A Novel Power Allocation Scheme under Outage Constraints in NOMA Systems

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***Abstract*—In this letter, we study a downlink non-orthogonal multiple access (NOMA) transmission system, where only average channel state information (CSI) is available at the transmitter. Two criteria in terms of transmit power and user fairness for NOMA systems are used to formulate two optimization problems, subjected to outage probabilistic constraints and the optimal decoding order. We first investigate the optimal decoding order when the transmitter knows only the average CSI, and then we develop the optimal power allocation schemes in closed-form by employing the feature of the NOMA principle for the two problems. Furthermore, the power difference between NOMA systems and OMA systems under outage constraints is attained.**

***Index Terms*—Non-orthogonal multiple access (NOMA), average channel state information, power allocation, probabilistic constraints**

I. INTRODUCTION

Recently, non-orthogonal multiple access (NOMA) based on successive interference cancellation (SIC) has been recognized as one of the promising multiple access technologies to be used for fifth generatin (5G) communication [1]. Different from conventional orthogonal multiple access (OMA), e.g. time-division multiple access (TDMA), NOMA introduces an new dimension – power domain for multiple access.

The works in [2] and [3] focused on analysing the performance of the NOMA scheme in terms of outage probability and user pairing, where the fixed power allocation scheme is characterized. To further exploit the potential gain of NOMA in the power domain, the problem to maximize the worst user throughput has been studied in [4] with the total throughput constraint and in [5] with the total transmission power constraint, respectively. In [6], a joint power and subcarrier allocation policy has been proposed in multicarrier NOMA system. Most of the existing works on NOMA assume that perfect channel state information (CSI) is available at the transmitter. However, such an assumption is considered idealistic [7] due to the limited CSI feedback. Generally, imperfect CSI can lead to substantial performance degradation, such as quality of service (QoS) outages, if not taken care of properly.

This paper derives the relationship between power control and QoS requirements based on outage probabilities in NOMA systems. While the problem of outage balancing under the power constraint has been studied [5] and [8], there is little literature that addresses power-allocation methods to minimize transmitter power subject to probabilistic constraints and maximize rate fairness under constraints using outage probabilities and power constraints in NOMA systems. In general, intelligent power allocation is critical in wireless networks for improving the spectral efficiency and realizing the users' QoS goals. Therefore, the two addressed problems in the paper provides an important insight of NOMA for future communications. Furthermore, the optimal SIC decoding order has been found under the outage constraints when the average CSI is unchangeable during a timeslot. Finally, the provided simulation results demonstrate that NOMA outperforms OMA with the two proposed power allocation schemes.

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II. SYSTEM MODEL

Consider a cellular downlink NOMA scenario with one BS and M users denoted as the set $\mathcal{M} = \{1, \cdots, M\}$. All nodes are equipped with a single antenna. The NOMA principle is used for transmission purposes. Therefore, the observation at the m-th user is given by

$$y_m = h_m \sum_{i=1}^{M} \sqrt{P_i} s_i + n_m, m \in \mathcal{M}, \tag{1}$$

where s_i is the message for the i-th user, h_m denotes the channel gain between the BS and the m-th user. We assume perfect CSI is available on the users, but only average CSI is available on the BS. In particular, it is assumed that $h_m = d_m^{-\gamma} g_m$, with d_m being the distance from the m-th user to the BS, where γ is the pass loss exponent, and $g_m \sim \mathcal{CN}(0, 1)$. Without loss of generality, the distances are sorted as $d_1 \geq d_2 \geq \cdots \geq d_M$. P_m is the transmission power allocated for the m-th user and n_m denotes zero-mean additive noise of the m-th user with variance σ_m^2.

In this case, each user employs SIC in a successive order to remove partial inter-user interference. As a result, the decoding order is an essential issue for the NOMA system. Denote by $\pi(m)$ the user index corresponding to the decoding order m. Let $R_{\pi(j)}^{\pi(m)}$ denote the rate for user $\pi(m)$ to detect the user $\pi(j)$'s message, $j \leq m$, which can be expressed as

$$R_{\pi(j)}^{\pi(m)} = \log \left(1 + \frac{|h_{\pi(m)}|^2 P_{\pi(j)}}{\sum_{i=j+1}^{M} |h_{\pi(m)}|^2 P_{\pi(i)} + \sigma_{\pi(m)}^2} \right). \tag{2}$$

Since the perfect CSI is not available at the BS, an outage event may happen in NOMA systems, which is defined as that user $\pi(m)$ is not able to decode its own message or the message of the weaker user $\pi(j),\ j < m$ [2].

