Performance Analysis of Cloud Radio Access Networks with Uniformly Distributed Base Stations

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***Abstract*—In this letter, we consider a cloud radio access networks (C-RANs) in which base stations (BSs) are uniformly distributed inside a disk. The outage probability and the ergodic rate achieved by this C-RANs are analysed, where stochastic geometry is used to capture the randomness of BS locations. Compared to existing studies to C-RANs, more accurate analytical results are developed by applying the Gaussian-Chebyshev integration, as confirmed by our simulation results.**

***Keywords*—Cloud radio access networks, randomly deployed base stations, Gauss-Chebyshev integration, distributed beamforming.**

I. INTRODUCTION

Cloud radio access networks (C-RANs) have been recognized as a key enabler for 5G networks, and one of the most promising solutions to address the bandwidth crunch problem in current cellular systems. For these reasons, the C-RANs technology has gradually drawn a lot of interests from mobile operators and the researchers from the communication community [1]. In C-RANs systems, distributed beamforming is applied to coordinate the transmissions from multiple BSs, and the challenge to analyse the performance of this C-RANs is that BSs are randomly deployed, which means that the distances between the BSs and the user are also random. Distributed beamforming can achieve performance similar to maximal ratio combining (MRC), and the performance of MRC has been studied in Nakagami-m and Rayleigh fading wireless channels [2]. However, this paper considers that BSs are randomly deployed in a disk, and the channel model contains both Rayleigh fading and distance random variable, different from [2]. In [3], the outage probability of C-RANs with randomly deployed BSs has been studied, where closed-form analytical results have been obtained by using two assumptions, one based on the high signal-to-noise ratio (SNR) and the other based on the special case that the path loss exponent is 2. The technology of joint demodulation and decoding for the uplink of multi-antenna C-RANs has been investigated in [4].

This paper considers a C-RANs scenario, in which the BSs are uniformly distributed in a disc and collaborate with each other by applying distributed beamforming to serve a user which is located at the center of the disc. The aim of this letter is to characterize the fundamental limits of the C-RANs by using two key performance metrics, the outage probability and the ergodic rate. The key step of this paper is to define the density function of the receive SNR, where the challenge is due to the fact that the distances between the BSs and the user are randomly distributed. Different from the existing study in [3], the analytical results developed in this paper are accurate with an arbitrary choice of the pass loss factor and at moderate SNRs. The reason for such accuracy improvement is due to the use of the Gaussian-Chebyshev integration, which can yield better accuracy compared to the uniform based approximation used in [3]. Computer simulations are provided to demonstrate the accuracy of the developed analytical results.

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II. SYSTEM MODEL

Consider a C-RANs system, where one user U is located at the center of a disk $\mathcal{D}$ with the radius D, and M BSs are located inside the disk. Assume that the M BSs are uniformly distributed in the disk $\mathcal{D}$. Let d_j $(j = 1, 2, \cdots, M)$ denote the distance between the j-th BS and the user U. It is assumed that all the BSs and the user U are equipped with a single antenna. Distributed beamforming has been previously applied to two-way relaying networks in [5]. In [6], a joint design of distributed beamforming and cooperative relaying has been proposed in cognitive radio relay networks, which can boost the spectrum efficiency and improve secondary user's performance. However, most existing works about distributed beamforming, such as the ones in [5] and [6], consider only small scale multi-path fading, whereas in this paper we will take large scale path loss into consideration, by assuming that low-cost C-RANs BSs are randomly deployed.

To be more accurate, both of small scale Rayleigh fading and large scale path loss are considered in the channel model. Thus, the channel gain between the j-th BS and the user U is modelled as follows:

$$X_j = \frac{|h_j|^2}{1 + d_j^{\alpha}}, \tag{1}$$

where $h_j \sim \mathcal{CN}(0, 1)$ represents Rayleigh fading between the j-th BS and the user U, and α is the path loss exponent. Note that we use the bounded path loss model in (1), which avoids the singularity issue when $d_j \to 0$. The distributed beamforming transmission strategy is as follows: the j-th BS will transmit the signal $\frac{\sqrt{P}h_j^*}{\sqrt{1+d_j^{\alpha}}}s \Big/ \sqrt{\sum_{j=1}^{M} X_j}$. The received signal at the user can be expressed as follows:

