Performance Analysis for Multi-Way Relaying in Rician Fading Channels

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Abstract

In this paper, the multi-way relaying scenario is considered with M users who want to exchange their information with each other with the help of N relays ($N \gg M$) among them. There are no direct transmission channels between any two users. Particularly all users transmit their signals to all relays in the first time slot and $M-1$ relays are selected later to broadcast their mixture signals during the following $M-1$ time slots to all users. Compared to the transmission with the help of single relay, the multi-way relaying scenario reduces the transmit time significantly from $2M$ to M time slots. Random and semiorthogonal relays selections are applied. Rician fading channels are considered between the users and relays, and analytical expressions for the outage probability and ergodic sum rate for the proposed relaying protocol are developed by first characterizing the statistical property of the effective channel gain based on random relays selection. Also, the approximation of ergodic sum rate at high signal-to-noise ratio (SNR) regime is derived. In addition, the diversity order of the system is investigated for both random and semiorthogonal relay selections. Meanwhile, it is shown that when the relays are randomly separated into L groups of $M-1$ relays, the group with maximum average channel gain can achieve the diversity order L which will increase when more relays considered in the scheme. Furthermore, when semiorthogonal selection (SS) algorithm is applied to select the relays with semiorthogonal channels, it is shown that the system will guarantee that all the users can decode the others information successfully. Moreover, the maximum of channel gain after semiorthogonal relays selection is investigated by using extreme value theory, and tight lower and upper bounds are derived. Simulation results demonstrate that the derived expressions are accurate.

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Index Terms

Cooperative communication, extreme value theory, multi-way relaying, semiorthogonal relays selection.

I. Introduction

Cooperative communications [1] has triggered enormous research interest in understanding the performance of different multi-way relay channels (MWRCs). The MWRC can be viewed as an extension of the two-way relay channel (TWRC) [2]–[4] where two users exchange their information via a relay. Such a channel was first introduced in [5], where the achievable information rates were developed. In multi-way relay scenario, several users try to exchange their information with each other with the help of relays, where direct links between the source nodes whether exist or are not considered either due to large scale path loss or shadowing effects. Similar to two-way relaying, self interference can be removed by exploring the priori information at the source nodes [6]–[9]. Various relaying protocols, such as amplify-and-forward (AF), decode-and-forward (DF) or compress-and-forward (CF), were considered and the achievable symmetric rate of all users were studied in [5]. In [8], the authors investigated the capacity of binary multi-way relay systems. Considering Nakagami-m fading, the performance of multi-way relay direct-sequence code-division multiple-access (MR-DS-CDMA) systems was analyzed in [10]. The capacity region of MWRC with functional-decode-forward (FDF) was studied in [11]. The outage performance of compute-and-forward (CPF) multi-way relay system was investigated in [12]. Recently, the authors studied secure performance of two way relaying scenario with one pair of source nodes, one relay and one eavesdropper in [13].

Meanwhile, different feasible types of multi-flow relaying strategies, network codings and cooperation schemes were analyzed in [14]–[17]. Multi-way relay communications were studied for a group of single-antenna users with regenerative relaying strategies in [7]. By using stochastic geometry and percolation theory, the authors analyzed the connectivity of cooperative ad hoc network with selfishness in [18]. Using non-coherent fast frequency hopping (FFH) techniques, information exchange among a group of users was studied in [9] where information exchange could be accomplished within only two time slots regardless of the number of users. The authors in [19], studied a matching framework for cooperative networks with multiple source and destination pairs. In [20], authors studied the capacity gap for different relaying techniques and showed that FDF results in a capacity gap less than $\frac{1}{2(M-1)}$ bit.

The physical layer network coding (PLNC) [14] was studied in [21], [22] and references therein which allow several transmitters to transmit signals simultaneously to the same receiver to improve overall performance. In [22], PLNC was investigated for multi-user relay channel with multiple source nodes, single relay and single destination, and a novel decoder was designed that offered the maximum possible diversity order of two. In [23], a novel cooperation protocol based on complex-field wireless network coding was developed in a network with multiple sources and one destination. Meanwhile, relay selection strategy is inseparable from the cooperative network coding problem [24], because the relay selection can indeed benefit the network performance from many aspects. In [25]–[27], relay selection schemes were studied in ad hoc network. The authors studied novel contention-free and contention-based relay selection algorithms for multiple source-destination system in [28] where the best relay selection is based on the channel gains. Considering the multiple relays system with multiple sources and one destination, the authors in [29] proposed optimal and sub-optimal relay selection schemes based on the sum capacity maximization criterion. In [30], a relay selection algorithm, called RSTRA (Relay Selection algorithm combined Throughput and Resource allocation), is proposed for IEEE 802.16m network in order to maximize the network throughput. However, proposing more effective and practical multi-way relaying protocols is still a hot topic requiring more investigations.

Motivated by the previous works, a new transmission strategy of multi-way relaying protocol has been proposed and investigated in this paper which can reduce the transmission time slots and increase the diversity order. We consider a multiple relaying scenario with multiple sources and relays, where sources exchange information with each other with the help of the selected relays. According to the cooperative transmission strategy proposed in this paper, the time consumption will be reduced significantly. Also, the diversity order is equal to the number of randomly separated relay groups which will increase with the total number of relays in this scheme. By utilizing the statistical property of Rician fading channels, we first find the density function of effective channel gains, from which the performance of the proposed multi-way relaying protocol can be analyzed by using two information theoretic criteria, outage probabilities and ergodic sum rates, respectively. Analytical results are also provided to demonstrate the superior performance of the proposed protocol. To guarantee that all the users can decode the others messages, semiorthogonal selection method [31] is applied in our scenario. The advantages of semiorthogonal selection can be seen by our simulation results compared to random selection. Moreover, the maximum channel gain after semiorthogonal relays selection is studied by extreme value theory when the number of total relays goes to infinity [32]–[34]. The maximum channel gain is bounded by $\log N + \log\log N + \mathcal{O}(\log\log\log N)$, where N is the number of all relays. In addition, the simulation results are shown to match the developed analytical results, which demonstrate the accuracy of the analytical results.