Therefore, the outage probability at user $\pi(m)$ can be expressed as

$$\begin{aligned} P_{\pi(m)}^{out} &= 1 - P\left(R_{\pi(1)}^{\pi(m)} \geq \tilde{R}_{\pi(1)}^{\pi(m)}, \cdots, R_{\pi(m)} \geq \tilde{R}_{\pi(m)} \right) \\ &= 1 - e^{-\lambda_m \max\limits_{j=1,\cdots m} \frac{\phi_{\pi(j)} \sigma^2}{P_{\pi(j)} - \phi_{\pi(j)} \sum_{i=j+1}^{M} P_{\pi(i)}}}, \end{aligned} \tag{3}$$

where $\lambda_{\pi(m)} = d_{\pi(m)}^{\gamma}$, $\phi_{\pi(j)} = 2^{\tilde{R}_{\pi(j)}} - 1$ and $\tilde{R}_{\pi(j)}$ is the targeted data rate of user $\pi(j)$. Note that $P_{\pi(j)} > \phi_{\pi(j)} \sum_{i=j+1}^{M} P_{\pi(i)}$. If $P_{\pi(j)} \leq \phi_{\pi(j)} \sum_{i=j+1}^{M} P_{\pi(i)}$, the outage probability of user $\pi(j)$ will be always one.

III. Problem formulation

In NOMA, power allocation is an important issue to enhance the achievable rate of each user due to power-domain multi-user multiplexing. In this section, two power allocation schemes based on two criteria–transmit power and fairness rate will be studied separately, where only average CSI is known at the BS.

A. Minimizing transmission power

The problem of interest here is to design the system, which should provide an acceptable QoS requiring as little transmit power as possible, and serving as many users as possible. In particular, the problem of minimizing the total transmit power subjected to individual outage constraints under the optimal decoding order, can be formulated as follows:

$$\min_{\{P_{\pi(m)}\},\pi} \quad \sum_{m=1}^{M} P_{\pi(m)} \tag{4a}$$

$$\text{s.t.} \quad P^{out}_{\pi(m)} \leq \epsilon_{\pi(m)},\ m \in \mathcal{M}, \tag{4b}$$

$$P_{\pi(m)} > \phi_{\pi(m)} \textstyle\sum_{i=m+1}^{M} P_{\pi(i)},\ m \in \mathcal{M}, \tag{4c}$$

$$P_{\pi(m)} \geq 0,\ m \in \mathcal{M}, \tag{4d}$$

$$\pi \in \Pi. \tag{4e}$$

where Π represents the set of all possible SIC decoding orders for the NOMA system. Specifically, $\epsilon_{\pi(m)} \in [0,1)$ is denoted by the maximum tolerable outage probability for user $\pi(m)$. By substituting (3) into (4b) and then using the basic properties of inequalities and the logarithm operator to (4b), the outage constraint for user $\pi(m)$ can be transformed into a linear constraint as follows:

$$\min_{j=1,\cdots,m} \frac{P_{\pi(j)} - \phi_{\pi(j)} \sum_{i=j+1}^{M} P_{\pi(i)}}{\phi_{\pi(j)}} \geq \rho_{\pi(m)}, \tag{5}$$

where $\rho_{\pi(m)} = \frac{\lambda_{\pi(m)}\sigma^2_{\pi(m)}}{\log\frac{1}{1-\epsilon_{\pi(m)}}}$. However, due to the combinational nature of decoding order, the problem of (5) can not be solved directly by a standard optimization solver [9]. Before the next step, we first introduce the following Proposition.

