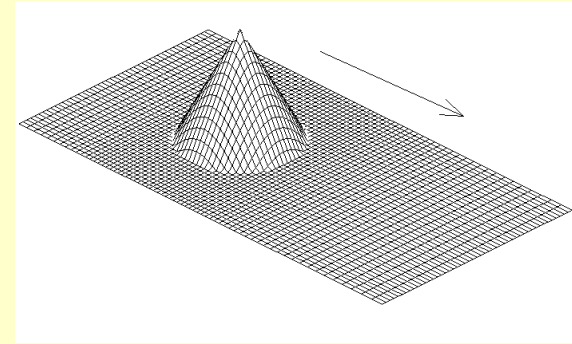


Wall Functions For Arbitrarily-Rough Surfaces

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Motivation

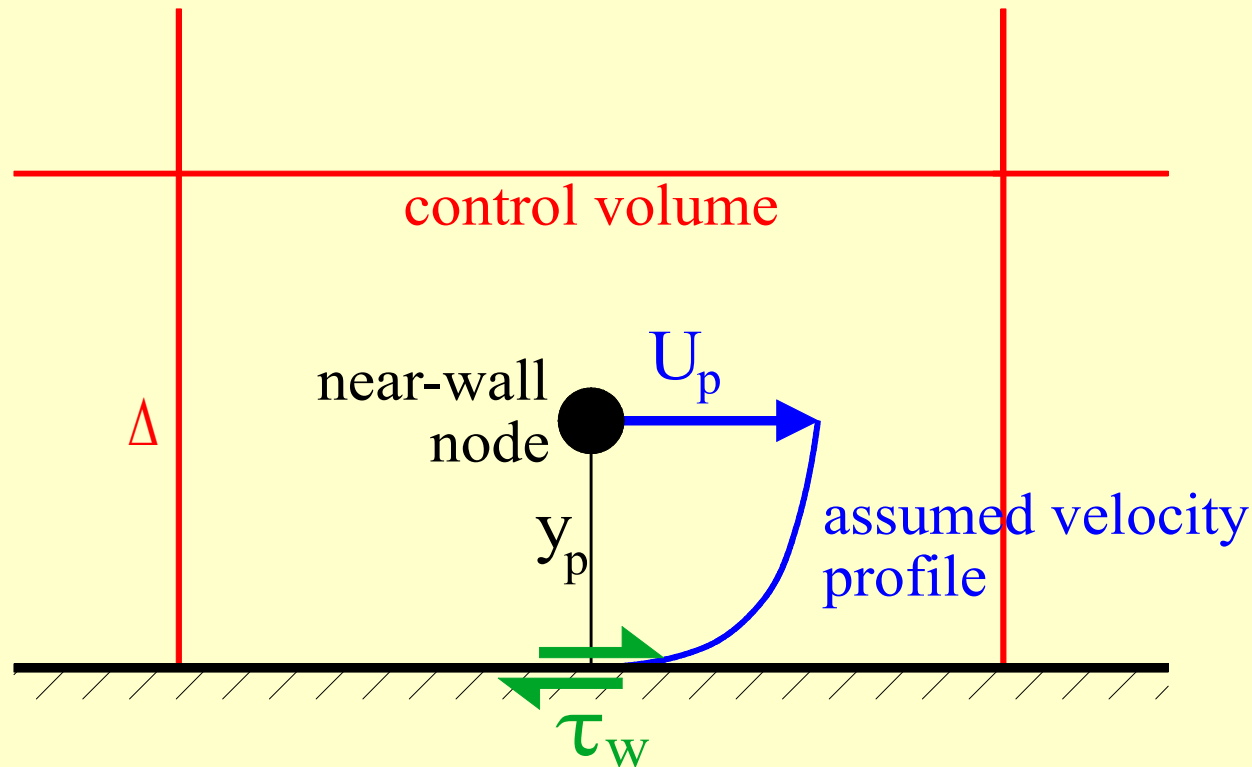
- Sediment morphodynamics



Outline

- Description of wall function
- Validation for pipe flow
- Sediment-transport modelling
- Applications
 - Scour in a channel bend
 - Evolution of sand mounds