Throughout the paper, following notations are adopted. Matrices and vectors are denoted by bold uppercase and bold lowercase letters. $\mathbf{I}_n$ denotes the $n \times n$ identity matrix and $[\mathbf{A}]_{i,j}$ is the (i,j)th element of matrix $\mathbf{A}$. $(\cdot)^\dagger$ denotes the conjugate transpose of a matrix or vector. $\mathcal{CN}(\mu, \sigma^2)$ denotes the circularly symmetric complex Gaussian distribution with mean μ and variance σ^2. $\mathrm{tr}(\cdot)$ and $\det(\cdot)$ denote the trace and determinant of a matrix, respectively. $\mathbb{E}[\cdot]$ and $\log(\cdot)$ denote the expectation operation and natural logarithm.

The rest of the paper is organized as follows. The system model is introduced in Section II. Section III presents the key analytical results of cooperative transmission. The semiorthogonal relay selection is introduced in Section IV. In Section V, the properties of maximum channel gain are investigated. Numerical results are discussed in Section VI. Finally, Section VII provides the conclusion of this paper.

II. System Model

Assuming there are M single antenna users, they plan to exchange their information with each other with help from relays, because there are no direct transmission channels between any two users. In order to compare the performance of multi-way relaying scenario with the single relay transmission scheme, at first, a benchmark scheme without cooperation (i.e., single relay transmission) will be described. Then, the cooperative scheme with AF strategy will be proposed and analyzed where M users communicate with each other via $M-1$ relays.

Firstly, considering the multi-user transmission scenario with help of single relay (only one relay[1] in the system), each user needs two time slots to transmit his own information to all the other users. In first time slot, one user transmits his signal to the relay and the relay broadcast this signal to all the other users in the second time slot. Following this strategy, $2M$ time slots are needed for M users sharing their own information.

Secondly, we consider a multi-way relaying scenario in which M users transmit their own signal in the first time slot simultaneously and N relays listen, where $N \gg M$ and PLNC scheme is used at all relays[2]. As shown in Fig. 1, The proposed cooperative transmission strategy consists of two phases. During the first phase, all sources broadcast their messages, where all the relays listen. During the second phase, $(M-1)$ relays are collaborating with the sources by broadcasting their observations during the $(M-1)$ time slots. Assuming there are M users, the total time consumption is reduced to M time slots, compared to $2M$ time slots of transmission with single relay. All the relays use amplify-and-forward (AF) strategy to transmit their received mixtures. Meanwhile, we assume that all nodes are equipped with single antenna and the full channel state information (CSI) is known by all the nodes.

III. Cooperative Transmission

Assuming there are M users, they need $2M$ time slots to exchange their messages with help from single relay described above. In our cooperative transmission, the transmission time consumption is reduced to just M time slots. Furthermore, the properties of the cooperative transmission will be investigated.

A. Outage Probability

In the cooperative transmission protocol, M users transmit their own signal in the first time slot and each relay receive the superposition of M signals. Hence, received signal at the nth

[1] In this paper, the single relay system has been considered only in two places. One is in here, where we consider the single relay system to compare the time consumption of transmission. Another one is in the section of numerical results where we compare the performance of ergodic rate in Fig. 5. In all the other places, "single relay" means one (arbitrary) relay from the selected relays.

[2] we assume that all the relays use physical layer network coding which allows all the users to transmit their signals simultaneously to the relays in the same time slot without interweaving with each other's signal. However, this topic is beyond the scope of this paper and more details can be found in [14], [21], [22] and references therein.

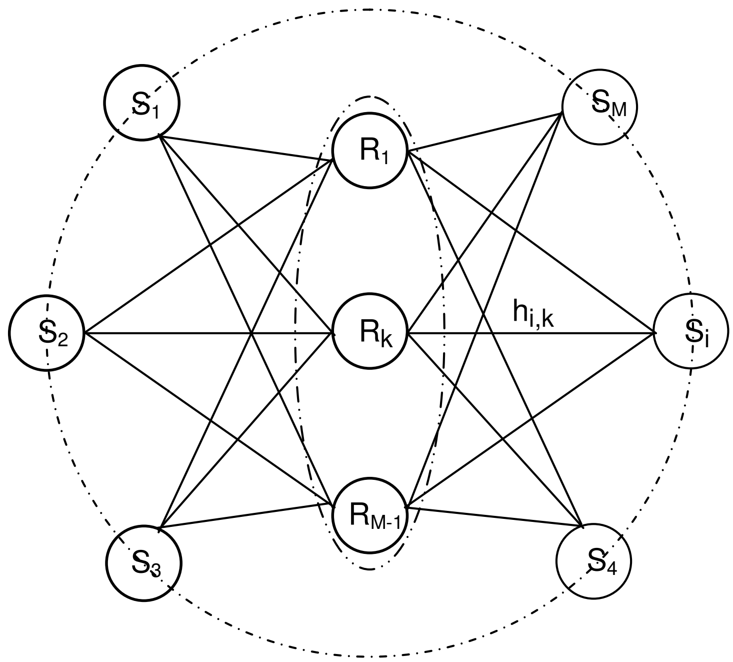

Fig. 1. System Model with multi-way relays.