Proposition *1:* Without loss of generality, we assume that $\pi(j) = j$. Let $x_j = P_j - \phi_j \sum_{i=j+1}^{M} P_i,\ j = 1,\cdots,M$, $x_j > 0$. The equations in (6) are guaranteed.

$$P_j = x_j + \phi_j \sum_{n=j+1}^{M} \prod_{l=j+1}^{n-1} (1+\phi_l) x_n, \tag{6a}$$

$$\textstyle\sum_{j=1}^{M} P_j = \sum_{j=1}^{M} \beta_j x_j, \quad \text{where} \tag{6b}$$

$$\beta_j = \begin{cases} 1,\ j = 1; \\ \prod_{i=1}^{j-1}(1+\phi_i),\ j = 2,\cdots,M. \end{cases} \tag{6c}$$

Proof: We introduce the auxiliary parameter $x_j = P_j - \phi_j \sum_{i=j+1}^{M} P_i,\ j \in \mathcal{M}$. It is easy to know that $x_j > 0,\ \forall j \in \mathcal{M}$ due to the the constraint of (4c). As a result, one can write the power P_j as

$$P_j = x_j + \phi_j \textstyle\sum_{i=j+1}^{M} P_i,\ j \in \mathcal{M}. \tag{7}$$

To obtain the closed-form expression for P_j, we calculate the sum, $\sum_{i=j+1}^{M} P_i$. By using the relationship in (7), we can obtain the following results for a fixed feasible j.

$$\sum_{i=j+1}^{M} P_i = x_{j+1} + (1+\phi_{j+1}) \sum_{i=j+2}^{M} P_i, \tag{8a}$$

$$= x_{j+1} + \sum_{n=j+2}^{M} \prod_{l=j+1}^{n-1} (1+\phi_l) x_n = \sum_{n=j+1}^{M} \beta_n x_n. \tag{8b}$$

where $\beta_n = \prod_{l=j+1}^{n-1}(1+\phi_l)$ and the right hand side (r.h.s) of (8b) is attained by forcing $\beta_n = 1$ if $l > n-1$. By substituting the left hand side (l.h.s) of (8b) into (7), (6a) can be obtained. (6c) can be derived directly by setting $j = 0$ in the r.h.s of (8b) and $\beta_1 = 1$. ■

From (6), the minimization transmit power problem in (4) can be reformulated as

$$\min_{\{x_{\pi(m)}\},\pi} \quad \sum_{m=1}^{M} \beta_{\pi(m)} x_{\pi(m)} \tag{9a}$$

$$\text{s.t.} \quad \min_{j=1,\cdots,m} \frac{x_{\pi(j)}}{\phi_{\pi(j)}} \geq \rho_{\pi(m)},\ m \in \mathcal{M}, \tag{9b}$$

$$x_{\pi(m)} > 0,\ m \in \mathcal{M}\ \&\ \text{(4e)}. \tag{9c}$$

Note that the objective function is monotonically increasing with $x_{\pi(m)}$. The optimal decoding order is given in the following theorem.

Theorem *1:* For the optimization problem in (4), the optimal decoding order π^* satisfies

$$\rho_{\pi^*(1)} \geq \rho_{\pi^*(2)} \geq \cdots \geq \rho_{\pi^*(M)}. \tag{10}$$

where $\rho_{\pi^*(m)} = \frac{\lambda_{\pi^*(m)}\sigma^2_{\pi^*(m)}}{\log\frac{1}{1-\epsilon_{\pi^*(m)}}}$.

Proof: See proof in the Appendix A. ■

For a known optimal decoding order π^*, the optimal power allocation for (9) can be derived in the following proposition.

Proposition *2:* The optimal solution for problem (9) is given by

$$P_{\pi^*(m)} = \phi_{\pi^*(m)} \cdot \left[\rho_{\pi^*(m)} + \sum_{j=m+1}^{M} \phi_{\pi^*(j)}\rho_{\pi^*(j)} \prod_{n=m+1}^{j-1} (\phi_{\pi^*(n)}+1)\right] \tag{11}$$

for any $m \in \mathcal{M}$.

Proof: The constraints in (9b) can be equivalently expressed as

$$\frac{x_{\pi^*(m)}}{\phi_{\pi^*(m)}} \geq \max\left\{\rho_{\pi^*(m)},\cdots,\rho_{\pi^*(M)}\right\}, m \in \mathcal{M}. \tag{12}$$

By applying the Theorem 1, it is easy to obtain

$$\frac{x_{\pi^*(m)}}{\phi_{\pi^*(m)}} \geq \rho_{\pi^*(m)}, m \in \mathcal{M}. \tag{13}$$

In addition, note that $x_{\pi^*(1)},\cdots,x_{\pi^*(M)}$ are independent and the objective function of (9) is linear. Therefore, the optimal solution of (9) will be obtained at all the constraints with equalities [10], [11]. Thus by substituting $x_{\pi^*(m)} = \phi_{\pi^*(m)}\rho_{\pi^*(m)}$ $m \in \mathcal{M}$ into (6a), (11) can be attained. ■

From Theorem 1, one can find that the optimal decoding order is independent of the choice of the targeted data rate. Therefore, it is easy to see from (11) the power allocation for user $\pi^*(m)$ is a monotonically increasing function of the targeted data rate of the users $\pi^*(j)$ satisfying $j \geq m$.