$$y = \sum_{j=1}^{M} \sqrt{P} \frac{X_j s}{\sqrt{\sum_{j=1}^{M} X_j}} + w,$$

where P is the BS transmission power, and w is the additive white Gaussian noise (AWGN), denoted by $w \sim \mathcal{CN}(0, \sigma^2)$. Therefore the use of distributed beamforming means that the achievable date rate at the user is $\log_2(1+\rho S_M)$, where $\rho = \frac{P}{\sigma^2}$ is the transmit SNR, and S_M is given by

$$S_M = \sum_{j=1}^{M} X_j. \tag{2}$$

The outage probability achieved by such a C-RANs scheme is $\mathrm{P_{out}} = \Pr\big(S_M < \frac{2^R-1}{\rho}\big)$, where R denotes the targeted data rate.

A. A straightforward approach for analysis

Since the small scale Rayleigh fading and large scale path loss are independent, a straightforward approach for analysis is to find the Laplace transform of X_j, then calculate the Laplace transform of the S_M, and finally find the inverse Laplace transform of the product. In this way, we can obtain the probability distribution function (PDF) of the channel gain S_M, from which the outage probability can be obtained easily.

The Laplace transform of S_M can be evaluated as

$$\begin{aligned}\mathcal{L}_{S_M}(s) &= \int_0^\infty e^{-st} f_{S_M}(t)dt \\ &= \Big(\int_0^\infty e^{-sz} f_{X_j}(z)dz\Big)^M \\ &\overset{(a)}{=} \Big(\int_0^D \int_0^\infty e^{-\frac{sx}{1+y^\alpha}} f_{|h_j|^2}(x)dx f_{d_j}(y)dy\Big)^M \\ &\overset{(b)}{=} \Big(\int_0^D \int_0^\infty e^{-\frac{sx}{1+y^\alpha}} e^{-x}dx \frac{2y}{D^2}dy\Big)^M \\ &= \Big(\frac{2}{D^2}\int_0^D \frac{(1+y^\alpha)y}{1+s+y^\alpha}dy\Big)^M,\end{aligned} \tag{3}$$

where (a) follows from the fact that $|h_j|^2$ and d_j are independent [7], (b) follows from the fact that the location of the j-th BS is uniformly distributed within the disc $\mathcal{D}$. Therefore the distance d_j has the PDF as follows [8]:

$$f_{d_j}(x) = \frac{2x}{D^2},\ 0 < x < D.$$

Let $t = D^{-\alpha}y^\alpha$, $\mathcal{L}_{S_M}(s)$ can be rewritten as follows:

$$\begin{aligned}\mathcal{L}_{S_M}(s) &= \Big(\frac{2D^\alpha}{\alpha(1+s)}\int_0^1 \frac{t^{\frac{2}{\alpha}}}{1+\frac{D^\alpha}{1+s}t}dt \\ &\quad + \frac{2}{\alpha(1+s)}\int_0^1 \frac{t^{\frac{2}{\alpha}-1}}{1+\frac{D^\alpha}{1+s}t}dt\Big)^M \\ &= \Big(\frac{2D^\alpha}{(1+s)(\alpha+2)}\ {}_2F_1\big(1, 1+\frac{2}{\alpha}; 2+\frac{2}{\alpha}; \frac{-D^{-\alpha}}{1+s}\big) \\ &\quad + \frac{1}{1+s}\ {}_2F_1\big(1, \frac{2}{\alpha}; 1+\frac{2}{\alpha}; \frac{-D^{-\alpha}}{1+s}\big)\Big)^M,\end{aligned} \tag{4}$$

where the last equation is obtained by using the definition of Gauss hypergeometric function ${}_2F_1\big(a, b; c; x\big)$ in [9, 9.111]. This special function has an integral representation as follows:

$${}_2F_1\big(a, b; c; x\big) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-xt)^{-a}dt.$$

We can use the binomial theorem to expand the result of $\mathcal{L}_{S_M}(s)$ in (4), and then apply the inverse Laplace transform in $\mathcal{L}_{S_M}(s)$ to obtain the PDF of S_M. However, it is difficult to obtain $f_{S_M}(z)$, since $\mathcal{L}_{S_M}(s)$ contains Gauss hypergeometric functions. Therefore, an exact analytical expression of $f_{S_M}(z)$ based on the above $\mathcal{L}_{S_M}(s)$ expression is hard to obtain and reveals little insight into the key quantities. Instead we will take an approach based on the Gauss-Chebyshev integration [10] as shown in the following sections.