relay is given by

$$y_{R_n} = \sum_{i=1}^{M} h_{i,n} x_i + \mu_n, \quad n = 1, \ldots, N. \tag{1}$$

where $\mu_n \sim \mathcal{CN}(0,1)$ denotes the background noise of nth relay and $h_{i,n} \sim \mathcal{CN}([\mathbf{\Theta}]_{i,n}, \varepsilon^2)$ denotes the channel coefficient between ith user and nth relay. Each relay normalizes the received signal and forwards the mixture which can be written as

$$\begin{aligned} r_n &= \sqrt{Q}\frac{y_{R_n}}{\sqrt{\mathbb{E}\{|y_{R_n}|^2\}}} \\ &= \eta\left(\sum_{i=1}^{M} h_{i,n} x_i + \mu_n\right), \end{aligned} \tag{2}$$

where

$$\eta = \frac{\sqrt{Q}}{\sqrt{\mathbb{E}\{|y_{R_n}|^2\}}} = \frac{\sqrt{Q}}{\sqrt{\sum_{i=1}^{M} \mathbb{E}\{|h_{i,n} x_i|^2\} + 1}}$$

denotes the scaling factor of each relay which is used to ensure $\mathbb{E}\{|r_n|^2\} = Q$.

During the next $M-1$ time slots, the selected $M-1$ relays are invited respectively to transmit their received mixtures. The details of relay selection will be described in next section. Hence, during the $(M-1)$ time slots, received signal at the ith user is given by

$$\begin{aligned} y_{k,i}^{(\mathrm{R})} &= h_{k,i} r_k + z_{k,i} \\ &= h_{k,i}\eta \left(\sum_{i=1}^{M} h_{i,k} x_i + \mu_k \right) + z_{k,i}, \quad k = 1, \ldots, M-1, \end{aligned} \tag{3}$$

where $h_{k,i} \sim \mathcal{CN}\left([\boldsymbol{\Theta}]_{k,i}, \varepsilon^2\right)$ denotes the channel coefficient between kth relay and ith user. $z_{k,i} \sim \mathcal{CN}(0,1)$ denotes noise imposed on the ith user at the time of receiving signal from kth relay. Eliminating the own signal of ith user, the received signal at the ith user is given by

$$\hat{y}_{k,i}^{(\mathrm{R})} = h_{k,i}\eta \left(\sum_{j=1, j\neq i}^{M} h_{j,k} x_j + \mu_k \right) + z_{k,i}. \tag{4}$$

After $(M-1)$ time slots, the observation at the ith user is expressed as,

$$\begin{aligned} \begin{bmatrix} y_{1,i} \\ \vdots \\ y_{M-1,i} \end{bmatrix} &= \begin{bmatrix} \eta h_{1,i} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \eta h_{M-1,i} \end{bmatrix} \begin{bmatrix} h_{1,1} & \cdots & h_{M(j\neq i),1} \\ \vdots & \ddots & \vdots \\ h_{1,M-1} & \cdots & h_{M(j\neq i),M-1} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{M(j\neq i)} \end{bmatrix} \\ &+ \begin{bmatrix} z_{1,i} + \eta\mu_1 h_{1,i} \\ \vdots \\ z_{M-1,i} + \eta\mu_{M-1} h_{M-1,i} \end{bmatrix} \end{aligned} \tag{5}$$

which is written as

$$\hat{\mathbf{y}}_i^{(\mathrm{R})} = \mathbf{D}_i \mathbf{G}_i \mathbf{s} + \mathbf{w}_i \tag{6}$$

where

$$\hat{\mathbf{y}}_i^{(\mathrm{R})} = \begin{bmatrix} y_{1,i} \\ \vdots \\ y_{M-1,i} \end{bmatrix}, \quad \mathbf{D}_i = \begin{bmatrix} h_{1,i}\eta & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & h_{M-1,i}\eta \end{bmatrix},$$

$$\mathbf{G}_i = \begin{bmatrix} h_{1,1} & \cdots & h_{M(j\neq i),1} \\ \vdots & \ddots & \vdots \\ h_{1,M-1} & \cdots & h_{M(j\neq i),M-1} \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} x_1 \\ \vdots \\ x_{M(j\neq i)} \end{bmatrix}$$

and

$$\mathbf{w}_i = \begin{bmatrix} z_{1,i} + \eta\mu_1 h_{1,i} \\ \vdots \\ z_{M-1,i} + \eta\mu_{M-1} h_{M-1,i} \end{bmatrix}. \tag{7}$$

The principle of zero-forcing (ZF) detection is considered in this system, because a cooperative network can outperform non-cooperative ones at moderate or high SNR, which motivates the use of ZF detection. In particular, at moderate or high SNR, ZF can achieve performance similar to MMSE, but the use of ZF can facilitate performance evaluation significantly [35]. Applying the ZF detection, we have

$$\begin{aligned} \left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1} \mathbf{G}_i^\dagger \mathbf{D}_i^{-1} \mathbf{y}_i^{(\mathrm{R})} &= \mathbf{s} + \left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1} \mathbf{G}_i^\dagger \mathbf{D}_i^{-1} \mathbf{w}_i \\ &= \mathbf{s} + \tilde{\mathbf{w}} \end{aligned} \tag{8}$$