B. Maximizing fairness rate

In this subsection, we consider to provide QoS guarantees within a tolerable outage probability for the users. Specifically, we consider the problem of maximizing the worst user's received rate giving the maximum tolerable outage probability for each user m, which can be formulated as

$$\max_{\{P_{\pi(m)}\},\{\tilde{R}_{\pi(m)}\},\pi} \quad \min_{m=1,\cdots,M} \tilde{R}_{\pi(m)} \tag{14a}$$

$$\text{s.t.} \quad \textstyle\sum_{m=1}^{M} P_{\pi(m)} \leq P, \tag{14b}$$

$$\text{(4b)} - \text{(4e)}. \tag{14c}$$

Note that the outage constraint in (5) is equivalent to (4b). Therefore, using the property of inequality like (12), (5) can be rewritten as

$$\phi_{\pi(m)} \leq \frac{P_{\pi(m)}}{\sum_{i=m+1}^{M} P_{\pi(i)} + \rho^*_{\pi(m)}}, \quad m \in \mathcal{M}, \tag{15}$$

where $\rho^*_{\pi(m)} = \max_{j=m,\cdots,M}\left\{\rho_{\pi(j)}\right\}$. $\rho_{\pi(j)}$ and $\phi_{\pi(m)}$ have been defined in section III-A.

Proposition *3:* The optimal decoding order for problem (4) is the same as problem (14).

The proof of Proposition 3 is similar to the proof of Theorem 1 and is omitted here. As a result, $\rho^*_{\pi(m)} = \rho_{\pi^*(m)}$. Given the optimal decoding order π^*, by introducing an additional variable t, the optimization problem can be reformulated as

$$\max \quad t \tag{16a}$$

$$\text{s.t. } t = \min_{m\in\mathcal{M}} \phi_{\pi^*(m)}, \tag{16b}$$

$$\phi_{\pi^*(m)}\left(\textstyle\sum_{i=m+1}^{M} P_{\pi^*(i)} + \rho_{\pi^*(m)}\right) \le P_{\pi^*(m)}, m\in\mathcal{M}, \tag{16c}$$

$$(14b) \ \& \ (4c)-(4e). \tag{16d}$$

with variables $\{P_{\pi^*(m)}\}, \{\phi_{\pi^*(m)}\}$ and t. From the definition of ϕ_m, it is easy to know that $\phi_m > 0$, $\forall m$, is always true, which implies $t > 0$ in (16). Further, we introduce Proposition 4 related to (16).

Proposition *4:* The optimal solution of (16) satisfies the constraints in (16c) and (14b) with strictly equality and the constraint in (4c) and (4d) with strictly inequality. In addition, for (16b) the condition $\phi_{\pi(1)} = \cdots = \phi_{\pi(M)} = t$ will be met at the optimal solution.

Proof: See proof in the Appendix B. ■

By applying Proposition 4 and using some manipulations similar to the procedures as shown in the proof of Proposition 1, we can obtain the following equations:

$$P_{\pi^*(m)} = t\rho_{\pi^*(m)} + \textstyle\sum_{i=m+1}^{M} t^2(1+t)^{i-m-1}\rho_{\pi^*(i)}, \tag{17a}$$

$$P = \textstyle\sum_{m=1}^{M} t(1+t)^{m-1}\rho_{\pi^*(m)}. \tag{17b}$$

It can be observed that (17b) is a non-linear equation, but it can be solved by Newton's method. Then, by substituting t into (17a), we can calculate individual $P_{\pi^*(m)}$, $\forall m$.