B. A Gaussian-Chebyshev based approximation

Note that an approximation of the outage probability of the distributed beamforming scheme using all BSs in this system model has been developed in [3]. However, this obtained optimization is only accurate at high SNR and when the path loss exponent is 2, i.e. $\alpha = 2$, since the uniform distribution is used to approximate the PDF of channel gain X_j. Therefore an important question to be answered in this paper is what is the outage performance with an arbitrary choice of the path loss exponent and at not so high SNR. In the following Theorem, a general expression for the outage probability achieved by C-RANs at moderate SNR and with an arbitrary path loss exponent is obtained.

Theorem 1: The outage probability achieved by distributed beamforming C-RANs transmissions can be approximated as follows:

$$\begin{aligned}\mathrm{P_{out}} \approx \big(\frac{\pi}{nD}\big)^M &\sum_{t_1+t_2+\cdots+t_n=M} \frac{M!}{t_1!t_2!\cdots t_n!} \\ &\times \sum_{i=1}^{n}\sum_{k=1}^{t_i} \frac{(1+x_i^\alpha)^{-k}\beta_k}{(k-1)!}\gamma(k, (1+x_i^\alpha)\varepsilon),\end{aligned} \tag{5}$$

where $\varepsilon = \frac{2^R-1}{\rho}$, $\gamma(a,b) = \int_0^b t^{a-1}e^{-t}dt$ is a lower incomplete Gamma function, $x_i = \frac{D}{2}(1+\cos\frac{2i-1}{2n}\pi)$, β_k is defined in (15), and n is a approximation parameter due to the use of the Gaussian-Chebyshev integration.

Proof: See Section III. ■

The parameter n is the key to achieve a trade-off between the approximation accuracy and computation complexity. For example, by letting $n \to \infty$, the result in Theorem 1 can be viewed as an exact expression of the outage probability, but with a lot of computation complexity. From (5), by using the definition of the diversity gain $d = -\lim\limits_{\mathrm{SNR}\to\infty}\frac{\log \mathrm{P_{out}}}{\log \mathrm{SNR}}$, it is straightforward to demonstrate that the diversity gain achieved by the number of BSs M is M.

In addition to the outage probability, the ergodic transmission rate is another important metric for performance evaluation in communication systems. The approximated ergodic rate achieved in the addressed C-RANs system is given by the following Theorem.

Theorem 2: The approximated ergodic rate achieved by C-RANs with distributed beamforming is given by

$$\begin{aligned} \mathrm{R}_{\text{ave}} &\approx \frac{\rho\big(\frac{\pi}{nD}\big)^M}{\ln 2}\sum_{t_1+t_2+\cdots+t_n=M}\frac{M!}{t_1!t_2!\cdots t_n!} \\ &\times \sum_{i=1}^{n}\sum_{k=1}^{t_i}\frac{\beta_k}{(1+x_i^\alpha)^k}\sum_{l=0}^{k-1}\frac{(1+x_i^\alpha)^l e^{\frac{1+x_i^\alpha}{\rho}}}{l!\rho^{l+1-m}} \\ &\times \sum_{m=0}^{l}(-1)^{l-m}\frac{\Gamma(m,\frac{1+x_i^\alpha}{\rho})}{(1+x_i^\alpha)^m}, \end{aligned} \quad (6)$$

where $\Gamma(a,b) = \int_b^\infty t^{a-1}e^{-t}dt$ is the upper incomplete Gamma function, $x_i = \frac{D}{2}(1+\cos\frac{2i-1}{2n}\pi)$, and β_k is defined in (15).

Proof: See Section IV. ■

It is worthy to point out that the results of the average rate in (6) still contain some upper incomplete functions; however, such obtained analytical results are much easier to use for performance evaluation compared to Monte Carlo simulations.