Requirements of a Wall Function



- For \mathbf{U} : τ_w wall shear stress, τ_w
- For k or $\overline{u_i u_j}$: cell-averaged production and dissipation
- For ε : value at near-wall node

Key Elements of Wall Function

- Velocity scale: $\tilde{u}_\tau = C_\mu^{1/4} k_p^{1/2}$

- Total viscosity: $\nu_{total} = \begin{cases} \nu & y \leq y_v \\ \nu + \kappa \tilde{u}_\tau (y - y_v), & y \geq y_v \end{cases}$

- Roughness-dependent zero-eddy-viscosity height:

$$y_v(\tilde{k}_s^+)$$

- Roughness-dependent ε formulation:

$$\varepsilon = \begin{cases} \frac{\tilde{u}_\tau^3}{\kappa(y - y_d)}, & y \geq y_\varepsilon \\ \varepsilon_w, & y \leq y_\varepsilon \end{cases} \quad y_\varepsilon, y_d \text{ functions of } \tilde{k}_s^+$$

Log-Law Velocity Profile

- Smooth: $U^+ = \frac{1}{\kappa} \ln y^+ + 5.2$ $y^+ = \frac{yu_\tau}{\nu}, \quad U^+ = \frac{U}{u_\tau}$
- Rough: $U^+ = \frac{1}{\kappa} \ln \frac{y}{k_s} + 8.0$
- Intermediate: $U^+ = \frac{1}{\kappa} \ln y^+ + B(k_s^+)$

Whole range covered by:

$$B = 8.0 - \frac{1}{\kappa} \ln(k_s^+ + 3.152)$$

Velocity / Shear-Stress Relation

Shear stress: $\tau = \rho v_{total} \frac{\partial U}{\partial y}$

Assume $\tau = \tau_w$ and integrate:

$$\frac{U}{\tilde{u}_\tau} = \frac{\tau_w}{\rho \tilde{u}_\tau^2} \times \begin{cases} y^+, & y^+ \leq y_v^+ \\ y_{v0}^+ + \frac{1}{\kappa} \ln \left[\frac{1 + \kappa(y^+ - y_v^+)}{1 + \kappa(y_{v0}^+ - y_v^+)} \right], & y^+ \geq y_v^+ \end{cases}$$
$$y_{v0}^+ = \max(y_v^+, 0)$$

Conveniently implemented via an effective wall viscosity:

$$\tau_w = \rho v_{eff,wall} \frac{U_p}{y_p}$$

Zero-Eddy-Viscosity Height, y_v^+

- Log law: $U^+ = \frac{1}{\kappa} \ln y^+ + B(k_s^+)$

- Velocity profile: $U^+ = y_v^+ + \frac{1}{\kappa} \ln[1 + \kappa(y^+ - y_v^+)]$

(case $y_v > 0$)