IV. Some discussions between NOMA and OMA

Note that, from the power expression (11) and (17a), to achieve the minimum power or the maximum fairness rate, the BS will allocate more power to the user with larger $\rho_{\pi^*(j)}$, for all $j \in \mathcal{M}$, since the user with the larger $\rho_{\pi^*(j)}$ will have a higher priority in decoding order. Therefore, it is easy to see from (11) and (17a) the power allocation coefficient for user $\pi^*(m)$ is a monotonically increasing function of the targeted data rate $\tilde{R}_{\pi^*(j)}$ and the parameter $\rho_{\pi^*(j)}$ for any j, $j \ge m$. To compare the total transmit power between NOMA and OMA from the perspective of outage probability constraints, the total transmit power is calculated by using (4) in NOMA and OMA systems separately. Without loss of generality, assume that $m = \pi^*(m)$ for convenience.

From (6) and (11), the total power satisfying the outage constraint in NOMA systems can be calculated as

$$P = \textstyle\sum_{m=1}^{M} 2^{\sum_{i=1}^{m-1}\tilde{R}_i}(2^{\tilde{R}_m}-1)\rho_m, \tag{18}$$

Obviously, the total transmit power of NOMA is also monotonically increasing with $\tilde{R}_m$ and ρ_m, for any $m \in \mathcal{M}$.

For comparison, we introduce OMA as a comparable scheme, which can support the data rate is given in the following under the same configuration. Note that the conventional OMA system, such as TDMA, requires M time slots to support M users, while NOMA can support M users during a single time slot. Thus, the achievable rate of OMA is

$$\bar{R}_m = \frac{1}{M}\log\left(1 + \frac{|h_m|^2\bar{P}_m}{\sigma^2}\right). \tag{19}$$

Similar to (3), the outage probability of OMA can be obtained. Note that the outage probability among each user in OMA is decoupled; hence, all the outage constraints must be equal at the optimal solution. Further, the total transmit power of OMA is given by

$$\bar{P} = \textstyle\sum_{m=1}^{M}(2^{M\tilde{R}_m}-1)\rho_m. \tag{20}$$

Therefore, the power difference between NOMA and OMA schemes can be calculated as

$$d_{gap} = \sum_{m=1}^{M}\left(2^{\sum_{i=1}^{m-1}\tilde{R}_i}\left(2^{\tilde{R}_m}-1\right) - \left(2^{M\tilde{R}_m}-1\right)\right)\rho_m \tag{21a}$$

$$\le \textstyle\sum_{m=1}^{M}\left(2^{\sum_{i=1}^{m}\tilde{R}_i} - 2^{\sum_{i=1}^{m-1}\tilde{R}_i} - 2^{M\tilde{R}_m} + 1\right)\rho_1, \tag{21b}$$

$$= \rho_1\left((M-1) + 2^{\sum_{m=1}^{M}\tilde{R}_m} - \sum_{m=1}^{M} 2^{M\tilde{R}_m}\right), \tag{21c}$$

$$\le \rho_1\left((M-1) + 2^{\sum_{m=1}^{M}\tilde{R}_m} - M2^{\sum_{m=1}^{M}\tilde{R}_m}\right), \tag{21d}$$

where $\rho_1 > 0$ if $\epsilon_1 \neq 1$. Theorem 1 is employed in (21b) and the inequality of arithmetic and geometric means is used to (21d). Now define a function of $g(M, \{\tilde{R}_m\})$ to denote the r.h.s in (21d). The partial derivative $\frac{\partial g(M,\{\tilde{R}_m\})}{\partial M} = \rho_1(1 - 2^{\sum_{m=1}^{M}\tilde{R}_m}) \le 0$ and $\frac{\partial g(M,\{\tilde{R}_m\})}{\partial \tilde{R}_m} = \rho_1(1 - M)2^{\sum_{m=1}^{M}\tilde{R}_m}\ln(2) \le 0$. Thus, $g(M, \{\tilde{R}_m\})$ is a monotonically decreasing function of M and $\{\tilde{R}_m\}$ for any m, and the maximum 0 is achieved when $M = 1$ or $\tilde{R}_m = 0$ for any m. In the downlink multiuser scenario, $M \ge 2$ and $\tilde{R}_m > 0$ is appropriate, as a result, d_{gap} is always negative, which indicates NOMA requires less power than OMA with the same outage requirements and configurations.

