Wall Functions For Arbitrarily-Rough Surfaces

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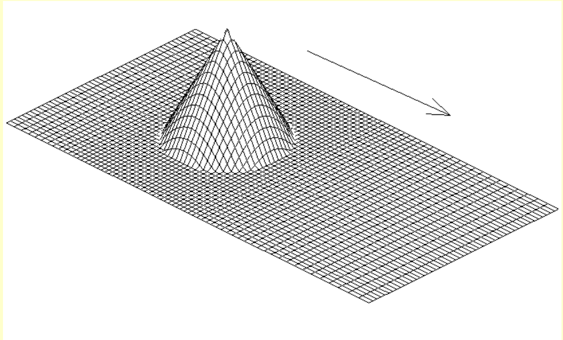
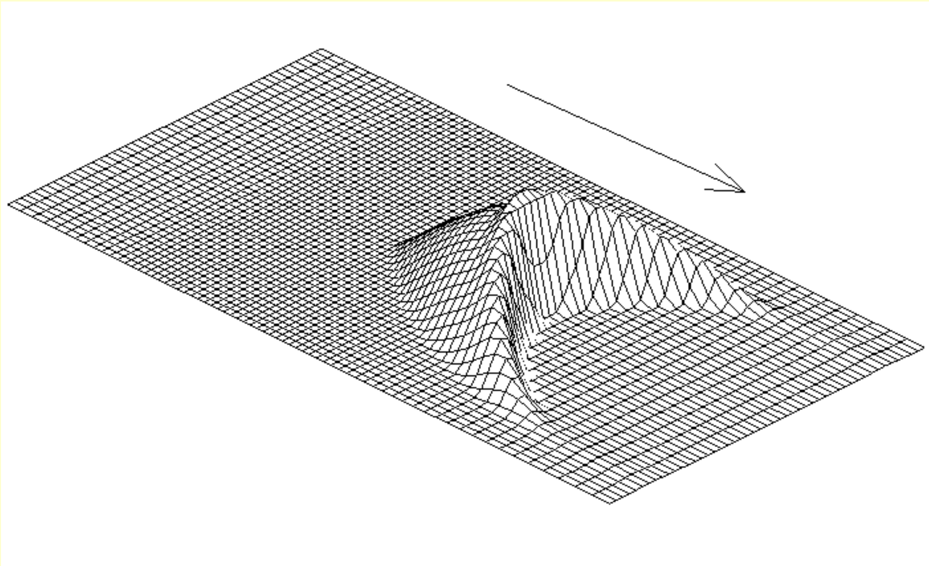
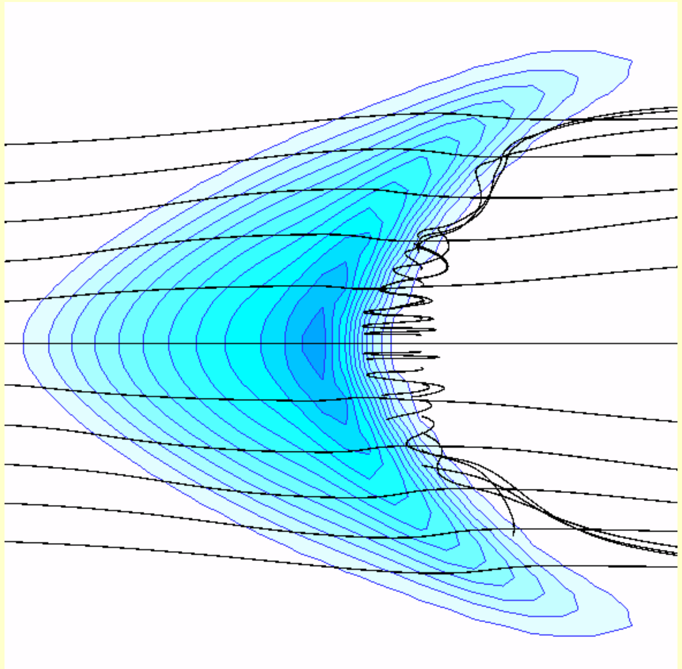
Motivation