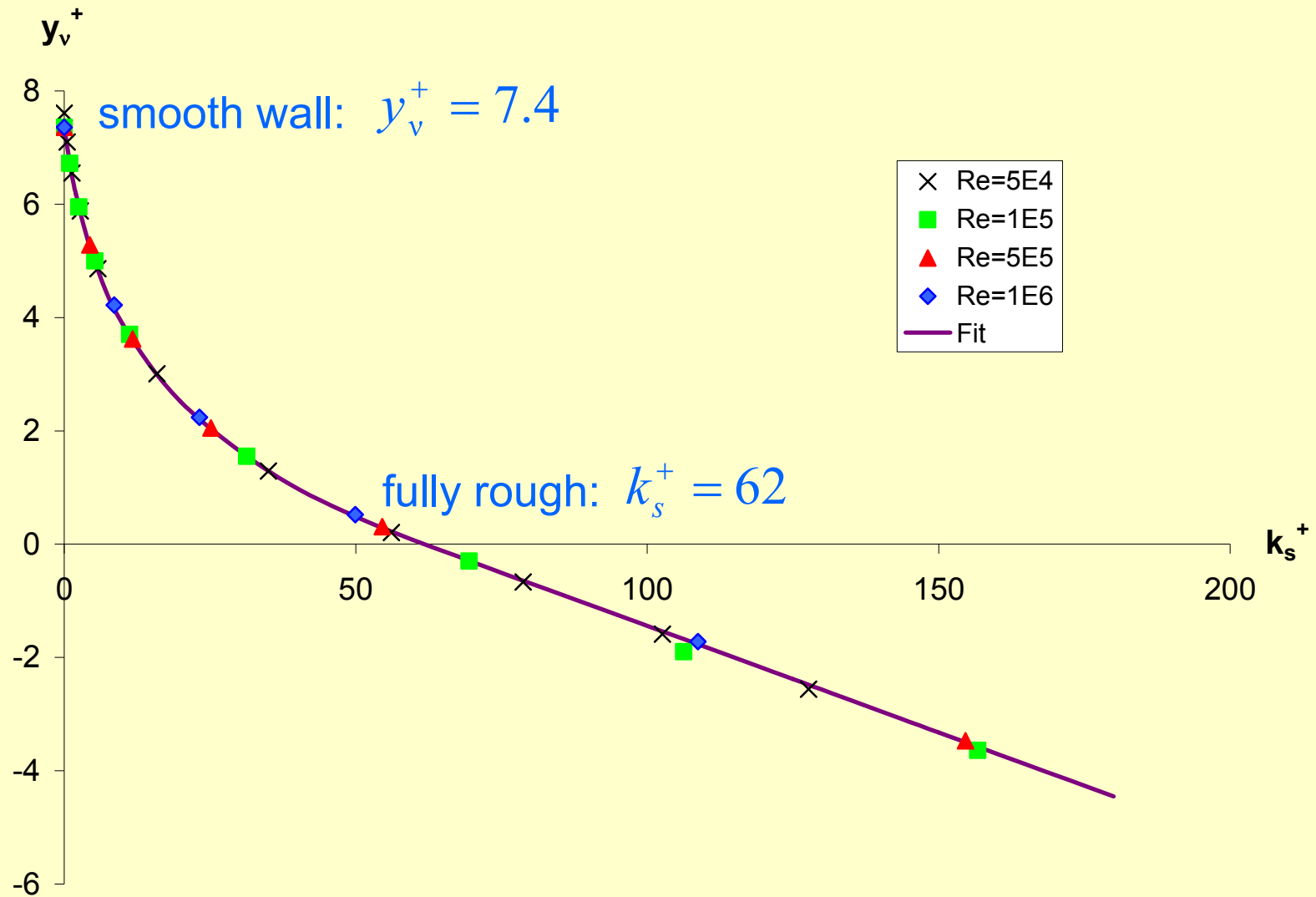
$$= \frac{1}{\kappa} \ln y^+ + (y_v^+ + \frac{1}{\kappa} \ln \kappa) + O(\frac{1}{y^+})$$

Enforcing higher-order consistency with the log law gives y_v^+

$$y_v^+ = \begin{cases} B - \frac{1}{\kappa} \ln \kappa, & B - \frac{1}{\kappa} \ln \kappa \geq 0 \\ \frac{1}{\kappa} (1 - e^{-\kappa(B - \frac{1}{\kappa} \ln \kappa)}), & B - \frac{1}{\kappa} \ln \kappa \leq 0 \end{cases}$$

$$B = 8.0 - \frac{1}{\kappa} \ln(k_s^+ + 3.152)$$

Roughness-Dependent y_v



Production and Dissipation of TKE

Profiles: $P^{(k)} \equiv -\overline{uv} \frac{\partial U}{\partial y} = v_t \left(\frac{\partial U}{\partial y} \right)^2$ $\varepsilon = \begin{cases} \frac{\tilde{u}_\tau^3}{\kappa(y - y_d)}, & y \geq y_\varepsilon \\ \varepsilon_w, & y \leq y_\varepsilon \end{cases}$

Average: $P_{av}^{(k)} \equiv \frac{1}{\Delta} \int_0^\Delta P^{(k)} dy = \left(\frac{\tau_w}{\rho \tilde{u}^2} \right)^2 \frac{\tilde{u}_\tau^3}{\kappa \Delta} \left[\ln \left(\frac{\Delta^+ - y_v^+ + 1/\kappa}{y_{v0}^+ - y_v^+ + 1/\kappa} \right) + \frac{1}{1 + \kappa(\Delta^+ - y_v^+)} - \frac{1}{1 + \kappa(y_{v0}^+ - y_v^+)} \right]$

