Joint Power and Time Allocation for NOMA-MEC Offloading

Zhiguo Ding, Jie Xu, Octavia A. Dobre, and H. Vincent Poor

***Abstract*—In this paper, non-orthogonal multiple access (NOMA) is applied to mobile edge computing (MEC), where the energy consumption for NOMA-MEC offloading is minimized by jointly optimizing the power and time allocation. By transforming the formulated NOMA-MEC offloading problem to a special case of geometric programming, closed-form expressions for the optimal power and time allocation solutions are obtained. Analytical and simulation results are provided to demonstrate the superior performance of NOMA-MEC over conventional MEC.**

I. Introduction

Both non-orthogonal multiple access (NOMA) and mobile edge computing (MEC) have been recognized as two important communication techniques in future wireless networks [1], [2]. Recall that MEC is crucial to the scenario in which computationally and energy constrained users need to carry out computationally intensive and latency-critical tasks. The key idea of MEC is that the users can seek help from MEC servers that have better computation capabilities, i.e., the users first offload their tasks to the servers and then download the outcomes of the tasks. By using orthogonal multiple access (OMA), such as time division multiple access (TDMA), the users have to take turns to offload their tasks to the server. By applying NOMA to MEC, not only can severe delay be avoided, but also energy consumption can be reduced [3], [4].

This letter studies the impact of NOMA on energy-efficient MEC offloading. Unlike the existing studies in [3] and [4], hybrid NOMA is considered in this paper, i.e., a user can first offload parts of its task by using a time slot allocated to another user and then offload the remainder of its task during a time slot solely occupied by itself. How long these time slots should be and how large powers allocated during the time slots should be are jointly studied in this paper, where closed-form expressions for the optimal time and power allocation solutions are obtained, by applying geometric programming (GP). The closed-form solutions not only facilitate low-complexity resource allocation, but also yield insight into NOMA-MEC. For example, by using the obtained closed-form solutions, hybrid-NOMA-MEC can be proved to be superior to OMA-MEC when users have demanding latency requirements for their task offloading, whereas OMA-MEC is preferred if a user's task is delay tolerant. It is worth pointing out that the pure NOMA strategy is not preferred for either of the two situations.

II. System Model

Consider an MEC offloading scenario, in which K users with different quality of service (QoS) requirements communicate with one access point with an integrated MEC server. Because of their limited computational capabilities, it is assumed that the users choose to offload their computationally intensive, latency-critical, and inseparable tasks to the server.

Each user's task is characterized by the parameter pair $\{N_k, \beta_k\}$, $k = 1, \ldots, K$, which is defined as follows:

- N_k denotes the number of nats contained in a task;
- D_k denotes the computation deadline of a task.

Without loss of generality, assume that $N_k = N$, $1 \leq k \leq K$, and the users are ordered according to their computation deadlines, i.e., $D_1 \leq \cdots \leq D_K$. To reduce the system complexity, it is further assumed that the MEC server schedules only two users, user m and user n, $m \leq n$, to be served at the same resource block. To better illustrate the benefit of NOMA, the OMA benchmark is illustrated first.

If OMA is used, each user is allocated a dedicated time slot for offloading[1]. Since user m has a more demanding deadline than user n, user m is served first. Therefore the users' transmit powers, denoted by P_m^{OMA} and P_n^{OMA}, need to satisfy $D_m \ln(1 + P_m^{\text{OMA}}|h_m|^2) = N$ and $(D_n - D_m)\ln(1 + P_n^{\text{OMA}}|h_n|^2) = N$, respectively, where h_i denotes user i's channel gain, $i = m, n$.

By using the principle of NOMA, the two users can offload their tasks simultaneously during D_m to the server. It is important to point out that user m experiences the same performance as in OMA if its message is decoded at the second stage of successive interference cancelation (SIC) and user n's data rate during D_m is constrained as follows:

$$R_n \leq \ln\left(1 + \frac{P_{n,1}|h_n|^2}{P_m^{\text{OMA}}|h_m|^2 + 1}\right), \tag{1}$$

where $P_{n,1}$ denotes the power used by user n during D_m.