- Sediment morphodynamics



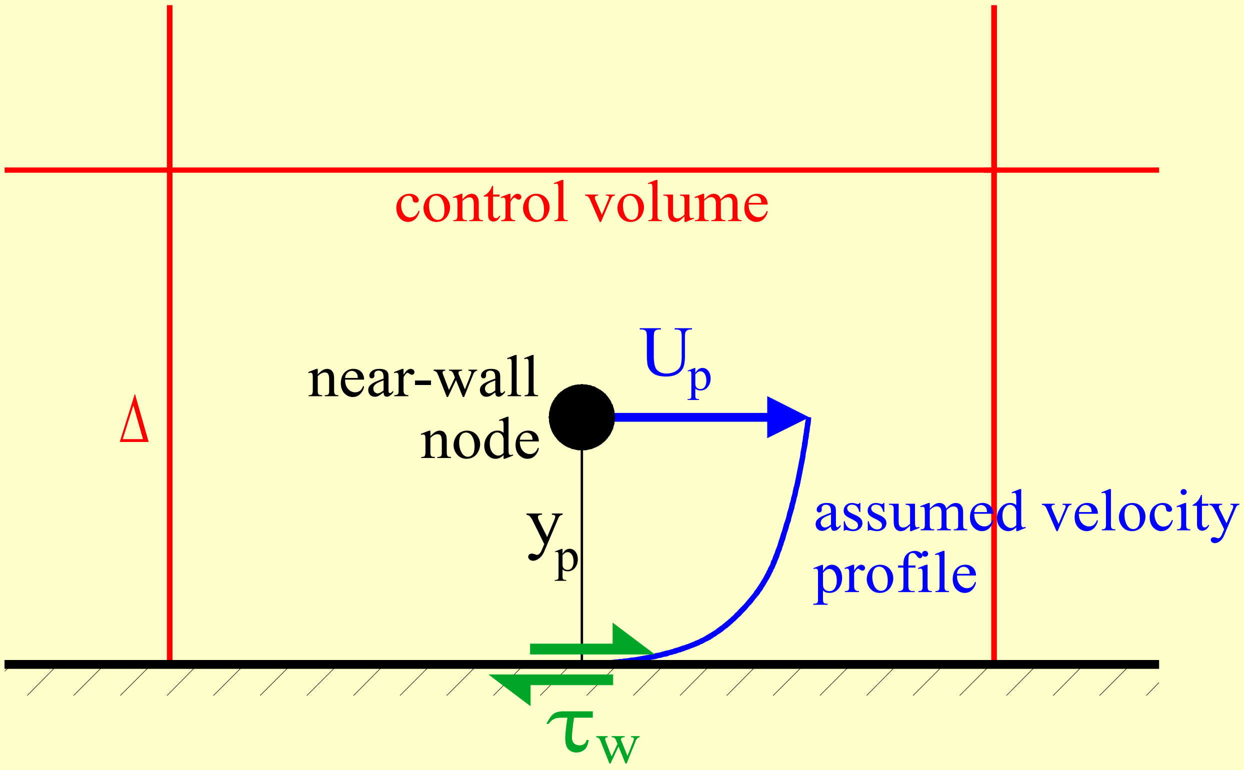




Outline

- Description of wall function
- Validation for pipe flow
- Sediment-transport modelling
- Applications
  - Scour in a channel bend
  - Evolution of sand mounds

Requirements of a Wall Function



- For **U**: wall shear stress, τ_w
- For k or $\overline{u_i u_j}$: cell-averaged production and dissipation
- For ε: value at near-wall node

Key Elements of Wall Function

- Velocity scale: $\widetilde{u}_\tau = C_\mu^{1/4} k_p^{1/2}$
- Total viscosity: $$\nu_{total} = \begin{cases} \nu & y \le y_\nu \\ \nu + \kappa \widetilde{u}_\tau (y - y_\nu), & y \ge y_\nu \end{cases}$$
- Roughness-dependent zero-eddy-viscosity height: $$y_\nu(\widetilde{k}_S^+)$$
- Roughness-dependent ε formulation: $$\varepsilon = \begin{cases} \dfrac{\widetilde{u}_\tau^3}{\kappa(y - y_d)}, & y \ge y_\varepsilon \\ \varepsilon_w, & y \le y_\varepsilon \end{cases}$$ y_ε, y_d functions of $\widetilde{k}_s^+$

Log-Law Velocity Profile

- Smooth: $$U^+ = \frac{1}{\kappa}\ln y^+ + 5.2$$ $$y^+ = \frac{y u_\tau}{\nu}, \quad U^+ = \frac{U}{u_\tau}$$
- Rough: $$U^+ = \frac{1}{\kappa}\ln \frac{y}{k_s} + 8.0$$
- Intermediate: $$U^+ = \frac{1}{\kappa}\ln y^+ + B(k_s^+)$$