Therefore, after $M-1$ time slots, the effective channel gain at the ith user due to the jth user's signal is given by

$$\begin{aligned} \gamma_{i,j}^{(\mathrm{R})} &= \frac{P}{\mathbb{E}\left\{\tilde{\mathbf{w}}\tilde{\mathbf{w}}^\dagger\right\}} \\ &= \frac{P}{\mathbb{E}\left\{\left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1} \mathbf{G}_i^\dagger \mathbf{D}_i^{-1} \mathbf{w}_i \mathbf{w}_i^\dagger \left(\mathbf{D}_i^{-1}\right)^\dagger \mathbf{G}_i \left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1}\right\}} \\ &= \frac{P}{\left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1} \mathbf{G}_i^\dagger \mathbf{D}_i^{-1} \mathbb{E}\left\{\mathbf{w}_i \mathbf{w}_i^\dagger\right\} \left(\mathbf{D}_i^{-1}\right)^\dagger \mathbf{G}_i \left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1}} \\ &= \frac{P}{\left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1} \mathbf{G}_i^\dagger \mathbf{D}_i^{-1} \boldsymbol{\Phi} \left(\mathbf{D}_i^{-1}\right)^\dagger \mathbf{G}_i \left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1}}, \end{aligned} \tag{9}$$

where P is the transmit power at each relay and

$$\begin{aligned} \boldsymbol{\Phi} &= \mathbb{E}\left\{\mathbf{w}_i \mathbf{w}_i^\dagger\right\} \\ &= \begin{bmatrix} 1+\eta^2|h_{1,i}|^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1+\eta^2|h_{M-1,i}|^2 \end{bmatrix}. \end{aligned}$$

Denoting $\mathbf{D}_i = \begin{bmatrix} h_{1,i}\eta & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & h_{M-1,i}\eta \end{bmatrix}$ and $\mathbf{D}_i^{-1} = \begin{bmatrix} \frac{1}{h_{1,i}\eta} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{h_{M-1,i}\eta} \end{bmatrix}$.

We have

$$\begin{aligned}
\mathbf{D}_i^{-1}\boldsymbol{\Phi}\left(\mathbf{D}_i^{-1}\right)^\dagger &= \begin{bmatrix} \frac{1}{h_{1,i}\eta} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{h_{M-1,i}\eta} \end{bmatrix} \begin{bmatrix} 1+\eta^2|h_{1,i}|^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1+\eta^2|h_{M-1,i}|^2 \end{bmatrix} \\
&\times \begin{bmatrix} \frac{1}{\eta^\dagger h_{1,i}^\dagger} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\eta^\dagger h_{M-1,i}^\dagger} \end{bmatrix} \\
&= \begin{bmatrix} \frac{1+\eta^2|h_{1,i}|^2}{h_{1,i}\eta} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1+\eta^2|h_{M-1,i}|^2}{h_{M-1,i}\eta} \end{bmatrix} \begin{bmatrix} \frac{1}{\eta^\dagger h_{1,i}^\dagger} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\eta^\dagger h_{M-1,i}^\dagger} \end{bmatrix} \\
&= \begin{bmatrix} \frac{1+\eta^2|h_{1,i}|^2}{\eta^2|h_{1,i}|^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1+\eta^2|h_{M-1,i}|^2}{\eta^2|h_{M-1,i}|^2} \end{bmatrix} \\
&= \begin{bmatrix} 1+\frac{1}{\eta^2}\times\frac{1}{|h_{1,i}|^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1+\frac{1}{\eta^2}\times\frac{1}{|h_{M-1,i}|^2} \end{bmatrix}
\end{aligned} \tag{10}$$

Under the assumptions that no relay is scheduled twice and that the used relays have good enough outgoing channels with unity channel gain[3], i.e., $\frac{1}{|h_{k,i}|^2} = 1$, $k = 1, \ldots, M-1$, we have

$$\mathbf{D}_i^{-1}\boldsymbol{\Phi}\left(\mathbf{D}_i^{-1}\right)^\dagger = \left(1+\frac{1}{\eta^2}\right)\mathbf{I}_{M-1}. \tag{11}$$

Therefore, to obtain the tractable analytical expression for the PDF of $\gamma_{i,j}^{(\mathrm{R})}$, we construct an auxiliary signal model as follows:

$$\hat{\mathbf{y}}_i^{(\mathrm{R})} = \mathbf{s} + \tilde{\mathbf{q}} \tag{12}$$

[3]The "unity gain" is assumed so that the used relay selection strategy in our paper can ensure that the channel gains of outgoing channels are equal or even larger than one (by assuming there are large number of relays, ideally the number of relays can go to infinity), to simplify the analysis. Meanwhile, we focus on the lower bound of the performance achieved by the proposed protocol.

which has the new noise covariance matrix as

$$\begin{aligned}\tilde{\mathbf{Q}} &= \left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1} \mathbf{G}_i^\dagger \left(1+\frac{1}{\eta^2}\right) \mathbf{I}_{M-1} \mathbf{G}_i \left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1} \\ &= \left(1+\frac{1}{\eta^2}\right) \left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1} \mathbf{G}_i^\dagger \mathbf{G}_i \left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1} \\ &= \left(1+\frac{1}{\eta^2}\right) \left[\left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1}\right]_{jj} \end{aligned} \tag{13}$$

Therefore, after $M-1$ time slots, the effective channel gain at the ith user due to the jth user's signal is given by

$$\begin{aligned}\gamma_{i,j}^{(\mathrm{R})} &= \frac{P}{\left(1+\frac{1}{\eta^2}\right)\left[\left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1}\right]_{jj}} \\ &= \frac{\rho}{\left[\left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1}\right]_{jj}}, \end{aligned} \tag{14}$$

where $\rho = \frac{P}{\left(1+\frac{1}{\eta^2}\right)}$.