V. Simulation results

In this section, we present the simulation results of the proposed power allocation algorithm for NOMA systems by using only the average CSI at the transmitter. For simulation, we assume that all simulation configurations follow the system model introduced in Section II with $\gamma = 3$. The distance set from the users to the BS, $\mathcal{D} = \{d_1, \cdots, d_M\}$, can be generated by an arithmetic sequence. Here, it is assumed that $\mathcal{D} = \{d_m | d_m = M - m + 1, \forall m\}$ for simplicity. Further, we suppose that all users have the same maximum outage constraint $\epsilon_m = \epsilon$, $\forall m$, the same target rate $\tilde{R}_m = R_{th}$, $\forall m$, and the same noise power σ^2 for convenience.

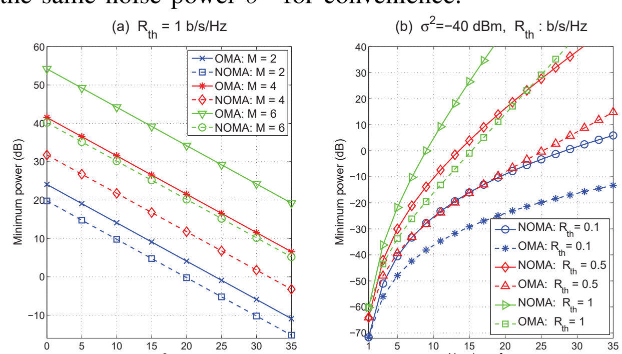

Fig. 1. Total transmission power comparisions versus the noiser power and the number of users.

Fig. 1 demonstrates the required minimum transmit power in both NOMA systems and OMA systems, respectively. In Fig. 1(a), the minimum transmit power is shown as a function of $\frac{1}{\sigma^2}$, where the targeted rate for NOMA $R_{th} = 1$ b/s/Hz. It can be observed from Fig. 1(a), the gap between NOMA and OMA becomes larger with increasing M. Another important observation is that the total transmit power has a linear relationship with $\frac{1}{\sigma^2}$ in dB, which can be derived from (18) and (20) in Section IV. Moreover, the impact of the number of users on NOMA and OMA is demonstrated in Fig. 1(b). From Fig. 1(b), the total transmit power will increase in order to guarantee the outage constraint when the number of the connected users increases. However, OMA will need more power to serve the same number of users. For example, when the transmit power is 0 dB and $R_{th} = 0.5$ b/s/Hz, NOMA can support 25 users while OMA can support 13 users only. Fig. 1 also reveals that with increasing the number of users the gap becomes a larger. Note also that as R_{th} increases, the slope of the curves becomes steeper because it is more difficult to ensure users with poor connections to be connected.

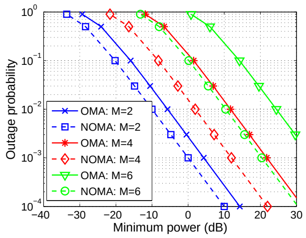

Fig. 2. Outage probility versus total transmission power and the number of users. Assume that $R_{th} = 1$ and $\sigma^2 = -10$dBm.

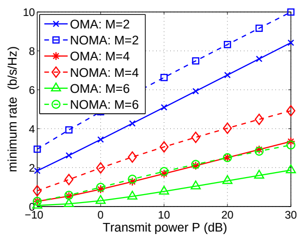

Fig. 3. Fairness rate comparisons versus the total transmit power under different user numbers with $\sigma^2 = -10$dBm and $\epsilon = 0.1$.

Fig. 2 provides a comparison between NOMA and OMA by depicting the achievable outage probability for different numbers of users with varying minimum transmit power. It can be seen from Fig. 2 that NOMA and OMA can achieve the same diversity order, but the outage performance of NOMA is always better than that of OMA. In fact, as the number of users increases, the performance gain of NOMA over OMA increases almost logarithmically. In Fig. 3, the impact of the total transmit power and the user number on the maximum fairness rate of NOMA and OMA can be observed. Clearly, NOMA always outperforms OMA. In addition, the minimum rate will be obtained with increasing the number of users.

VI. Conclusions

In this letter, the problem of optimal power allocation when the transmitter only has average CSI has been studied in downlink NOMA systems. Firstly, the problem to minimize transmit power and to maximize fairness rate are investigated. Furthermore, the power difference between NOMA and OMA is derived. The main results of this work show that NOMA can decrease the transmit power and enhance the fairness rate by careful power allocation compared with OMA and is thus a promising candidate technology in future 5G systems.