Whole range covered by: $$B = 8.0 - \frac{1}{\kappa}\ln(k_s^+ + 3.152)$$

Velocity / Shear-Stress Relation

Shear stress: $\tau = \rho \nu_{total} \dfrac{\partial U}{\partial y}$

Assume $\tau = \tau_w$ and integrate:

$$\frac{U}{\tilde{u}_\tau} = \frac{\tau_w}{\rho \tilde{u}_\tau^2} \times \begin{cases} y^+, & y^+ \le y_\nu^+ \\ y_{\nu 0}^+ + \dfrac{1}{\kappa} \ln[\dfrac{1+\kappa(y^+ - y_\nu^+)}{1+\kappa(y_{\nu 0}^+ - y_\nu^+)}], & y^+ \ge y_\nu^+ \end{cases}$$

$$y_{\nu 0}^+ = \max(y_\nu^+, 0)$$

Conveniently implemented via an effective wall viscosity:

$$\tau_w = \rho \nu_{eff,wall} \frac{U_p}{y_p}$$

Zero-Eddy-Viscosity Height, y_ν^+

- Log law: $$U^+ = \frac{1}{\kappa}\ln y^+ + B(k_s^+)$$

- Velocity profile: (case $y_\nu > 0$)

$$U^+ = y_\nu^+ + \frac{1}{\kappa}\ln[1+\kappa(y^+ - y_\nu^+)]$$

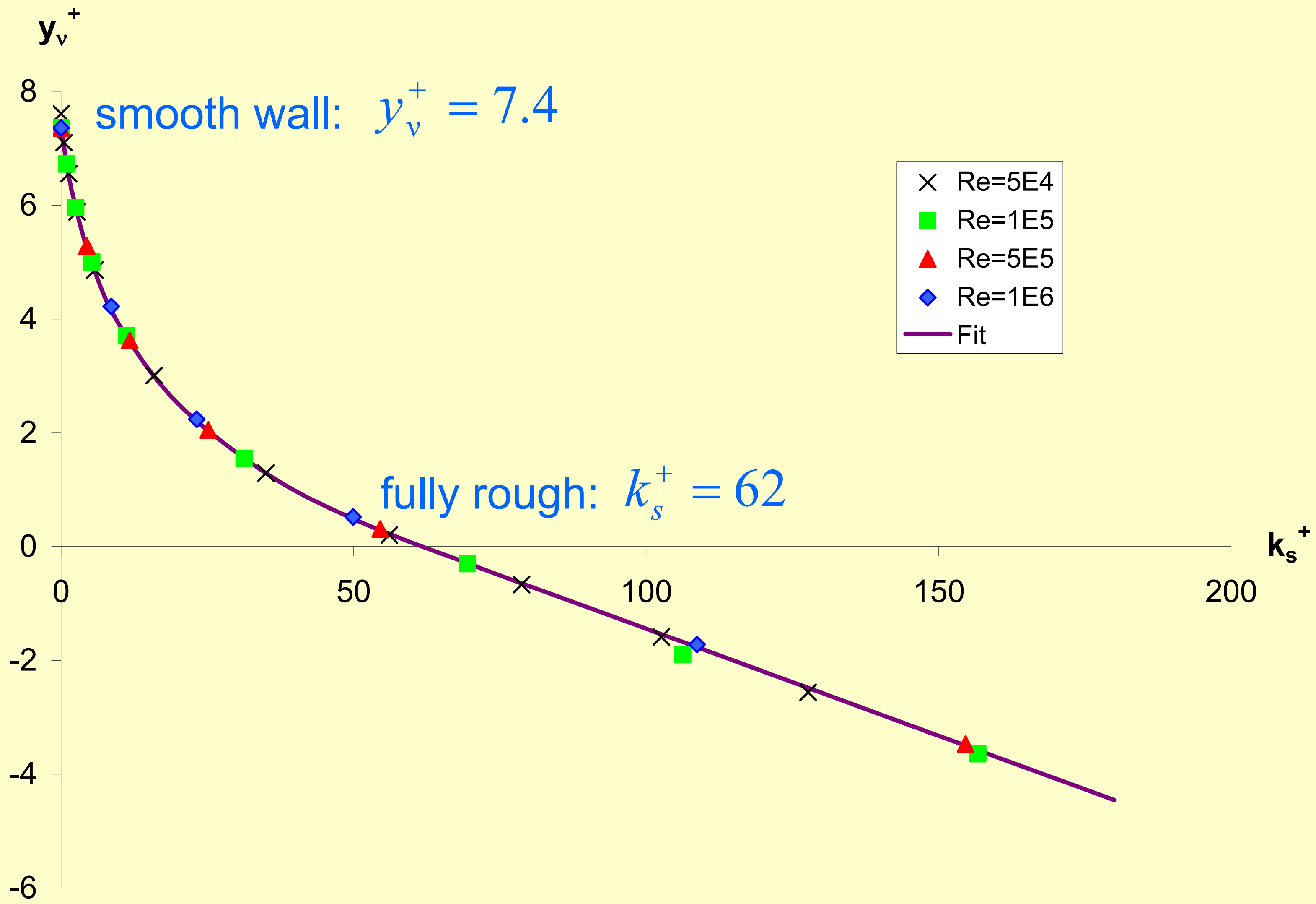
$$= \frac{1}{\kappa}\ln y^+ + (y_\nu^+ + \frac{1}{\kappa}\ln\kappa) + O(\frac{1}{y^+})$$