As pointed out in [5], user n needs to consume more energy in NOMA than in OMA if the user completely relies on D_m. Therefore, hybrid NOMA is considered, i.e., user n shares D_m with user m, and then continuously transmits for another time interval, denoted by T_n, after D_m. Denote the power used by user n during T_n by $P_{n,2}$. As user m experiences the same as in OMA, we focus only on user n's performance in this letter.

III. NOMA-Assisted MEC Offloading

The problem for minimizing the energy consumption of NOMA-MEC offloading can be formulated as follows:

$$\begin{aligned}
\min_{T_n, P_{n,1}, P_{n,2}} \quad & D_m P_{n,1} + T_n P_{n,2} && \text{(2a)}\\
s.t. \quad & D_m \ln\left(1 + \frac{P_{n,1}|h_n|^2}{P_m^{\text{OMA}}|h_m|^2 + 1}\right) && \text{(2b)}\\
& + T_n \ln\left(1 + |h_n|^2 P_{n,2}\right) \geq N \\
& 0 \leq T_n \leq D_n - D_m && \text{(2c)}\\
& P_{n,i} \geq 0, \forall i \in \{1, 2\}. && \text{(2d)}
\end{aligned}$$

[1]In this paper, the time and the energy costs for the server to send the outcomes of the tasks to the users are omitted, since the size of the outcomes is typically very small. The energy consumption for the computation at the server is also omitted, as the server is not energy constrained.

Z. Ding and H. V. Poor are with the Department of Electrical Engineering, Princeton University, Princeton, USA. Z. Ding is also with the School of Electrical and Electronic Engineering, the University of Manchester, Manchester, UK. J. Xu is with School of Information Engineering, Guangdong University of Technology, China. O. A. Dobre is with the Department of Electrical and Computer Engineering, Memorial University, St. John's, NL, Canada.

The objective function (2a) denotes user n's energy consumption for MEC offloading, (2b) denotes the rate constraint to ensure that user n's N nats are offloaded within D_m+T_n, and (2c) denotes the deadline constraint, i.e., $T_n + D_m \le D_n$. It is worth noting that the benefit of using NOMA is obvious for the case of $D_n = D_m$, where the power required by the OMA case becomes infinite while the power in NOMA is finite.

In the first two subsections of this section, we will focus on the scenario where $D_n < 2D_m$, in order to avoid the trivial case with OMA solutions. In particular, we first obtain the optimal solutions for $P_{n,1}$ and $P_{n,2}$ as explicit functions of T_n by applying GP, and then find the optimal solution of T_n. The scenario $D_n \ge 2D_m$ is also discussed at the end of this section.

A. Finding the Optimal Solutions for $P_{n,1}$ and $P_{n,2}$

In order to make GP applicable, the objective function and the constraints in (2) need to be transformed as follows. By using the fact $D_m \ln(1 + P_m^{\text{OMA}}|h_m|^2) = N$, constraint (2b) can be simplified as follows:

$$\ln\left(1 + e^{-\frac{N}{D_m}}|h_n|^2 P_{n,1}\right)^{D_m}\left(1 + |h_n|^2 P_{n,2}\right)^{T_n} \ge N. \quad (3)$$

Define $x_1 = 1 + e^{-\frac{N}{D_m}}|h_n|^2 P_{n,1}$ and $x_2 = 1 + |h_n|^2 P_{n,2}$. Problem (2) is transformed to the following equivalent form:

$$\begin{aligned} \min_{T_n, x_1, x_2} \quad & D_m e^{\frac{N}{D_m}} x_1 + T_n (x_2 - 1) && (4a)\\ s.t. \quad & e^N x_1^{-D_m} x_2^{-T_n} \le 1 && (4b)\\ & 0 \le T_n \le D_n - D_m && (4c)\\ & x_i \ge 1, \forall i \in \{1, 2\}. && (4d) \end{aligned}$$

Define $y_i = \ln x_i$, $i = 1, 2$. By fixing T_n, problem (4) can be transformed to the following equivalent form:

$$\begin{aligned} \min_{y_1, y_2} \quad & D_m e^{\frac{N}{D_m}} e^{y_1} + T_n e^{y_2} && (5a)\\ s.t. \quad & e^{-D_m y_1 - T_n y_2 + N} \le 1 && (5b)\\ & y_i \ge 0, \forall i \in \{1, 2\}. && (5c) \end{aligned}$$