Proposition 1. *The effective channel gains* $\gamma_{i,j}^{(\mathrm{R})} = \frac{\rho}{\left[\left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1}\right]_{jj}}$, $j = 1, \ldots, M, j \neq i$ *follow noncentral Chi-squared distribution and the probability density function (p.d.f.) can be expressed as*

$$f_{\gamma_{i,j}^{(\mathrm{R})}}(\gamma) = \frac{1}{2\rho\varepsilon^2} \left(\frac{\gamma}{\rho[\mathbf{\Theta}]_{i,j}^2}\right)^{-\frac{1}{4}} e^{-\frac{[\mathbf{\Theta}]_{i,j}^2 + \frac{\gamma}{\rho}}{2\varepsilon^2}} I_{-\frac{1}{2}} \left(\frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon^2} \sqrt{\frac{\gamma}{\rho}}\right) \tag{15}$$

where $I_a(x)$ *is the modified Bessel function of the first kind.*

Proof: See Appendix A. ∎

This proposition is the basis of following analysis in this paper and was derived by the random matrix theory of noncentral Wishart matrix.

Proposition 2. *The cumulative distribution function (c.d.f.) of effective channel gains,* $\gamma_{i,j}^{(\mathrm{R})}$*, is given by*

$$F\left(\gamma_{i,j}^{(\mathrm{R})} \leq x\right) = 1 - Q_{\frac{1}{2}}\left(\frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon}, \frac{\sqrt{x/\rho}}{\varepsilon}\right) \tag{16}$$

where $Q_\beta(a,b)$ *is the generalized Marcum Q-function.*

Proof: See Appendix B. ∎

By using Proposition 2, the following proposition can be derived.

Proposition 3. *The outage probability of* $\gamma_{i,j}^{(\mathrm{R})}$ *with threshold* γ_{th} *is given by*

$$P_{out}\left(\gamma_{i,j}^{(\mathrm{R})} \leq \gamma_{th}\right) = 1 - Q_{\frac{1}{2}}\left(\frac{[\boldsymbol{\Theta}]_{i,j}}{\varepsilon}, \frac{\sqrt{\gamma_{th}/\rho}}{\varepsilon}\right). \tag{17}$$

Proof: This can be derived easily from Proposition 2. ■

B. Ergodic Achievable Rate

The ergodic achievable rate at the ith user due the jth user's signal is given by

$$R_{i,j}^{\mathrm{zf-relay}} = \mathbb{E}\left\{\log_2\left(1+\gamma_{i,j}^{(\mathrm{R})}\right)\right\}. \tag{18}$$

The following proposition presents the analytical expression of the ergodic sum rate of the ith user considering all the selected relays.

Proposition 4. *The ergodic sum rate of* i*th user is given by*

$$\mathcal{C} = \frac{M-1}{\ln 2} e^{-\frac{[\boldsymbol{\Theta}]_{i,j}^2}{2\varepsilon^2}} \sum_{k=0}^{\infty} \frac{2^{-k}\varepsilon^{-2k}[\boldsymbol{\Theta}]_{i,j}^{2k}}{k!\Gamma\left(k+\frac{1}{2}\right)} G_{3,2}^{1,3}\left[2\rho\varepsilon^2 \middle| \begin{array}{c} \frac{1}{2}-k,1,1 \\ 1,0 \end{array}\right], \tag{19}$$

where $G_{p,q}^{m,n}\left[x \middle| \begin{array}{c} a_1,\cdots,a_p \\ b_1,\cdots,b_q \end{array}\right]$ *is the Meijer's G-function [36, Eq. (9.3)] and* $\Gamma(x)$ *is the gamma function [36, Eq. (8.31)].*

Proof: We know that

$$\begin{aligned}\mathcal{C} =& E\left\{\sum_{j=1}^{M-1}\log_2\left(1+\gamma_i^{(\mathrm{R})}\right)\right\} \\ =& (M-1)E\left\{\log_2\left(1+\gamma_i^{(\mathrm{R})}\right)\right\} \\ =& \frac{(M-1)}{\ln 2}\int_0^{\infty} G_{2,2}^{1,2}\left[\gamma_i^{(\mathrm{R})} \middle| \begin{array}{c} 1,1 \\ 1,0 \end{array}\right] f_{\gamma_{i,j}^{(\mathrm{R})}}\left(\gamma_i^{(\mathrm{R})}\right) d\gamma_i^{(\mathrm{R})}.\end{aligned} \tag{20}$$

Using [36, Eq. (8.445)], the modied Bessel function $I_{-\frac{1}{2}}\left(\frac{[\boldsymbol{\Theta}]_{i,j}}{\varepsilon^2}\sqrt{\frac{\gamma}{\rho}}\right)$ can be expressed as

$$I_{-\frac{1}{2}}\left(\frac{[\boldsymbol{\Theta}]_{i,j}}{\varepsilon^2}\sqrt{\frac{\gamma}{\rho}}\right) = \sum_{k=0}^{\infty}\frac{1}{k!\Gamma(k+\frac{1}{2})}\left(\frac{[\boldsymbol{\Theta}]_{i,j}}{2\varepsilon^2}\sqrt{\frac{\gamma}{\rho}}\right)^{2k-\frac{1}{2}}.$$

Therefore, we derive Proposition 4 with the help of [36, Eq. (7.813.1)]. ■

To gain better insight into the ergodic sum rate performance and reduce the computation complexity, we investigate the ergodic sum rate at the high SNR regime in the following proposition.

Proposition 5. *At high SNR regime, the ergodic sum rate can be approximated by*

$$\mathcal{C}^{high-SNR} = \frac{(M-1)}{\ln 2} \sum_{k=0}^{\infty} 2^{-k} \varepsilon^{-2k} [\mathbf{\Theta}]_{i,j}^{2k} e^{-\frac{[\mathbf{\Theta}]_{i,j}^2}{2\varepsilon^2}} \frac{1}{k!} \left[\psi\left(k+\frac{1}{2}\right) + \ln(2\rho\varepsilon^2) \right], \tag{21}$$

where $\psi(x)$ is the Euler psi function [36, Eq. (8.36)].