III. Proof of the Outage Performance of the C-RANs Scheme

In this section, the proof of the outage probability approximation shown in Theorem 1 is described. Particularly, the proof follows from three steps. Firstly, we use the Gauss-Chebyshev integration to approximate the PDF of the channel gain X_j. Secondly, applying the approximated PDF to find the Laplace transform of $S_M = \sum_{j=1}^M X_j$. Finally, finding the inverse Laplace transform of the product.

Recall that the closed-form expression of the cumulative distribution function (CDF) of X_j has been obtained in [11]

$$F_{X_j}(x) = 1 - \frac{2e^{-x}}{\alpha D^2 x^{\frac{2}{\alpha}}}\gamma(\frac{2}{\alpha}, D^\alpha x). \quad (7)$$

This expression is quite difficult to use for the calculation of the outage probability since it contains lower incomplete functions, exponential functions, and polynomial functions. Fortunately, we can use the Gauss-Chebyshev integration to approximate the above expression, and the resultant expression contains only exponential functions, as described in the following.

Based on the exact distribution of $|h_j|^2$ and d_j, the CDF of X_j can be evaluated as

$$\begin{aligned} F_{X_j}(z) &= \Pr\{\frac{|h_j|^2}{1+d_j^\alpha} < z\} \\ &= \int_0^D \big(1-e^{-z(1+x^\alpha)}\big)\frac{2x}{D^2}dx \\ &= 1 - \frac{2}{D^2}\int_0^D xe^{-z(1+x^\alpha)}dx. \end{aligned} \quad (8)$$

The Gauss-Chebyshev integration [10] will be used to approximate the above integral as follow:

$$\int_0^D xe^{-z(1+x^\alpha)}dx \approx \frac{\pi D}{2n}\sum_{i=1}^{n}\big|\sin\frac{2i-1}{2n}\pi\big|x_i e^{-z(1+x_i^\alpha)}, \quad (9)$$

where $x_i = \frac{D}{2}(1+\cos\frac{2i-1}{2n}\pi)$, and n is the number of terms included in the summation.

Thus, the CDF of X_j can be approximated as follows:

$$F_{X_j}(z) \approx 1 - \frac{\pi}{nD}\sum_{i=1}^{n}\big|\sin\frac{2i-1}{2n}\pi\big|x_i e^{-z(1+x_i^\alpha)}. \quad (10)$$

Note that the above expression $F_{X_j}(z)$ contains the summation of some simple exponential functions, which is much easier to be used than the exact expression in (7).

As a consequence, the PDF of X_j is obtained by applying derivative as follow:

$$f_{X_j}(z) \approx \frac{\pi}{nD}\sum_{i=1}^{n}\big|\sin\frac{2i-1}{2n}\pi\big|x_i(1+x_i^\alpha)e^{-z(1+x_i^\alpha)}. \quad (11)$$

Applying the approximate result in (11), the approximate Laplace transform of X_j can be evaluated as follows:

$$\begin{aligned} \mathcal{L}_{X_j}(s) &= \int_0^\infty e^{-sz}f_{X_j}(z)dz \\ &\approx \frac{\pi}{nD}\sum_{i=1}^{n}\big|\sin\frac{2i-1}{2n}\pi\big|x_i(1+x_i^\alpha) \\ &\quad\times\int_0^\infty e^{-sz}e^{-z(1+x_i^\alpha)}dz \\ &= \frac{\pi}{nD}\sum_{i=1}^{n}\big|\sin\frac{2i-1}{2n}\pi\big|\frac{x_i(1+x_i^\alpha)}{1+x_i^\alpha+s}. \end{aligned} \quad (12)$$

Since the channel gain X_j $(1 \le j \le M)$ are independent and identically (i.i.d.), the Laplace transform of $S_M = \sum_{j=1}^M X_j$ is given by the power M of the Laplace transforms of individual terms X_j

$$\begin{aligned} \mathcal{L}_{S_M}(s) &\approx \Big(\frac{\pi}{nD}\sum_{i=1}^{n}\big|\sin\frac{2i-1}{2n}\pi\big|\frac{x_i(1+x_i^\alpha)}{1+x_i^\alpha+s}\Big)^M \\ &= \big(\frac{\pi}{nD}\big)^M\sum_{t_1+t_2+\cdots+t_n=M}\frac{M!}{t_1!t_2!\cdots t_n!} \\ &\quad\times\underbrace{\prod_{i=1}^{n}\Big(\big|\sin\frac{2i-1}{2n}\pi\big|\frac{x_i(1+x_i^\alpha)}{1+x_i^\alpha+s}\Big)^{t_i}}_{Q(i,n)}, \end{aligned} \quad (13)$$

where the summation is taken over all sequences of non-negative integer indices t_1 through t_n such that the sum of all t_i is M.