Enforcing higher-order consistency with the log law gives y_ν^+

$$y_\nu^+ = \begin{cases} B - \frac{1}{\kappa}\ln\kappa, & B - \frac{1}{\kappa}\ln\kappa \geq 0 \\ \frac{1}{\kappa}(1 - e^{-\kappa(B - \frac{1}{\kappa}\ln\kappa)}), & B - \frac{1}{\kappa}\ln\kappa \leq 0 \end{cases}$$

$$B = 8.0 - \frac{1}{\kappa}\ln(k_s^+ + 3.152)$$

Roughness-Dependent y_ν



Production and Dissipation of TKE

Profiles: $P^{(k)} \equiv -\overline{uv}\dfrac{\partial U}{\partial y} = \nu_t\left(\dfrac{\partial U}{\partial y}\right)^2$ $\varepsilon = \begin{cases} \dfrac{\tilde{u}_\tau^3}{\kappa(y-y_d)}, & y \ge y_\varepsilon \\ \varepsilon_w, & y \le y_\varepsilon \end{cases}$

Average:

$$P_{av}^{(k)} \equiv \frac{1}{\Delta}\int_0^\Delta P^{(k)}\ \mathrm{d}y = \left(\frac{\tau_w}{\rho\tilde{u}^2}\right)^2 \frac{\tilde{u}_\tau^3}{\kappa\Delta}\left[\ln\left(\frac{\Delta^+ - y_\nu^+ + 1/\kappa}{y_{\nu 0}^+ - y_\nu^+ + 1/\kappa}\right) + \frac{1}{1+\kappa(\Delta^+ - y_\nu^+)} - \frac{1}{1+\kappa(y_{\nu 0}^+ - y_\nu^+)}\right]$$

$$\varepsilon_{av} \equiv \frac{1}{\Delta}\int_0^\Delta \varepsilon\ \mathrm{d}y = \frac{\tilde{u}_\tau^3}{\kappa\Delta}\left[\ln\left(\frac{\Delta^+ - y_d^+}{y_\varepsilon^+ - y_d^+}\right) + \frac{y_\varepsilon^+}{y_\varepsilon^+ - y_d^+}\right]$$

In an equilibrium boundary layer: $\rho\tilde{u}_\tau^2 \approx \tau_w$

$$\int_0^\infty (P^{(k)} - \varepsilon)\ \mathrm{d}y = 0$$

Matching: $y_d^+ = y_\nu^+ - \dfrac{1}{\kappa}$

$$y_\varepsilon^+ = y_d^+ + \frac{s_1}{\kappa}\exp\left(\frac{y_\varepsilon^+}{y_\varepsilon^+ - y_d^+} + \frac{1}{s_1}\right) \qquad s_1 = 1 + \kappa(y_{\nu 0}^+ - y_\nu^+) = 1 + \kappa\max(0, -y_\nu^+)$$

Smooth wall: $y_d^+ = 4.9$ $y_\varepsilon^+ = 27.4$

Reynolds-Stress Equations

Use cell-averaged production and dissipation

In simple shear:

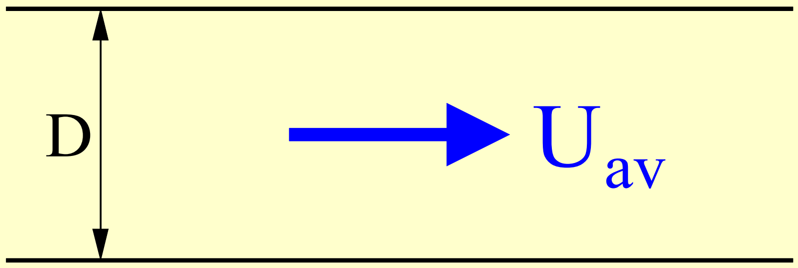
$$P_{tt} = -2\overline{u_t u_n}\frac{\partial U_t}{\partial n} = 2P^{(k)}, \qquad P_{nn} = P_{bb} = 0$$

$$P_{tn} = -\overline{u_n^2}\frac{\partial U_t}{\partial n} = \frac{\overline{u_n^2}}{\overline{u_t u_n}}P^{(k)} \qquad P_{nb} = P_{bt} = 0$$