Proof: At high SNR regime,

$$\mathcal{C}^{high-SNR} \approx \frac{(M-1)}{\ln 2} \int_0^{\infty} \ln\left(\gamma_i^{(\mathrm{R})}\right) f_{\gamma_{i,j}^{(\mathrm{R})}}(\gamma_i^{(\mathrm{R})}) d\gamma_i^{(\mathrm{R})}, \tag{22}$$

with the help of [36, Eq. (8.445), Eq. (4.352.1)], Proposition 5 can be derived after some algebraic manipulations. ■

C. Diversity Order

Considering the number of relays to be large enough in this scenario, we can randomly separate the relays into L different groups, $L = \left\lfloor \frac{N}{M-1} \right\rfloor$ where $\lfloor x \rfloor$ denotes the largest integer which is smaller than x, and there are $M-1$ relays in each group. Based on this, L groups of relays are independent of each other. We denote the average channel gain in each group as $\left\{\gamma_{i,1}^{(\mathrm{R})}, \gamma_{i,2}^{(\mathrm{R})}, \ldots, \gamma_{i,L}^{(\mathrm{R})}\right\}$ where $\gamma_{i,n}^{(\mathrm{R})}$ denote the average channel gains (which can be seen as the channel gain of the channel between an arbitrary grouped relay and the ith user) in group n. Considering the order statistics and assuming $\gamma_{min} = \min\left\{\gamma_{i,1}^{(\mathrm{R})}, \gamma_{i,2}^{(\mathrm{R})}, \ldots, \gamma_{i,L}^{(\mathrm{R})}\right\}$ and $\gamma_{max} = \max\left\{\gamma_{i,1}^{(\mathrm{R})}, \gamma_{i,2}^{(\mathrm{R})}, \ldots, \gamma_{i,L}^{(\mathrm{R})}\right\}$, the p.d.f. of γ_{min} and γ_{max} are given by

$$\begin{aligned} f_{\gamma_{i,j}^{(\mathrm{R})}}(\gamma_{min}) &= L f_{\gamma_{i,j}^{(\mathrm{R})}}(\gamma) \left[1 - F_{\gamma_{i,j}^{(\mathrm{R})}}(\gamma)\right]^{L-1}; \\ f_{\gamma_{i,j}^{(\mathrm{R})}}(\gamma_{max}) &= L f_{\gamma_{i,j}^{(\mathrm{R})}}(\gamma) \left[F_{\gamma_{i,j}^{(\mathrm{R})}}(\gamma)\right]^{L-1}. \end{aligned} \tag{23}$$

Proposition 6. *Considering random separation, the outage probability of the relay group with maximum channel gain γ_{max} is given by*

$$P_{out}(\gamma_{max} \leq \gamma_{th}) = \left[1 - Q_{\frac{1}{2}}\left(\frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon}, \frac{\sqrt{\gamma_{th}/\rho}}{\varepsilon}\right)\right]^L. \tag{24}$$

Proof: This can be derived by using eq. (23). ∎

Considering the definition of marcum Q-function and the basic integration property of boundary, there exist real numbers $t \leq T$, so that

$$t\left(\frac{\sqrt{\gamma_{th}/\rho}}{\varepsilon}\right)^{L} \leq \left[1 - Q_{\frac{1}{2}}\left(\frac{[\boldsymbol{\Theta}]_{i,j}}{\varepsilon}, \frac{\sqrt{\gamma_{th}/\rho}}{\varepsilon}\right)\right]^{L} \leq T\left(\frac{\sqrt{\gamma_{th}/\rho}}{\varepsilon}\right)^{L}$$

where it shows the diversity order is L. The algebraic manipulations and proof are omitted here. It is worth to notice that the diversity order will increase if more relays are involved in the scheme. Moreover, the diversity order based on the random selection is a lower bound of the diversity order of using semiorthogonal selection.

IV. Semiorthogonal Relay selection

In this section, we consider how to select $M-1$ relays to construct the full rank channel matrix $\mathbf{H}_S$. Semiorthogonal selection (SS) is applied which can select the relay with the best channel gain and all the selected relays orthogonal to each other as much as possible. Full rank channel matrix will guarantee all the users can decode the messages of others successfully and can potentially provide some fairness among the multiple source nodes. Because of these, the semiorthogonal selection algorithm [31] is applied in the form of pseudo-code.

Algorithm 1 Semiorthogonal Relays Selection

1: **procedure** Semiorthogonal Relays Selection

2: **Initial**: $R = \emptyset$, $\mathbf{H} = \mathbf{h}_1, \ldots, \mathbf{h}_N$, where R is the set of selected relays, $\emptyset$ is the empty set, S_β is the set of index of subchannel in the βth selection and $\mathbf{h}_i$ is the subchannel vector from each relay to all users;

3: **Calculation**: $\mathbf{g}_1 = \mathbf{h}_1$, $\mathbf{g}_i = \mathbf{h}_i - \sum_{j=1}^{i-1} \frac{\mathbf{g}_j^\dagger \mathbf{h}_i}{\|\mathbf{g}_j\|^2}\mathbf{g}_j$, where the component of $\mathbf{h}_i$ orthogonal to the subspace which is spanned by vectors $\{\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g}_{i-1}\}$;

4: **Select the βth relay**: $k = \text{argmax}_{i \in S_\beta} \|\mathbf{g}_i\|$, $R \leftarrow R \cup k$, $\mathbf{H}_S(:, \beta) = \mathbf{h}_k$ is the βth column of $\mathbf{H}_S$, $\mathbf{H}(:, k) = \mathbf{0}$, $\mathbf{g}_\beta = \mathbf{g}_k$;