$\varepsilon_{av} \equiv \frac{1}{\Delta} \int_0^\Delta \varepsilon dy = \frac{\tilde{u}_\tau^3}{\kappa \Delta} \left[\ln \left(\frac{\Delta^+ - y_d^+}{y_\varepsilon^+ - y_d^+} \right) + \frac{y_\varepsilon^+}{y_\varepsilon^+ - y_d^+} \right]$

In an equilibrium boundary layer: $\rho \tilde{u}_\tau^2 \approx \tau_w$

$$\int_0^\infty (P^{(k)} - \varepsilon) dy = 0$$

Matching: $y_d^+ = y_v^+ - \frac{1}{\kappa}$

$y_\varepsilon^+ = y_d^+ + \frac{s_1}{\kappa} \exp\left(\frac{y_\varepsilon^+}{y_\varepsilon^+ - y_d^+} + \frac{1}{s_1}\right)$

$s_1 = 1 + \kappa(y_{v0}^+ - y_v^+) = 1 + \kappa \max(0, -y_v^+)$

Smooth wall: $y_d^+ = 4.9$ $y_\varepsilon^+ = 27.4$

Reynolds-Stress Equations

Use cell-averaged production and dissipation

In simple shear:

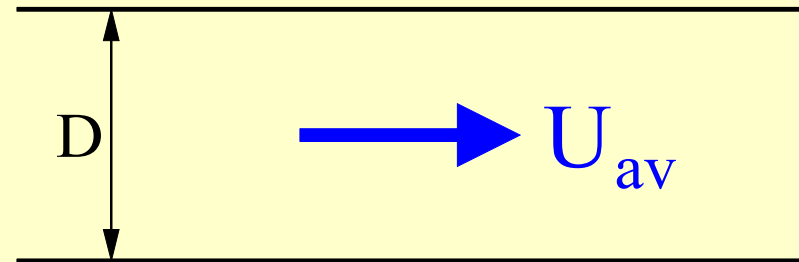
$$P_{tt} = -2\overline{u_t u_n} \frac{\partial U_t}{\partial n} = 2P^{(k)}, \quad P_{nn} = P_{bb} = 0$$

$$P_{tn} = -\overline{u_n^2} \frac{\partial U_t}{\partial n} = \frac{\overline{u_n^2}}{\overline{u_t u_n}} P^{(k)} \quad P_{nb} = P_{bt} = 0$$

Pipe Flow

Head loss:

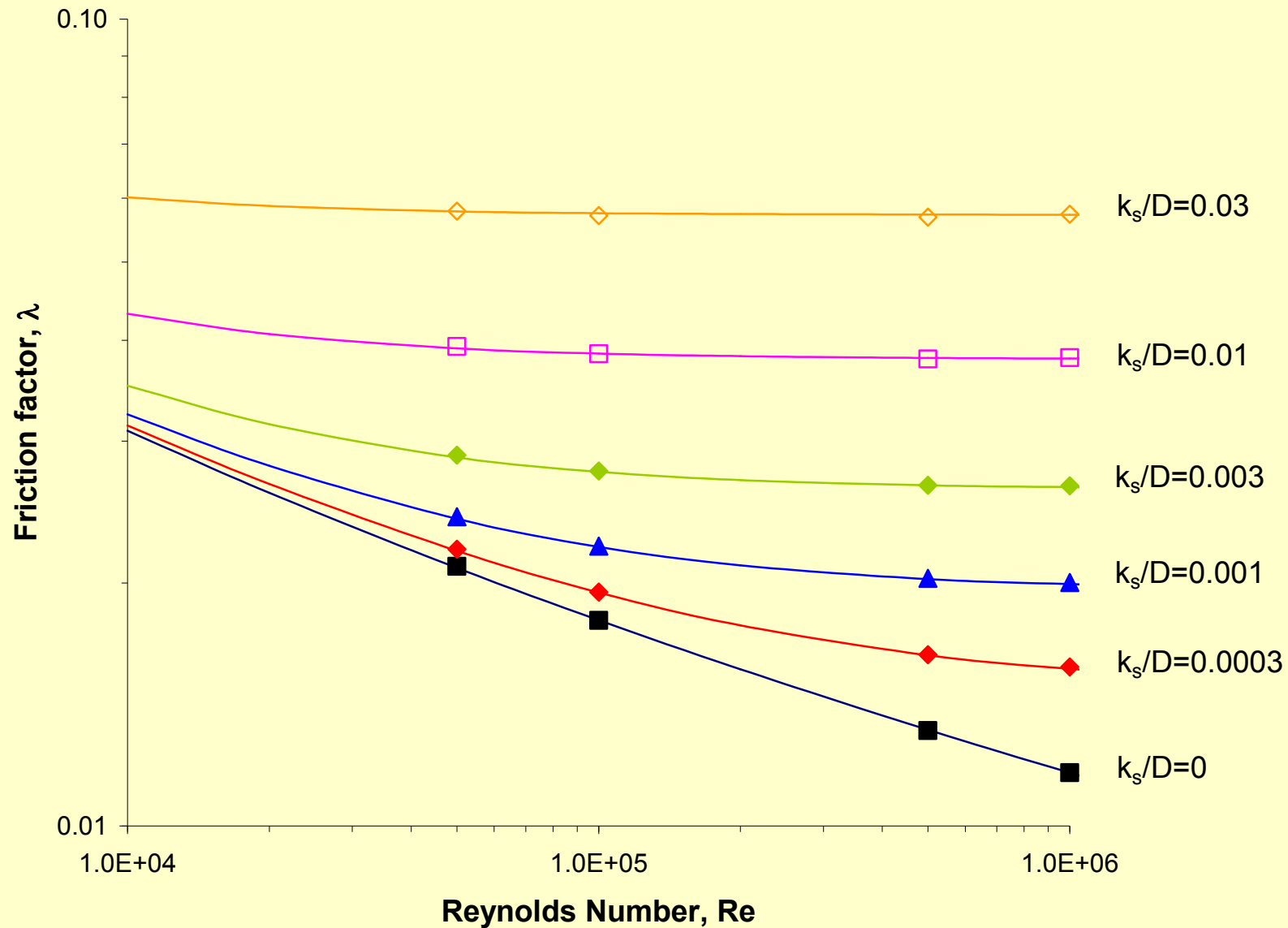
$$h = \lambda \frac{L}{D} \frac{U_{av}^2}{2g}$$



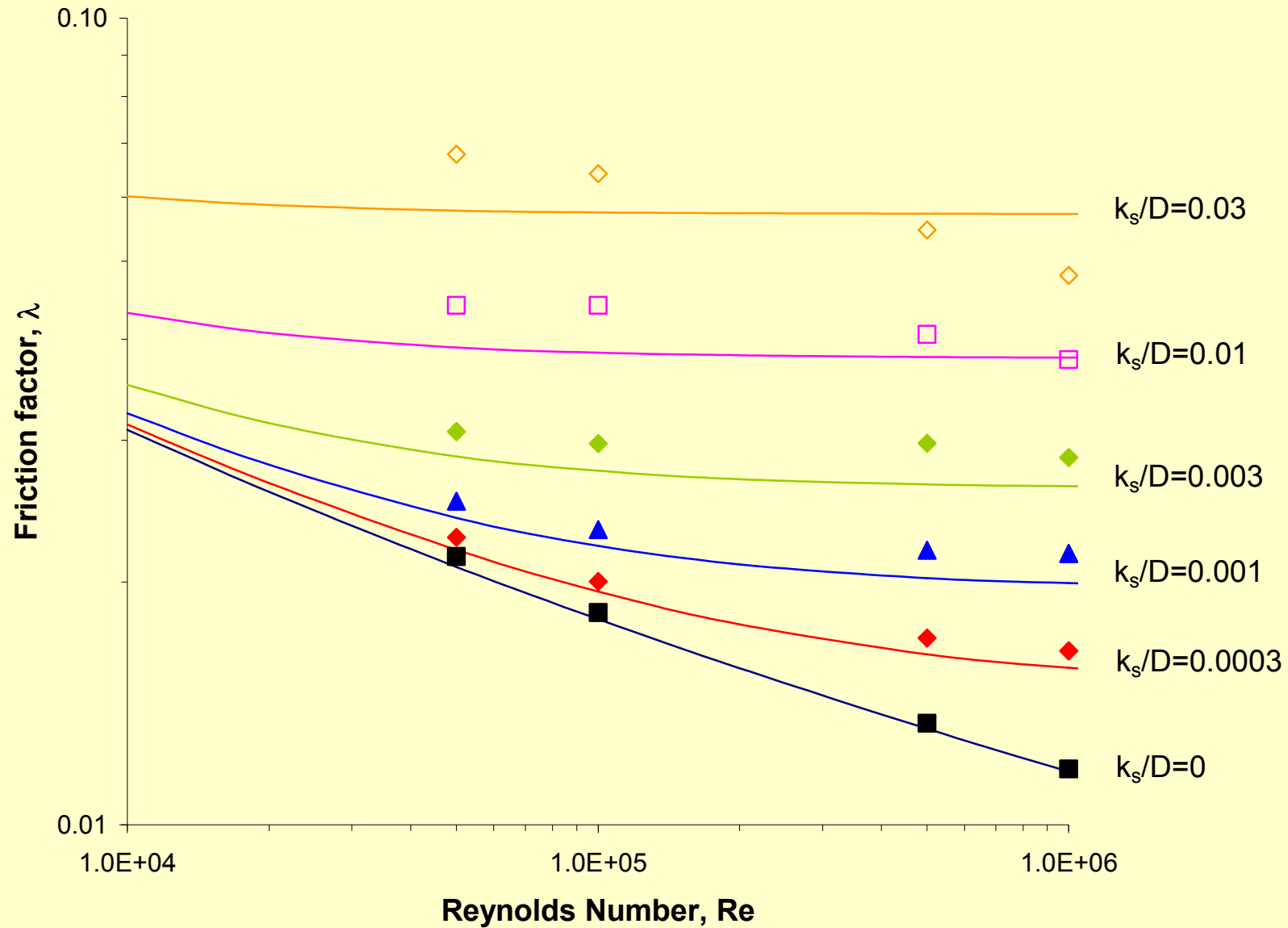
Friction factor (Colebrook-White equation):