Pipe Flow

Head loss:

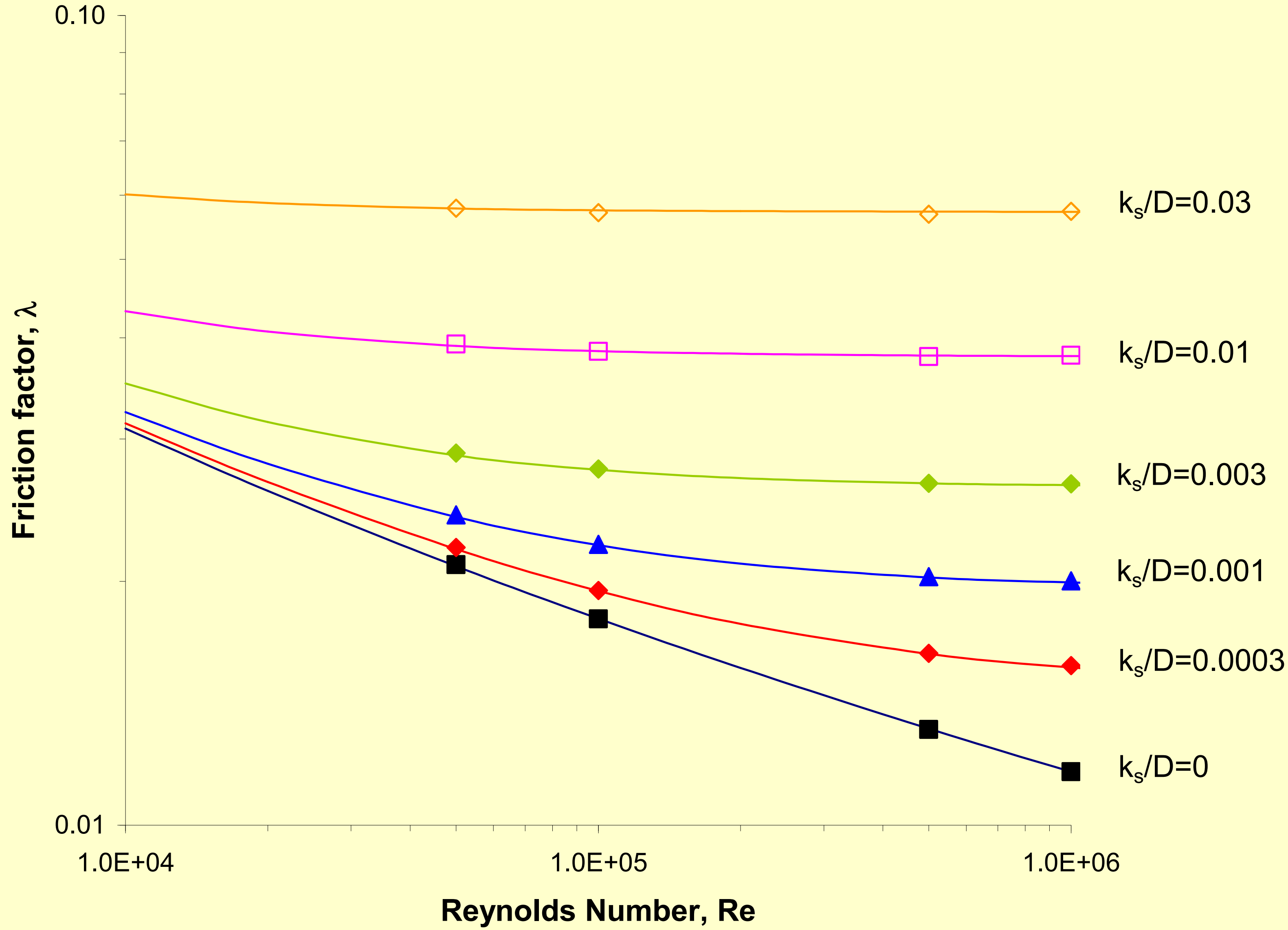
$$h = \lambda \frac{L}{D} \frac{U_{av}^2}{2g}$$