5: **If** size of set R is less than $M-1$, **improve** the set of index $S_{\beta+1}$ for next selection by $S_{\beta+1} = \left\{\lambda \in S_\beta, \lambda \neq k, \frac{|\mathbf{h}_\lambda^\dagger \mathbf{g}_\beta|}{\|\mathbf{h}_\lambda\|\|\mathbf{g}_\beta\|} < \alpha\right\}$, $\beta \leftarrow \beta + 1$, where $\alpha = 0.4$ [31]. **If** $S_{\beta+1} \neq \emptyset$, go to Step 3);

6: **Else Quit**

After repeating step two to four for $M-1$ times, we select $M-1$ relays from N relays and construct the full rank channel matrix $\mathbf{H}_S$. The reason is that $\mathbf{g}_i, i = 1, 2, \ldots, M-1$ are orthogonal vectors created by step 3) in the pseudo-code. In step 5), we improve the selection index set S_β by dropping off the subchannels that are not semiorthogonal to one of the $\mathbf{g}_1, \ldots, \mathbf{g}_{\beta-1}$ by the condition[4] $\frac{|\mathbf{h}_\lambda^\dagger \mathbf{g}_\beta|}{\|\mathbf{h}_\lambda\|\|\mathbf{g}_\beta\|} = \cos\theta < \alpha$, where θ is the angle between vectors $\mathbf{h}_\lambda$ and $\mathbf{g}_\beta$. When $\mathbf{H}_S$ is full rank, all the users can decode the other $M-1$ users' information by solving the $M-1$ equations of received mixed signals.

In the following, we analyze the computational complexity of the relays selection above. The computational complexity in each step can be given by

- It takes $12M(i-1)$ real multiplications and $(10M-3)(i-1)+2M$ real additions to compute $\mathbf{g}_i$ in step 3). Assuming N_i is the size of S_β, The total flop count in this step is $(22M-3)(i-1)N_i + 2MN_i$.
- In step 4), it takes $N_i(2M+1)$ real multiplications and $N_i(2M-1)$ real additions to compute all $\|\mathbf{g}_i\|$ for $i \in S_\beta$. In addition, it takes $N_i - 1$ real comparisons to select a relay. The total flop count is $(4N_iM) + (N_i - 1)$ in this step.
- In step 5), during the ith relay selection, it takes $(N_i-1)(8M+4)$ real multiplications, $(N_i-1)(8M-3)$ real additions and $N_i - 1$ real comparisons to compute $S_{\beta+1}$. Thus the flop count is $(N_i-1)(16M+1)$ in this step.

Since the exact closed-form expression of N_i is unknown, the exact flop count of the relays selection could be calculated by simulation. However, it should be noted that $N_i \le N$ and $N \ge M$ in our system. In this way, the upper bound of the flop count of the relays selection can be given by

$$\begin{aligned}\varepsilon \le& \sum_{i=1}^{M-1} \left[(22M-3)(i-1)N + (N-1)(16M+1) + 6NM + N - 1\right] \\ =&\frac{1}{2}\left(4 + 28M - 32M^2 - 10N + 13MN - 25M^2N + 22M^3N\right) \\ =&\mathcal{O}\left(M^3N\right), \end{aligned} \tag{25}$$

where ε denotes the flop count of the relays selection.

[4]It should be noticed that α changes from 0.2 to 0.4 when the total number of relays changes from 100000 to 100. It means we should relax the condition of α when the searching space is just hundreds or less relays. $\alpha = 0.4$ has been chosen in this paper according to the system assumption and the results have shown that this condition can be satisfied in this system.

According to the Lemma 2 in [31], the average channel gain between the βth selected relay and the users, γ_β, is lower bounded by

$$\gamma_\beta > \frac{\|\mathbf{g}_\beta\|^2}{1 + \frac{(M-2)^4\alpha^2}{1-(M-2)\alpha^2}}. \quad (26)$$

Considering the lower bound in-eq. (26) and the channel gain of eq. (14), the semiorthogonal selection always chooses the ith relay which has $\|\mathbf{g}_{max}\|$ and γ_{max} first. Analyzing the statistical characters of γ_{max} can help us to understand the performance of the system, because it will determine the properties of the whole system when the number of relays is large enough. In next section, extreme value theory will be applied to get deep insight of the properties of γ_{max} based on semiorthogonal relays selection.

V. Maximum Channel Gain Analysis

In the following, the asymptotic behavior of the distribution of the maximum channel gain γ_{max} of the best relay is investigated. Extreme value theory [34], [38]–[40] is used to evaluate the upper and lower bounds of γ_{max}. First, it is proved that the p.d.f. of γ_{max} converges to Gumbel distribution as a sufficient condition of using extreme value theory. Second, the unique root x^* for the equation $1 - F_{\gamma_{i,j}}(x^*) = \frac{1}{N}$ is derived. Finally, the value of γ_{max} can be bounded by the unique solution of x^*. Meanwhile, the bounds of ergodic rate is derived based on the bounds of γ_{max}.

Generally speaking, extreme value theory is used to deal with extreme values, such as maxima or minima of asymptotic distributions. Assuming $\gamma_{i,j}, j = 1, \ldots, N$ are N i.i.d random variables of the effective channel gains from the ith user to jth relay (equally as the channel from the relay to the user). Different to the previous works the addressed variable is not a Chi-square variable, but the non-central Chi-square variable.

By extreme value theory [39], [40], if there exist constants $a \in R$, $b > 0$, and some non-degenerate distribution function $G(x)$ such that the distribution of $\frac{\gamma_{max}-a}{b}$ converges to $G(x)$, then $G(x)$ converges to one of the three standard extreme value distributions: Frechet, Webull, and Gumbel distributions, where $\gamma_{max} = \max\{\gamma_{i,1}, \ldots, \gamma_{i,N}\}$.