Appendix A

Let $\frac{x_{\pi(m')}}{\phi_{\pi(m')}} = \min\limits_{j=1,\cdots,m} \frac{x_{\pi(j)}}{\phi_{\pi(j)}}$, then the outage constraint of (9b) in user $\pi(m)$ and $\pi(m+1)$ can be written as

$$\frac{x_{\pi(m')}}{\phi_{\pi(m')}} \geq \rho_{\pi(m)}, \tag{22a}$$

$$\min\left\{\frac{x_{\pi(m')}}{\phi_{\pi(m')}}, \frac{x_{\pi(m+1)}}{\phi(m+1)}\right\} \geq \rho_{\pi(m+1)}, \tag{22b}$$

From (22a) and (22b), one can observe that for a given π, if $\rho_{\pi(m)} \leq \rho_{\pi(m+1)}$, we can exchange the decoding order for user $\pi(m)$ and user $\pi(m+1)$. The value of $\frac{x_{\pi(m')}}{\phi_{\pi(m')}}$ is unchanged while the value of $\frac{x_{\pi(m+1)}}{\phi(m+1)}$ will decrease or is kept unchanged. As a result, the transmission power in problem (4) will either decrease or is kept the same while not affecting other users' outage constraints. Therefore, by iteratively optimizing any two adjacent users the optimal decoding order can be found with $\rho_{\pi(m)}$ in a decreasing order.

Appendix B

By replacing t with $-t$ in the objective function and (16b) with $-t = \max_{m=1,\cdots,M}\{-\phi_{\pi^*(m)}\}$, the optimization problem in (16) becomes convex. Therefore, the following Karush-Kuhn-Tucker (KKT) conditions [9] are necessary and sufficient for optimality of (16):

$$\sum_{i=1}^{m-1} \lambda_{\pi^*(i)}\phi_{\pi^*(i)} + \nu = \lambda_{\pi^*(m)} + \mu_{\pi^*(m)}, \tag{23a}$$

$$\phi_{\pi^*(m)}\left(\sum_{j=m+1}^{M} P_{\pi^*(j)} + \rho^*_{\pi^*(m)}\right) \leq P_{\pi^*(m)}, \quad \forall m, \tag{23b}$$

$$\lambda_{\pi^*(m)}(\phi_{\pi^*(m)}\left(\sum_{j=m+1}^{M} P_{\pi^*(j)} + \rho^*_{\pi^*(m)}\right) - P_{\pi^*(m)}) = 0, \quad \forall m, \tag{23c}$$

$$\sum_{m=1}^{M} P_{\pi^*(m)} \leq P, \tag{23d}$$

$$\nu(\sum_{m=1}^{M} P_{\pi^*(m)} - P) = 0, \tag{23e}$$

$$\mu_{\pi^*(m)} P_{\pi^*(m)} = 0, \tag{23f}$$

$$P_{\pi^*(m)} \geq 0, \lambda_{\pi^*(m)} \geq 0, \nu \geq 0, \mu_{\pi^*(m)} \geq 0. \tag{23g}$$

where $\lambda_{\pi^*(m)}$, $\mu_{\pi^*(m)}$, $\forall m$, and ν are the Lagrange multipliers for constraints (16c), (4d) and (14b) respectively. The right hand side of (23b) is strictly positive for all m as $\rho^*_{\pi^*(m)} > 0$, $\phi_{\pi^*(m)} > 0$ and $P_{\pi^*(m)} \geq 0$; hence, the left hand side has to be strictly positive which implies that the optimal $P^*_{\pi^*(m)} > 0$ and $\mu^*_{\pi^*(m)} = 0$ (from (23f)).

Now we show that the optimal $\nu^* > 0$ and $\lambda^*_{\pi^*(m)} > 0$, $\forall m$, by contradiction. If $\nu^* = 0$, from (23a), it follows that $\lambda^*_{\pi^*(m)} = \frac{\sum_{i=1}^{m-1} \lambda_{\pi^*(i)}\phi_{\pi^*(i)}}{\tau^*_{\pi^*(m)}}$, which implies $\lambda^*_{\pi^*(m)} = 0$, $\forall m$. Obviously, this assumption contradicts with strong duality – Slater's condition [9]. Therefore, $\nu^* > 0$, it follows that $\lambda^*_{\pi^*(m)} = \frac{\sum_{i=1}^{m-1} \lambda_{\pi^*(i)}\phi_{\pi^*(i)}+\nu}{\tau^*_{\pi^*(m)}} > 0$.