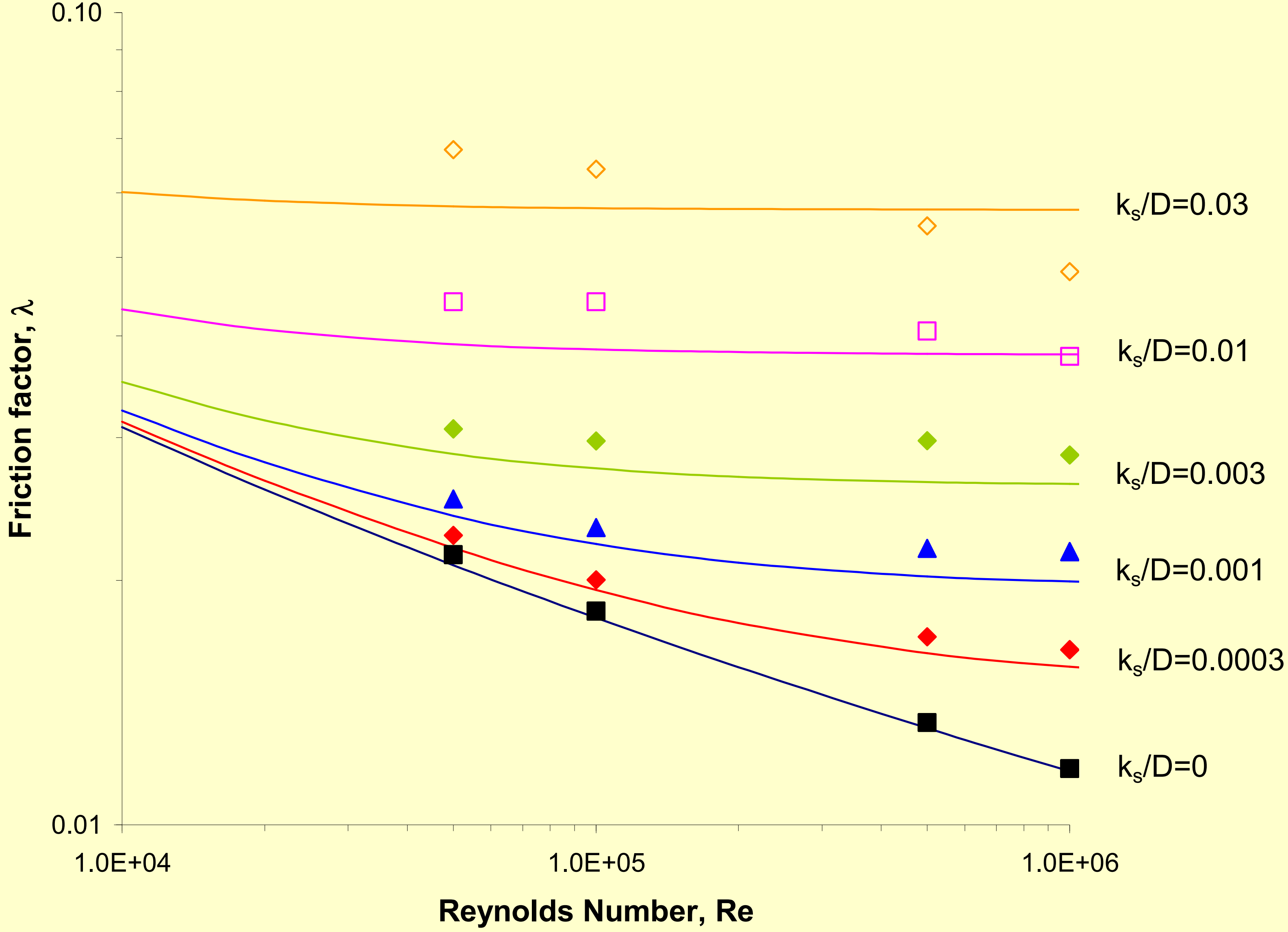
Friction factor (Colebrook-White equation):

$$\frac{1}{\sqrt{\lambda}} = -2.0 \log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}} \right)$$

Friction Factor (*k*-ε Model)



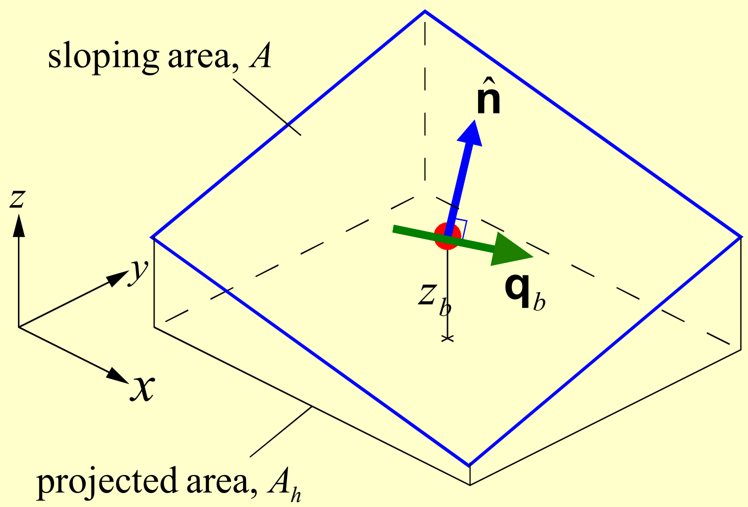
Friction Factor (Reynolds Stress Model)