By treating problem (5) as a special case of GP and applying logarithm to (5), the Karush-Kuhn-Tucker (KKT) conditions can be applied to find the optimal solution as follows:

$$\begin{cases} \frac{D_m e^{\frac{N}{D_m}} e^{y_1}}{D_m e^{\frac{N}{D_m}} e^{y_1} + T_n e^{y_2}} - \lambda_1 - \lambda_3 D_m &= 0\\ \frac{T_n e^{y_2}}{D_m e^{\frac{N}{D_m}} e^{y_1} + T_n e^{y_2}} - \lambda_2 - \lambda_3 T_n &= 0\\ N - D_m y_1 - T_n y_2 &\le 0\\ \lambda_3 \left(-D_m y_1 - T_n y_2 + N\right) &= 0\\ -y_i &\le 0, \forall i \in \{1, 2\}\\ \lambda_i y_i &= 0, \forall i \in \{1, 2\}\\ \lambda_i &\ge 0, \forall i \in \{1, 2, 3\} \end{cases}, \quad (6)$$

where λ_i are Lagrange multipliers. The optimal solutions of $P_{n,1}$ and $P_{n,2}$ can be obtained as in the following lemma.

Lemma 1. *Assume $D_n < 2D_m$. The optimal solutions for $P_{n,1}$ and $P_{n,2}$ in problem* (2) *can be expressed as the following closed-form functions of T_n:*

$$\begin{cases} P_{n,1}^* = & |h_n|^{-2} e^{\frac{N}{D_m}} \left(e^{\frac{N(D_m - T_n)}{D_m(D_m + T_n)}} - 1\right)\\ P_{n,2}^* = & |h_n|^{-2} \left(e^{\frac{N(D_m - T_n)}{D_m(D_m + T_n)} + \frac{N}{D_m}} - 1\right) \end{cases}. \quad (7)$$

Proof. Please refer to the appendix. □

B. Finding the Optimal Solution for T_n

By substituting the optimal solution obtained in Lemma 1 into problem (2), the original problem can be written in an equivalent form as follows:

$$\begin{aligned} \min_{T_n} \quad & g_{T_n} \triangleq D_m \left(e^{y_1^*} - 1\right) e^{\frac{N}{D_m}} + T_n \left(e^{y_2^*} - 1\right), && (8)\\ s.t. \quad & T_n \le D_n - D_m, \end{aligned}$$

where g_{T_n} is the energy consumption normalized by omitting the constant $|h_n|^{-2}$ in the objective function (2a). Note that both y_1^* and y_2^* are functions of T_n as defined in (21).

The derivative of g_{T_n} with respect to T_n can be expressed as follows:

$$\begin{aligned} \frac{dg_{T_n}}{dT_n} =& D_m e^{\frac{N}{D_m}} e^{y_1^*} \frac{(-2N)}{(D_m + T_n)^2} + \left(e^{y_2^*} - 1\right) && (9)\\ &+ T_n e^{y_2^*} \frac{(-2N)}{(D_m + T_n)^2}. \end{aligned}$$

Recall that $y_2^* = y_1^* + \frac{N}{D_m}$. Therefore, the derivative of g_{T_n} can be rewritten as follows:

$$\begin{aligned} \frac{dg_{T_n}}{dT_n} =& D_m e^{y_2^*} \frac{(-2N)}{(D_m + T_n)^2} + \left(e^{y_2^*} - 1\right) && (10)\\ &+ T_n e^{y_2^*} \frac{(-2N)}{(D_m + T_n)^2}\\ =& e^{y_2^*} \left(1 - \frac{2N}{D_m + T_n}\right) - 1. \end{aligned}$$