$$\frac{1}{\sqrt{\lambda}} = -2.0 \log_{10} \left(\frac{k_s}{3.7D} + \frac{2.51}{\text{Re} \sqrt{\lambda}} \right)$$

Friction Factor (k - ε Model)



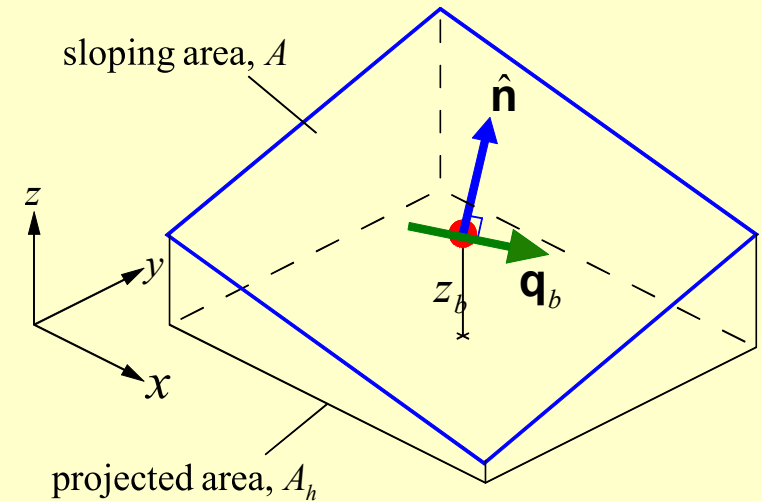
Friction Factor (Reynolds Stress Model)