Sediment Morphodynamics

Change of volume = - net flux

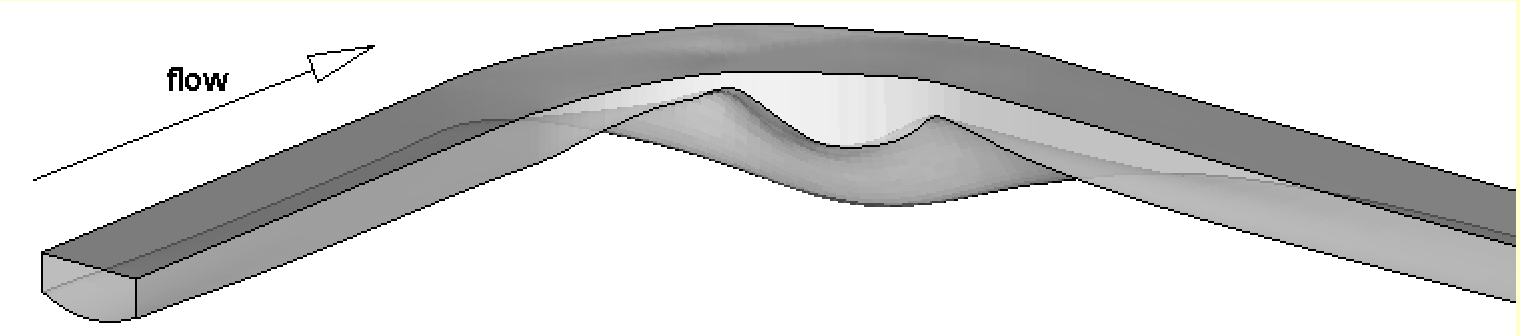
$$(1-p)A_h \frac{\Delta z_b}{\Delta t} = -\oint_{\partial A} \mathbf{q}_b \bullet \mathrm{d}\mathbf{s} \wedge \hat{\mathbf{n}}$$