Further, recall that $y_2^* = \frac{N(D_m - T_n)}{D_m(D_m + T_n)} + \frac{N}{D_m} = \frac{2N}{D_m + T_n}$. Thus, the derivative of g_{T_n} can be expressed as follows:

$$\frac{dg_{T_n}}{dT_n} = g_x\left(\frac{2N}{D_m + T_n}\right), \quad (11)$$

where

$$g_x(x) \triangleq e^x (1 - x) - 1. \quad (12)$$

$g_x(x)$ is a monotonically non-increasing function since $\frac{dg_x(x)}{dx} = -xe^{-x} \le 0$ for $x \ge 0$. Therefore, $\frac{dg_{T_n}}{dT_n} \le 0$ since

$$\frac{dg_{T_n}}{dT_n} \le g_x(0) = 0, \quad (13)$$

which means that g_{T_n} is monotonically non-increasing. Hence, the optimal solution of T_n for problem (2) is given by

$$T_n^* = D_n - D_m. \quad (14)$$

It is worth pointing out that $T_n^* < D_m$, since the case $D_n < 2D_m$ is considered in this subsection.

C. Remarks and Discussions

1) For the superiority of NOMA over OMA: we can show that OMA cannot outperform NOMA, as presented in the following. The energy consumption gap between NOMA-MEC and OMA-MEC is given by

$$\begin{aligned} \Delta \triangleq& D_m \left(e^{y_1^*} - 1\right) e^{\frac{N}{D_m}} |h_n|^{-2} + T_n \left(e^{y_2^*} - 1\right) |h_n|^{-2} && (15)\\ &- T_n \left(e^{\frac{N}{T_n}} - 1\right) |h_n|^{-2}. \end{aligned}$$

By using (21), the gap can be further expressed as follows:

$$
\begin{aligned}
|h_n|^2\Delta \triangleq & D_m e^{y_2^*}(D_m+T_n) - D_m e^{\frac{N}{D_m}} - T_n e^{\frac{N}{T_n}} \qquad (16)\\
=& e^{\frac{2N}{D_m+T_n}}(D_m+T_n) - D_m e^{\frac{N}{D_m}} - T_n e^{\frac{N}{T_n}} = f_{T_n}(T_n).
\end{aligned}
$$

As shown in (33), $f_{T_n}(T_n) \leq 0$, which means that the use of NOMA outperforms or at least yields the same performance as OMA, under the condition $D_n < 2D_m$.

2) For the case $D_n \geq 2D_m$: this case corresponds to a scenario in which user n has less demanding latency requirements. Compared to the case $D_n < 2D_m$, T_n can be larger than D_m for the case $D_n \geq 2D_m$, since $T_n = D_n - D_m$. In this case, OMA yields the best performance, as shown in the following. Since the hybrid NOMA solutions in Lemma 1 are feasible only if $T_n < D_m$ and the energy consumption of hybrid NOMA, i.e., g_{T_n} in (8), is a monotonically non-increasing function of T_n, g_{T_n} is always strictly lower bounded by

$$
D_m |h_n|^{-2}\left(e^{\frac{N}{D_m}} - 1\right). \qquad (17)
$$

On the other hand, the lower bound in (17) can be achieved by OMA when $D_n \geq 2D_m$, i.e., the solution obtained with $\lambda_1 \neq 0$, $\lambda_2 = 0$ and $T_n = D_m$, as shown in (24). In other words, when $D_n \geq 2D_m$, OMA requires less energy consumption than hybrid NOMA. Furthermore, OMA can also outperform pure NOMA since

$$
\begin{aligned}
\frac{E_{\text{OMA}} - E_{\text{NOMA}}}{|h_n|^{-2}} \underset{(a)}{\leq} & D_m\left(e^{\frac{N}{D_m}} - 1\right) - D_m e^{\frac{N}{D_m}}\left(e^{\frac{N}{D_m}} - 1\right)\\
=& -D_m\left(e^{\frac{N}{D_m}} - 1\right)^2 \leq 0, \qquad (18)
\end{aligned}
$$

where step (a) is due to the fact that the minimal energy required by OMA is no less than that in (17). Therefore, it is concluded that OMA outperforms hybrid NOMA and pure NOMA when $D_n \geq 2D_m$. This conclusion is reasonable, since a more relaxed deadline makes it possible to use only the interference-free time slot $(D_n - D_m)$ for offloading.