Using partial fractions, $Q(i,n)$ in (13) is given by

$$Q(i,n) = \sum_{i=1}^{n}\sum_{k=1}^{t_i}\beta_k\big(1+x_i^\alpha+s\big)^{-k}, \quad (14)$$

where

$$\begin{aligned} \beta_k &= \frac{1}{(t_i-k)!}\Bigg[\frac{d^{(t_i-k)}}{ds^{(t_i-k)}}\big\{(1+x_i^\alpha+s)^{t_i} \\ &\times\prod_{i=1}^{n}\Big(\big|\sin\frac{2i-1}{2n}\pi\big|\frac{x_i(1+x_i^\alpha)}{1+x_i^\alpha+s}\Big)^{t_i}\big\}\Bigg]_{s=-1-x_i^\alpha}. \end{aligned} \quad (15)$$

Substituting (14) and (15) into (13), $\mathcal{L}_{S_M}(s)$ can be expressed as follows:

$$\mathcal{L}_{S_M}(s) \approx \big(\frac{\pi}{nD}\big)^M \sum_{t_1+t_2+\cdots+t_n=M} \frac{M!}{t_1!t_2!\cdots t_n!} \times \sum_{i=1}^{n}\sum_{k=1}^{t_i} \beta_k \big(1+x_i^\alpha+s\big)^{-k}. \quad (16)$$

Using the inverse Laplace transform of $\mathcal{L}_{S_M}(s)$ in (16), and also the fact that

$$\mathcal{L}^{-1}\big(1+x_i^\alpha+s\big)^{-k} = \frac{t^{k-1}e^{-(1+x_i^\alpha)t}}{(k-1)!},$$

the PDF of S_M is given by

$$f_{S_M}(z) \approx \big(\frac{\pi}{nD}\big)^M \sum_{t_1+t_2+\cdots+t_n=M} \frac{M!}{t_1!t_2!\cdots t_n!} \times \sum_{i=1}^{n}\sum_{k=1}^{t_i} \frac{\beta_k}{(k-1)!} z^{k-1}e^{-(1+x_i^\alpha)z}. \quad (17)$$

Then, the CDF of S_M is given as

$$F_{S_M}(z) \approx \big(\frac{\pi}{nD}\big)^M \sum_{t_1+t_2+\cdots+t_n=M} \frac{M!}{t_1!t_2!\cdots t_n!} \times \sum_{i=1}^{n}\sum_{k=1}^{t_i} (1+x_i^\alpha)^{-k}\beta_k \frac{\gamma\big(k,(1+x_i^\alpha)z\big)}{(k-1)!}. \quad (18)$$

An outage event occurs when the instantaneous rate $\log_2(1+\rho S_M)$ is less than a targeted data rate R. Thus, substituting $\varepsilon = \frac{2^R-1}{\rho}$ into (18), the proof is completed.

IV. Proof of the Ergodic Rate of the C-RANs Scheme

In this section, we prove the analytical results about the ergodic rate shown in Theorem 2.

The proof follows from two steps. Firstly, conditioned on the lower gamma function expanding result, we obtain a new approximated expression of the CDF of S_M. Secondly, using the definition of average rate yields the final result.

The CDF of S_M obtained in the previous section is not able to use for calculating the ergodic rate directly due to the following reason. The CDF shown in (18) contains a term of $\frac{1}{1+x^\alpha}$, and an integration of $\int_0^\infty \frac{1}{1+x^\alpha}\log_2(1+x)dx$ does not exist. As a result, the CDF needs to be rewritten as follows.