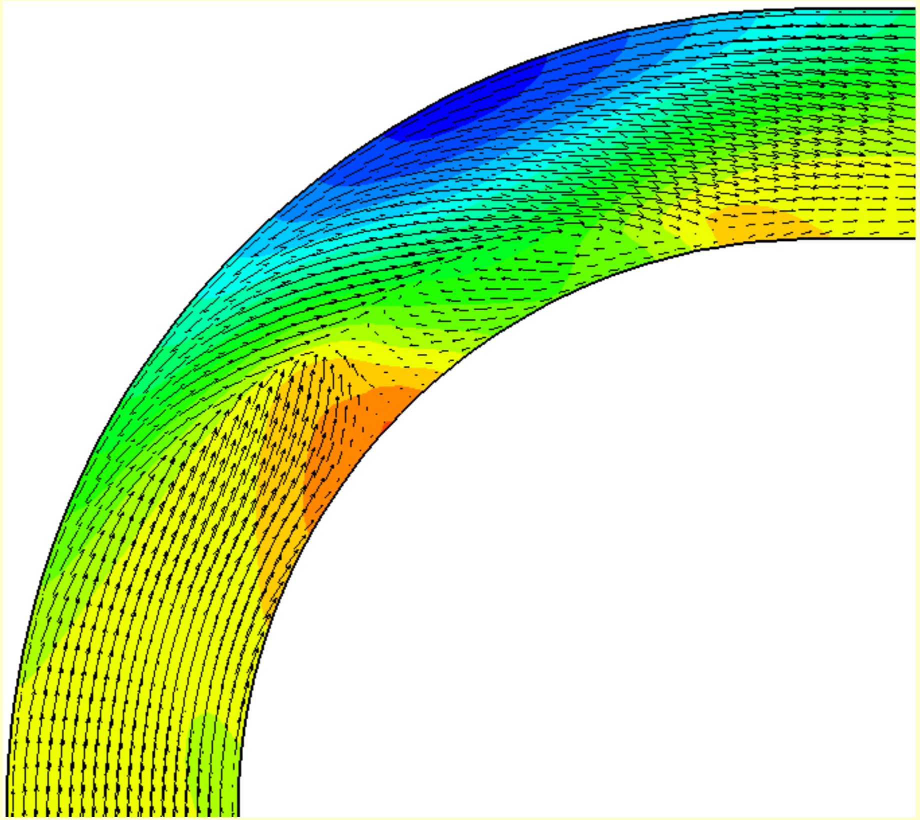
(Bed-load) sediment flux

$$\mathbf{q}_b = f(\textit{surface shear stress, surface orientation})$$

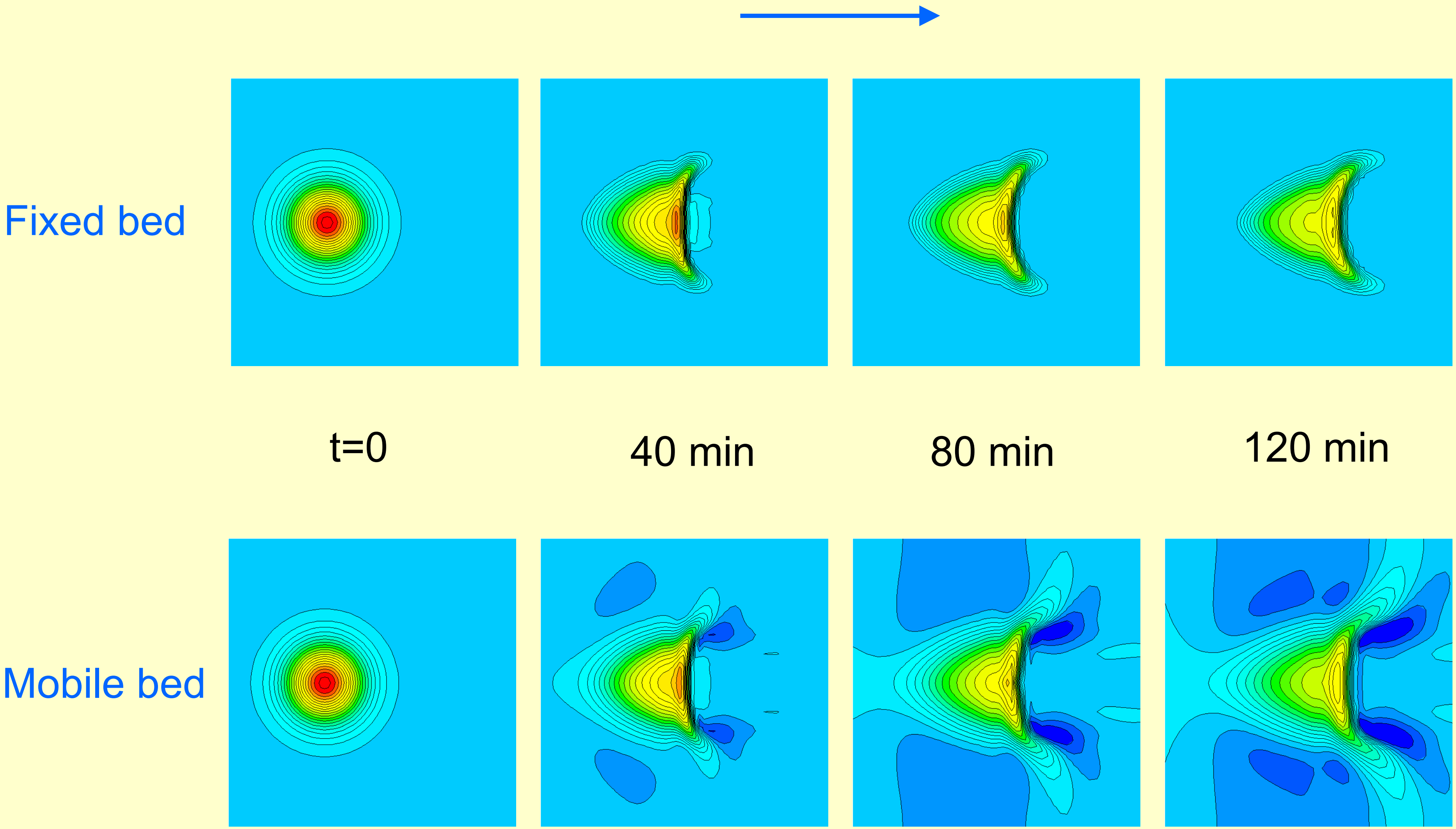
Scour in a Channel Bend



Data: Kawai and Julien, 1996



Transport of a Gaussian Mound



Reference

- Apsley, D.D., 2007, CFD calculation of turbulent flow with arbitrary wall roughness, *Flow, Turbulence and Combustion*, **78**, 153-175.