IV. Numerical Results

In this section, the performance of the proposed NOMA-MEC scheme is evaluated via simulation results, where the normalized energy consumption in (8) is used. As can be observed from Fig. 1, the use of NOMA-MEC can yield a significant performance gain over OMA-MEC, particularly when D_n is small. This is because OMA-MEC relies on the short period $(D_n - D_m)$ for offloading. Take $D_n \to D_m$ as an example. $(D_n - D_m)$ becomes close to zero, and hence the energy consumed by OMA-MEC becomes prohibitively large, as shown in the figure. On the other hand, NOMA-MEC uses not only $(D_n - D_m)$ but also D_m for offloading, which makes the energy consumed by NOMA-MEC more stable.

To better illustrate the optimality of the solutions obtained in Lemma 1, the energy consumption is shown as a function of different choices of $(P_{n,1}, P_{n,2})$ in Fig. 2. The figure clearly demonstrates that among all the possible power allocation choices, the one provided in Lemma 1 yields the lowest energy consumption. As discussed in Section III-C, the performance of NOMA and OMA becomes quite similar when D_n becomes large, which is confirmed by Fig. 1, while further details about this aspect are provided in Fig. 3. As can be seen from this figure, when D_n increases, the power allocated to D_m approaches zero, which means that hybrid NOMA is degraded relative to OMA, as pointed out in Section III-C.

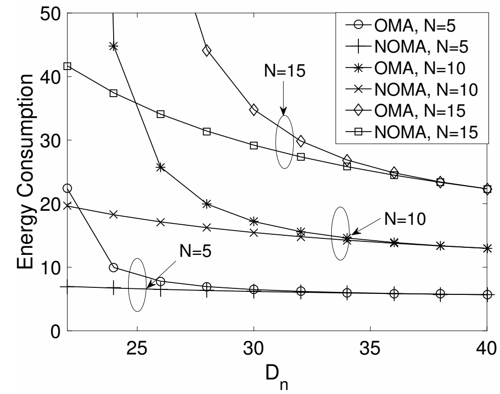

Fig. 1. Performance comparison between NOMA-MEC and OMA-MEC, where $D_m = 20$.

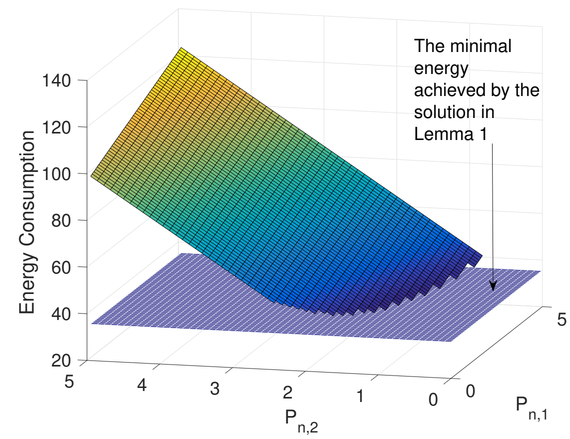

Fig. 2. Optimality of the solutions obtained in Lemma 1, where $N = 15$, $D_m = 20$, and $T_n = \frac{1}{4}D_m$.

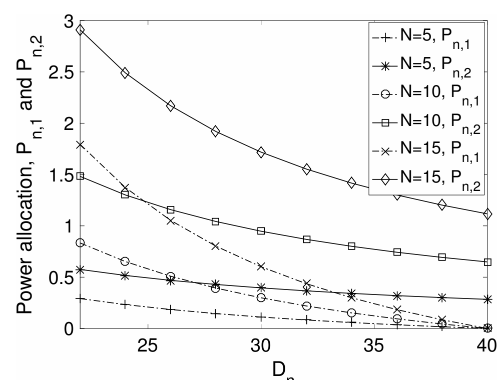

Fig. 3. The performance of NOMA-MEC, where $D_m = 20$.

V. Conclusions

In this paper, the principle of NOMA has been applied to MEC, and optimal solutions for the power and time allocation have been obtained by applying GP. Analytical and simulation results have also been provided to demonstrate the superior performance of NOMA-MEC over OMA-MEC.