Since k is a positive integer, the lower gamma function $\frac{\gamma(k,(1+x_i^\alpha)z)}{(k-1)!}$ in (18) can be rewritten as

$$\frac{\gamma\big(k,(1+x_i^\alpha)z\big)}{(k-1)!} = 1 - \sum_{l=0}^{k-1}\frac{(1+x_i^\alpha)^l}{l!} z^l e^{-(1+x_i^\alpha)z}. \quad (19)$$

Thus, $F_{S_M}(z)$ in (18) can be expressed as

$$F_{S_M}(z) \approx \big(\frac{\pi}{nD}\big)^M \sum_{t_1+t_2+\cdots+t_n=M} \frac{M!}{t_1!t_2!\cdots t_n!} \times \sum_{i=1}^{n}\sum_{k=1}^{t_i} \frac{\beta_k}{(1+x_i^\alpha)^k}\Big(1-\sum_{l=0}^{k-1}\frac{(1+x_i^\alpha)^l z^l e^{-(1+x_i^\alpha)z}}{l!}\Big). \quad (20)$$

Since $\lim_{z\to\infty} F_{S_M}(z) = 1$, we have

$$\big(\frac{\pi}{nD}\big)^M \sum_{t_1+t_2+\cdots+t_n=M} \frac{M!}{t_1!t_2!\cdots t_n!} \sum_{i=1}^{n}\sum_{k=1}^{t_i} \frac{\beta_k}{(1+x_i^\alpha)^k} = 1. \quad (21)$$

Therefore, $F_{S_M}(z)$ can be further expressed as

$$F_{S_M}(z) \approx 1 - \big(\frac{\pi}{nD}\big)^M \sum_{t_1+t_2+\cdots+t_n=M} \frac{M!}{t_1!t_2!\cdots t_n!} \times \sum_{i=1}^{n}\sum_{k=1}^{t_i} \frac{\beta_k}{(1+x_i^\alpha)^k}\sum_{l=0}^{k-1}\frac{(1+x_i^\alpha)^l z^l e^{-(1+x_i^\alpha)z}}{l!}. \quad (22)$$

In the above expression, the term of $\frac{1}{1+x^\alpha}$ has been removed, which facilitates the following calculations about the ergodic rate. Based on the above approximate result of $F_{S_M}(z)$, the ergodic rate of S_M can be evaluated as follows:

$$\begin{aligned}
\mathrm{R}_{\mathrm{ave}} &= \int_0^\infty \log_2(1+\rho z) f_{S_M}(z)dz \\
&= \frac{\rho}{\ln 2}\int_0^\infty \frac{1-F_{S_M}(z)}{1+\rho z}dz \\
&\approx \frac{\rho\big(\frac{\pi}{nD}\big)^M}{\ln 2} \sum_{t_1+t_2+\cdots+t_n=M} \frac{M!}{t_1!t_2!\cdots t_n!} \\
&\times \sum_{i=1}^{n}\sum_{k=1}^{t_i} \frac{\beta_k}{(1+x_i^\alpha)^k}\sum_{l=0}^{k-1}\frac{(1+x_i^\alpha)^l}{l!} \\
&\times \underbrace{\int_0^\infty \frac{z^l e^{-(1+x_i^\alpha)z}}{1+\rho z}dz}_{Q_1}.
\end{aligned} \quad (23)$$

Let $t = \rho z + 1$, the integral Q_1 in (23) can be evaluated as

$$\begin{aligned}
Q_1 &= \frac{1}{\rho^{l+1}}\int_1^\infty \frac{(t-1)^l e^{-(1+x_i^\alpha)\frac{t-1}{\rho}}}{t}dt \\
&\overset{(a)}{=} \frac{e^{\frac{1+x_i^\alpha}{\rho}}}{\rho^{l+1}}\int_1^\infty \frac{\sum_{m=0}^{l} t^m(-1)^{l-m}e^{-\frac{1+x_i^\alpha}{\rho}t}}{t}dt \\
&= \frac{e^{\frac{1+x_i^\alpha}{\rho}}}{\rho^{l+1}}\sum_{m=0}^{l}(-1)^{l-m}\int_1^\infty t^{m-1}e^{-\frac{1+x_i^\alpha}{\rho}t}dt \\
&= \frac{e^{\frac{1+x_i^\alpha}{\rho}}}{\rho^{l+1-m}}\sum_{m=0}^{l}(-1)^{l-m}\frac{\Gamma(m,\frac{1+x_i^\alpha}{\rho})}{(1+x_i^\alpha)^m},
\end{aligned} \quad (24)$$

where (a) follows from the fact that l is a positive positive integer and using the binomial theorem. $\Gamma(a,b) = \int_b^\infty t^{a-1}e^{-t}dt$ is an upper incomplete function. Note that $\Gamma(0,b) = \int_b^\infty \frac{e^{-t}}{t}dt = \boldsymbol{E}_1(b)$ is the exponential integral. Substituting (24) into (23), the proof is completed.