With $\lambda^*_{\pi^*(m)} > 0$, $\nu^* > 0$ and $\mu^*_{\pi^*(m)} = 0$ as proved above, conditions in (23c), (23e) and (23g) imply that all constraints in (16c) and (14b) must be enforced with equality and in (4d) with strictly inequality.

Finally, we show that $\phi^*_{\pi^*(m)} = \cdots = \phi^*_{\pi^*(M)} = t^*$ at the optimal solution. From the above proofs, (16) can be formulated as an equality constrained minimization problem. Furthermore, note that $\phi_{\pi^*(m)}$ is not a function of $P_{\pi^*(j)}$, $j < m$, and $\phi_{\pi^*(m)}$ is monotonically increasing with $P_{\pi^*(m)}$ and monotonically decreasing with $P_{\pi^*(j)}$, $j > m$. Assume that $t = \phi_{\pi^*(m')}$ with existing $\phi_{\pi^*(m')} \leq \phi_{\pi^*(m_1)}$, $m_1 \neq m'$. If $m_1 > m'$, $\phi_{\pi^*(m_1)}$ has no relationship with $P_{\pi^*(m')}$. At this time, there exists an $t_1 > t$ can be attained by decreasing $P_{\pi^*(m_1)}$ and increasing $P_{\pi^*(m')}$ under the total power constraint. Otherwise, if $m_1 < m'$, we can also find an $t_1 > t$ by similar operations. Combining the two cases above, one can conclude that $\phi^*_{\pi^*(m)} = t^*$, $\forall m$. Proposition 4 is thus proved.

References

[1] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, and K. Higuchi, "Non-orthogonal multiple access (NOMA) for cellular future radio access," in *Proc. IEEE Vehicular Technology Conference (VTC Spring), Dresden, Germany*, Jun. 2013, pp. 1–5.

[2] Z. Ding, Z. Yang, P. Fan, and H. Poor, "On the performance of non-orthogonal multiple access in 5G systems with randomly deployed users," *IEEE Signal Process. Lett.*, vol. 21, no. 12, pp. 1501–1505, Dec. 2014.

[3] Z. Ding, P. Fan, and V. Poor, "Impact of user pairing on 5g non-orthogonal multiple access downlink transmissions," *IEEE Trans. on Veh. Technol.*, to be published.

[4] S. Tomida and K. Higuchi, "Non-orthogonal access with SIC in cellular downlink for user fairness enhancement," in *Proc. IEEE International Symposium on Intelligent Signal Processing and Communications Systems (ISPACS), Chiang Mai, Thailand*, Dec. 2011, pp. 1–6.

[5] S. Timotheou and I. Krikidis, "Fairness for non-orthogonal multiple access in 5G systems," *IEEE Signal Process. Lett.*, vol. 22, no. 10, pp. 1647–1651, Oct. 2015.

[6] Y. L. Sun, D. W. K. Ng, Z. Ding, and R. Schober, "Optimal joint power and subcarrier allocation for MC-NOMA systems." [Online]. Available: http://arxiv.org/abs/1603.08132

[7] D. Love, R. Heath, V. Lau, D. Gesbert, B. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341–1365, Oct. 2008.

[8] S. Shi, L. Yang, and H. Zhu, "Outage balancing in downlink non-orthogonal multiple access with statistical channel state information," *IEEE Trans. on Wireless Commun.*, to be published.

[9] S. Boyd and L. Vandenberghe, *Convex optimization*. New York,USA: Cambridge Univ. Press, 2004.

[10] S. Kandukuri and S. Boyd, "Optimal power control in interference-limited fading wireless channels with outage-probability specifications," *IEEE Trans. Wireless Commun.*, vol. 1, no. 1, pp. 46–55, Jan 2002.

[11] J. Papandriopoulos, J. Evans, and S. Dey, "Optimal power control for rayleigh-faded multiuser systems with outage constraints," *IEEE Trans. Wireless Commun.*, vol. 4, no. 6, pp. 2705–2715, Nov 2005.