Appendix A
Proof of Lemma 1

The proof of the lemma can be completed by studying the possible choices of λ_i, $i = 1, 2, 3$, and showing that the solutions for the case with $\lambda_i = 0$, $\forall i \in \{1, 2\}$, yield the smallest energy consumption.

1) Hybrid NOMA ($\lambda_i = 0$, $\forall i \in \{1, 2\}$): since $\lambda_i = 0$, $\forall i \in \{1, 2\}$, $y_i > 0$ and hence $P_{n,1}$ and $P_{n,2}$ are non-zero, which is the reason why this case is termed hybrid NOMA. For this case, we can show that $\lambda_3 \neq 0$ as follows. If $\lambda_3 = 0$, the KKT conditions lead to the following two equations:

$$\begin{cases} \frac{D_m e^{\frac{N}{D_m}} e^{y_1}}{D_m e^{\frac{N}{D_m}} e^{y_1} + T_n e^{y_2}} & = 0 \\ \frac{T_n e^{y_2}}{D_m e^{\frac{N}{D_m}} e^{y_1} + T_n e^{y_2}} & = 0 \end{cases}, \tag{19}$$

which cannot be true. Therefore, $\lambda_3 \neq 0$ follows, which means that the KKT conditions can be rewritten as follows:

$$\begin{cases} \frac{e^{\frac{N}{D_m}} e^{y_1}}{D_m e^{\frac{N}{D_m}} e^{y_1} + T_n e^{y_2}} - \lambda_3 & = 0 \\ \frac{e^{y_2}}{D_m e^{\frac{N}{D_m}} e^{y_1} + T_n e^{y_2}} - \lambda_3 & = 0 \\ (-D_m y_1 - T_n y_2 + N) & = 0 \\ y_i & > 0, \forall i \in \{1, 2\} \end{cases}. \tag{20}$$

With some algebraic manipulations, the optimal solutions for y_1 and y_2 can be obtained as follows:

$$\begin{cases} y_1^* = & \frac{N(D_m - T_n)}{D_m(D_m + T_n)} \\ y_2^* = & \frac{N(D_m - T_n)}{D_m(D_m + T_n)} + \frac{N}{D_m} \end{cases}. \tag{21}$$

Since $D_n < 2D_m$, $T_n \leq D_n - D_m < D_m$, and the solutions y_i^*'s satisfy the constraints $y_i > 0$, which mean that the solutions shown in (21) are feasible. With the power allocation solutions in (21), the overall energy consumption is given by

$$\begin{aligned} E_{\text{H-NOMA}} = & D_m |h_n|^{-2} e^{\frac{N}{D_m}} \left(e^{\frac{N(D_m - T_n)}{D_m(D_m + T_n)}} - 1 \right) \\ & + T_n |h_n|^{-2} \left(e^{\frac{N(D_m - T_n)}{D_m(D_m + T_n)} + \frac{N}{D_m}} - 1 \right). \end{aligned} \tag{22}$$

2) Pure NOMA ($\lambda_1 = 0$ and $\lambda_2 \neq 0$): since $\lambda_1 = 0$ and $\lambda_2 \neq 0$, we have $y_1 \neq 0$ and $y_2 = 0$, and hence $P_{n,1} \neq 0$ and $P_{n,2} = 0$, which is the reason to term this case pure NOMA. Since $y_2 = 0$ corresponds to an extreme situation in which all the power is allocated to D_m, the use of the rate constraint in (3) yields the following choice of $P_{n,1}$:

$$\tilde{P}_{n,1}^* = \left(e^{\frac{N}{D_m}} - 1 \right) e^{\frac{N}{D_m}} |h_n|^{-2}, \tag{23}$$

which means that the overall energy consumption becomes

$$E_{\text{NOMA}} = D_m \left(e^{\frac{N}{D_m}} - 1 \right) e^{\frac{N}{D_m}} |h_n|^{-2}. \tag{24}$$