Sediment Morphodynamics

Change of volume = - net flux

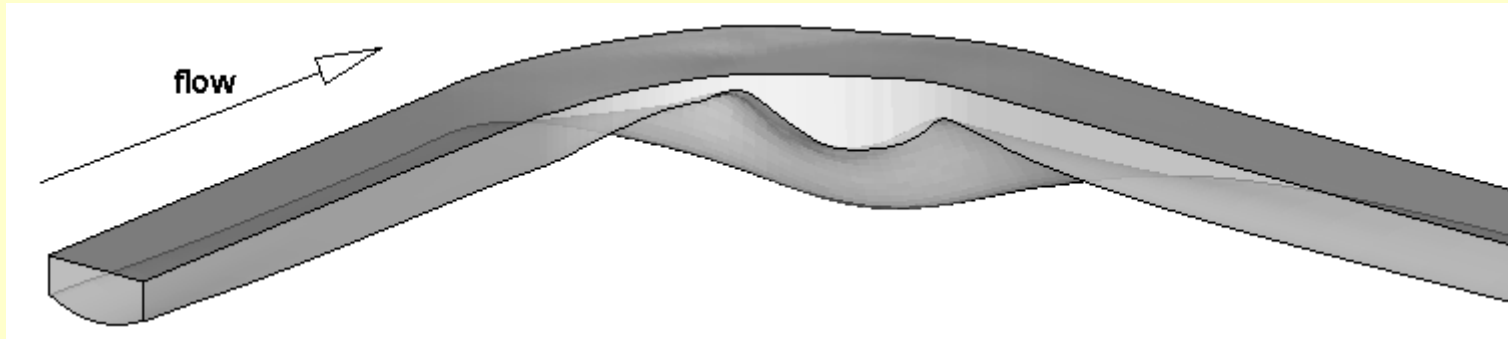
$$(1 - p)A_h \frac{\Delta z_b}{\Delta t} = - \oint_{\partial A} \mathbf{q}_b \cdot d\mathbf{s} \wedge \hat{\mathbf{n}}$$