V. Simulation and Numerical results

In this section, we present the simulation results to validate the analytical results obtained in this paper. We assume the following set of parameters: the targeted data rate is $R = 1$ bit/s/Hz in Figs. 1-2, Figs. 4-5, while $R = 3$ bits/s/Hz in

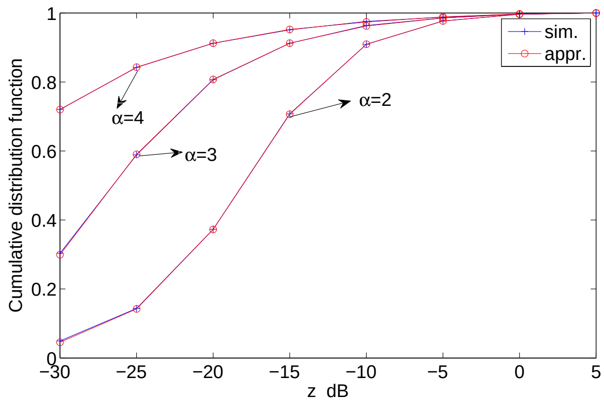

Fig. 1. Approximation of the CDF of X_j vs Monte Carlo simulations.

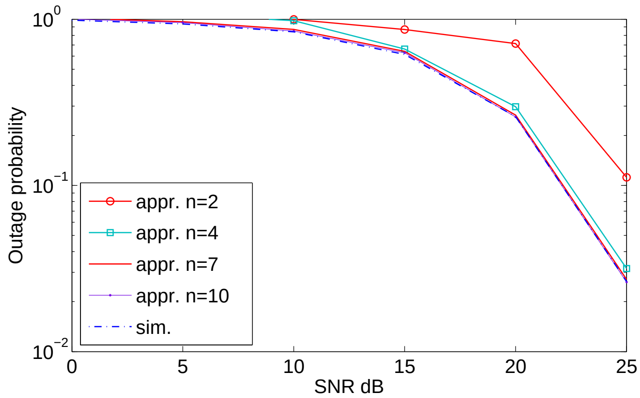

Fig. 2. The impact of the approximated parameter n on the accuracy of outage probability. BSs number $M = 4$ and the path loss exponent is $\alpha = 3$.

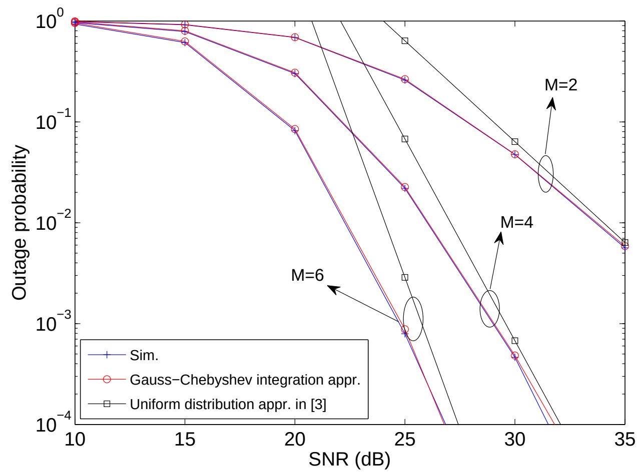

Fig. 3. Comparison of the uniform distribution based approximation in [3] and Gauss-Chebyshev integration based approximation. The path loss exponent is $\alpha = 2$.