3) OMA ($\lambda_1 \neq 0$ and $\lambda_2 = 0$): since $\lambda_1 \neq 0$ and $\lambda_2 = 0$, we have $y_1 = 0$ and $y_2 \neq 0$, and hence $P_{n,1} = 0$ and $P_{n,2} \neq 0$, which is the reason to term this case as OMA. Since all the power is allocated to T_n, the use of the rate constraint in (3) yields the following choice of $P_{n,2}$:

$$\tilde{P}_{n,2}^* = \left(e^{\frac{N}{T_n}} - 1 \right) |h_n|^{-2}, \tag{25}$$

which means that the overall energy consumption becomes

$$E_{\text{OMA}} = T_n \left(e^{\frac{N}{T_n}} - 1 \right) |h_n|^{-2}. \tag{26}$$

4) Comparisons among the three cases: in the following, we can show that hybrid NOMA requires the smallest energy. As discussed in Subsection III-B, the overall energy is a monotonically non-increasing function of T_n when $\lambda_i = 0$, $\forall i \in \{1, 2\}$. Therefore, $E_{\text{H-NOMA}}$ is upper bounded by

$$E_{\text{H-NOMA}} \leq D_m |h_n|^{-2} e^{\frac{N}{D_m}} \left(e^{\frac{N}{D_m}} - 1 \right) = E_{\text{NOMA}}, \tag{27}$$

since $T_n \geq 0$. Hence, the use of hybrid NOMA requires less energy consumption than pure NOMA.

The difference between $E_{\text{H-NOMA}}$ and E_{OMA} can be expressed as follows:

$$\begin{aligned} \frac{E_{\text{H-NOMA}} - E_{\text{OMA}}}{|h_n|^{-2}} = & D_m e^{\frac{N}{D_m}} \left(e^{\frac{N(D_m - T_n)}{D_m(D_m + T_n)}} - 1 \right) \\ + T_n \left(e^{\frac{2N}{(D_m + T_n)}} - 1 \right) - & T_n \left(e^{\frac{N}{T_n}} - 1 \right) = f_{T_n}(T_n), \end{aligned} \tag{28}$$

where $f_{T_n}(x)$ is defined as follows:

$$f_{T_n}(x) \triangleq (D_m + x) e^{\frac{2N}{(D_m + x)}} - D_m e^{\frac{N}{D_m}} - x e^{\frac{N}{x}}. \tag{29}$$

Note that $f_{T_n}(x)$ is a monotonically non-decreasing function for $x < D_m$, as shown in the following. The derivative of $f_{T_n}(x)$ is given by

$$\frac{d f_{T_n}(x)}{dx} = e^{\frac{2N}{D_m + x}} \left(1 - \frac{2N}{D_m + x} \right) - e^{\frac{N}{x}} \left(1 - \frac{N}{x} \right). \tag{30}$$

Now define $f_y(y) = e^{\frac{N}{y}} \left(1 - \frac{N}{y} \right)$, and the derivative $f_{T_n}(x)$ can be expressed as follows:

$$\frac{d f_{T_n}(x)}{dx} = f_y \left(\frac{D_m + x}{2} \right) - f_y(x). \tag{31}$$

Note that $f_y(y)$ is a monotonically increasing function since $\frac{d f_y(y)}{dy} = \frac{N^2 e^{\frac{N}{y}}}{y^3} > 0$. Since $x < D_m$, $\frac{D_m + x}{2} > x$. Therefore, the derivative $f_{T_n}(x)$ is non-negative, i.e.,

$$\frac{d f_{T_n}(x)}{dx} = f_y \left(\frac{D_m + T_n}{2} \right) - f_y(T_n) \geq 0, \tag{32}$$

which means that $f_{T_n}(x)$ is a monotonically non-decreasing function. Since $T_n < D_m$, we have

$$\frac{E_{\text{H-NOMA}} - E_{\text{OMA}}}{|h_n|^{-2}} = f_{T_n}(T_n) \leq f_{T_n}(D_m) = 0. \tag{33}$$

Combining (27) and (33), hybrid NOMA, i.e., the solutions obtained with $\lambda_i = 0$, $\forall i \in \{1, 2\}$, yields the smallest energy consumption. By using y_i^* in (21), the required powers during D_m and T_n can be obtained, and the proof is complete.

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