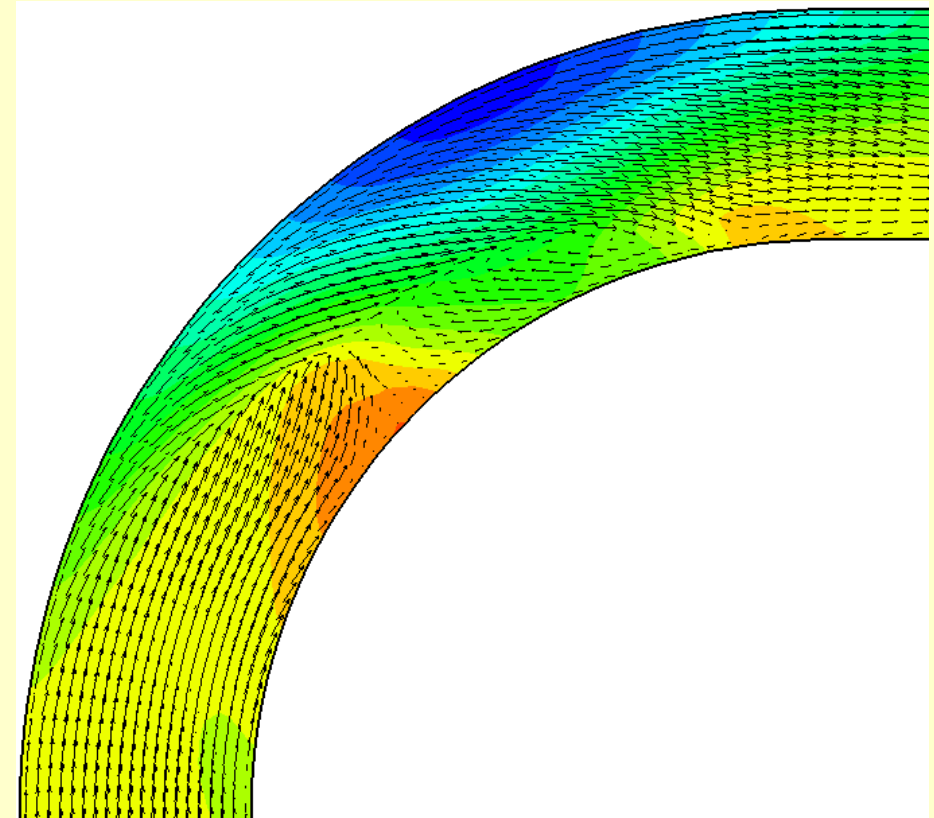
(Bed-load) sediment flux

$$\mathbf{q}_b = f(\text{surface shear stress, surface orientation})$$

Scour in a Channel Bend



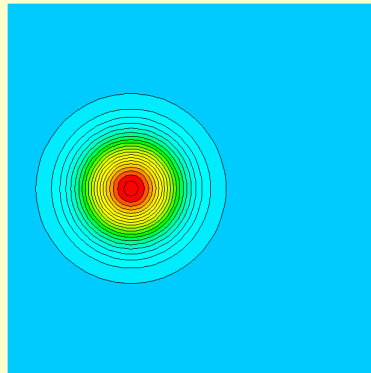
Data: Kawai and Julien, 1996



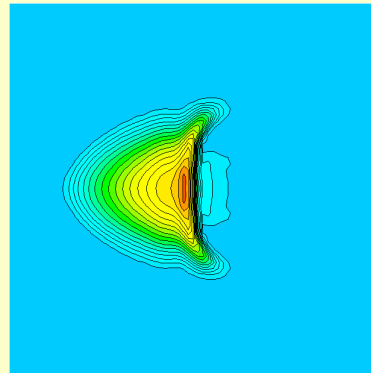
Transport of a Gaussian Mound



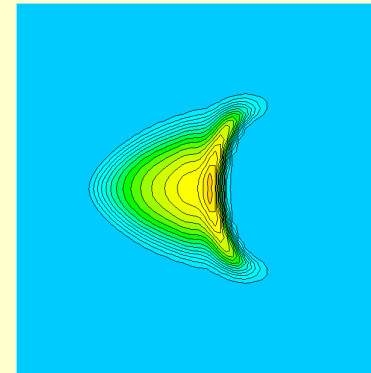
Fixed bed



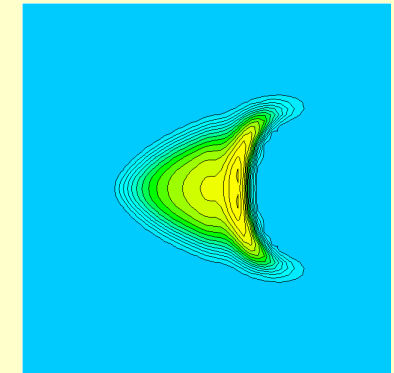
t=0



40 min

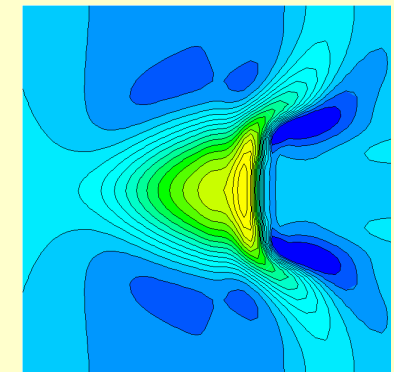
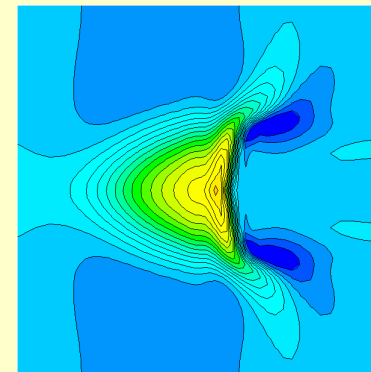
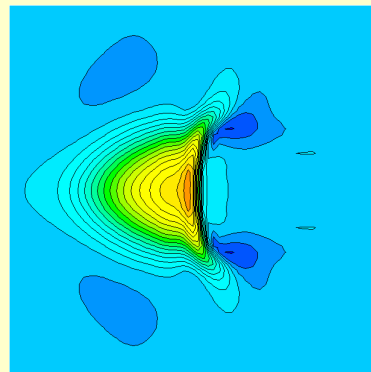
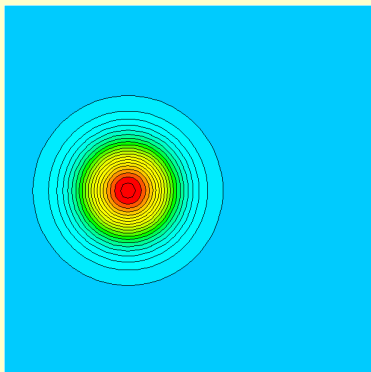


80 min



120 min

Mobile bed



Reference

- Apsley, D.D., 2007, CFD calculation of turbulent flow with arbitrary wall roughness, *Flow, Turbulence and Combustion*, **78**, 153-175.