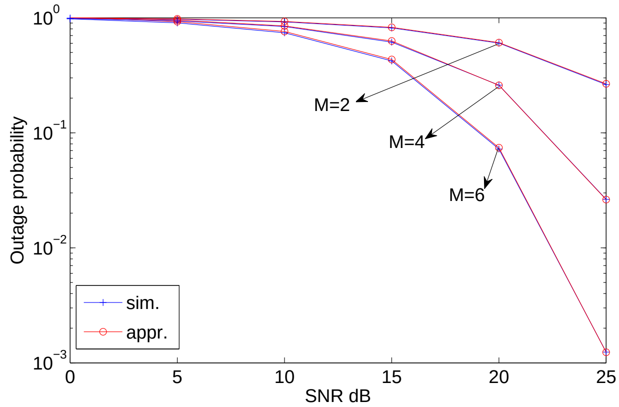

Fig. 4. Comparison of the analytical and Monte Carlo simulation results. The path loss exponent is $\alpha = 3$.

Fig. 3, the disk radius is $D = 10$ m, and the number of the approximation terms is $n = 10$.

In Fig. 1, we compare the analytical results of $F_{X_j}(z)$ provided at (10) with the actual CDF of X_j obtained from Monte Carlo simulations with different choices of the path loss exponent α. As can be seen from Fig. 1, the approximation for X_j is quite accurate for the whole range of the variable, which means the Gauss-Chebyshev integration is a good method to approximate the integral expression.

Fig. 2 shows the impact of the number of the summation n for the Gauss-Chebyshev approximation on the outage performance of the C-RANs. As can be seen from the Fig. 2, when the approximated parameter n increases, the accuracy of approximated results is improved, i.e., the gap between the analytical results and the simulation ones is reduced. In particular, when $n = 7$, the approximated result matches quite well with the Monte Carlo simulation result. Therefore it is reasonable for us to choose $n = 10$ in the previous example.

The analytical results about the outage probability are compared to Monte Carlo simulations in Fig. 3 and Fig. 4 for different choices of the path loss exponent. We also compare the analytical results obtained in this paper to the ones obtained in [3] which are based on the uniform distribution approximation method in Fig. 3. It can be observed from Fig. 3 that the approximation results from [3] are accurate only at high SNR while the analytical result developed in this paper is more accuracy even in moderate SNR. Fig. 3 and Fig. 4 also illustrate that the diversity gain achieved by C-RANs is proportional to the number of BSs in the system, and the outage probability decrease as the number of BSs increase. Since a larger number of BSs service the user U which is located at the center of the disk, a higher chance there is to successfully detect the signal. In addition, it is worth to point out that the approximated results match well with the Monte Carlo simulations in moderate SNR and with different path loss exponents.

In Fig. 5, we show the approximation of the ergodic rate and Monte Carlo simulation results achieved by the C-RANs scheme. Fig. 5 also demonstrates that the approximated results match quite well with the simulation results with different

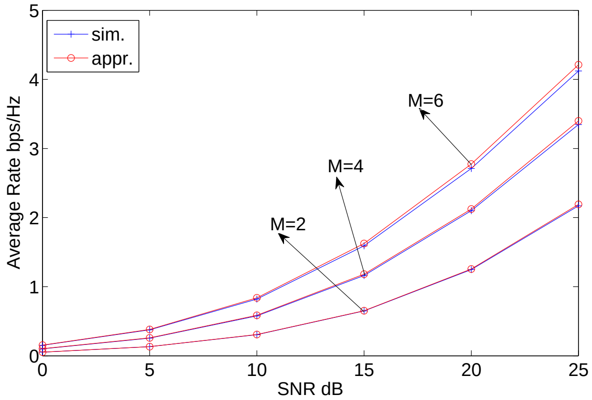

Fig. 5. Comparison of the analytical and Monte Carlo simulation results. The path loss exponent is $\alpha = 3$.

number of BSs. Furthermore, the ergodic rate improves when increasing the BSs number M. This is because a larger number of BSs in cooperation can boost the receive SNR, which in turn increases the average rate.

VI. Conclusions

In this letter, the Gauss-Chebyshev integration method was used to tightly approximate the outage probability and ergodic rate achieved by C-RANs with randomly deployed BSs. Compared to the existing work, the analytical results are more accurate and applicable to any choices of the path loss exponent. We verified our analytical results by using Monte Carlo simulations, showing that the analytical results match quite well in moderate SNR and with different choices of the path loss exponent.

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