There are only three possible non-degenerate limiting distributions for maxima, which can be expressed as

- $G(x) = e^{-e^{-x}}$;

- $G(x) = e^{-x^{-\alpha}} u(x),\ \alpha > 0;$
- $G(x) = \begin{cases} e^{-(-x)^{\alpha}}, & \alpha > 0, \quad x \leq 0; \\ 1, & x \geq 0. \end{cases}$

where $u(x)$ is the step function.

The distribution of $\gamma_{i,j}$, $F(x)$, determines the exact limiting distribution. A distribution function $F(x)$ belongs to the domain of attraction of the limiting distribution, if that distribution function $F(x)$ results in one limiting distribution for extreme.

Lemma 1. *(Gnedenko, 1947) Assume $x_1, x_2, \ldots, x_n$ are i.i.d. random variables with distribution function $F(x)$. Define $\psi(x) = \sup\{x : F(x) < 1\}$. Let there be a real number x_1 such that, for all $x_1 \leq x \leq \psi(x)$, $f(x) = F'(x)$ and $F''(x)$ exist and $f(x) \neq 0$. If*

$$\lim_{x \to \psi(x)} \frac{d}{dx} \left[\frac{1 - F(x)}{f(x)} \right] = 0, \tag{27}$$

then there exist constants a and $b > 0$ such that $\frac{\gamma_{max} - a}{b}$ uniformly converges in distribution to a normalized Gumbel random variable as $n \to \infty$. The normalizing constants a and b are determined by

$$a = F^{-1}\left(1 - \frac{1}{N}\right),$$
$$b = F^{-1}\left(1 - \frac{1}{Ne}\right) - F^{-1}\left(1 - \frac{1}{N}\right). \tag{28}$$

where $F^{-1}(x) = \inf\{y : F(y) \geq x\}$.

For a random variable X with the normalized Gumbel distribution, whose distribution function is given by

$$G(x) = e^{-e^{-x}}, \quad -\infty < x < \infty, \tag{29}$$

it follows that

$$E(X) = E_0, \tag{30}$$

$$Var(X) = \frac{\pi^2}{6}, \tag{31}$$

where $E_0 \approx 0.5772$ is the Euler constant [38].

To ensure applicability of those extreme value results, we define the growth function as $g(x) = \frac{1 - F_{\gamma_{i,j}}(x)}{f_{\gamma_{i,j}}(x)}$, where $F_{\gamma_{i,j}}(x)$ and $f_{\gamma_{i,j}}(x)$ are the c.d.f. and p.d.f. of the random variable $\gamma_{i,j}$ given by Proposition 1 and 2, respectively.

It is very important to verify that the growth function can converge to a constant when $x \to \infty$.

Proposition 7. *In our scenario, the growth function*

$$g(x) = \frac{1 - F_{\gamma_{i,j}}(x)}{f_{\gamma_{i,j}}(x)} \to \frac{1 - 2\rho\varepsilon^2}{2\rho\varepsilon^2},$$

when $x \to \infty$.

Proof: See Appendix C. ∎

According to Proposition 7, $\lim_{x\to\infty} g(x) = c > 0$ when $\rho < \frac{1}{2\varepsilon^2} \Leftrightarrow \frac{P}{\left(1+\frac{1}{\eta^2}\right)} < \frac{1}{2\varepsilon^2}$.

It turns out that the class of distribution functions for our scenario in this paper is the type of normalized Gumbel distribution as $N \to \infty$. Therefore, we further look into sufficient conditions on the distribution of $\gamma_{i.j}$, such that the distribution of maximum is Gumbel distribution.

Given the existence of limit of the growth function, we also need to find x^* which is the unique root for the equation $1 - F_{\gamma_{i,j}}(x^*) = \frac{1}{N}$ and it will be used to bound the value of $\gamma_{i,j}$ [39]. It should be noticed that x^* is unique because the c.d.f. $F_{\gamma_{i,j}}(x)$ is continuous and strictly increasing for $x \geq 0$.

Proposition 8. *The maximum value of channel gains,* $\gamma_{i,j}, j = 1, \ldots, N$ *which are i.i.d. random variables satisfies*

$$P\Bigg(\log N - \log\log N + \mathcal{O}(\log\log\log N) \\ \leq \max_{1\leq j\leq N} \gamma_{i,j} \leq \log N + \log\log N + \mathcal{O}(\log\log\log N)\Bigg) > 1 - \mathcal{O}\left(\frac{1}{\log N}\right). \tag{32}$$

Proof: See Appendix D. ∎

The proposition derives lower and upper bound of the maximum value of the channel gain after semiorthogonal relays selection. It is obvious that the performance of the system is a monotone increasing function depending on the number of total relays. The bounds, derived in Proposition 8, are significant for analyzing the properties of the system. Such extreme value results can be used to bound the outage probability and ergodic rate, such as the rate based on the maximum channel gain is bounded by

$$P\Bigg(\log(1 + \log N - \log\log N) \leq C\left(\max_{1\leq j\leq N} \gamma_{i,j}\right) \\ \leq \log(1 + \log N + \log\log N)\Bigg) > 1 - \mathcal{O}\left(-\log\log N\right). \tag{33}$$

VI. Numerical results

In this section, we provide the analytical results derived in the previous sections which are verified by Monte Carlo simulations. Note that in all simulations, unless otherwise specified, we assume that $K = 10$, $[\boldsymbol{\Theta}]_{i,j} = \sqrt{\frac{K}{K+1}}$ and $\varepsilon = \sqrt{\frac{1}{K+1}}$.

Fig. 2 shows the outage probability of a random relay in the multi-way relaying system for different values of factor K with threshold $\gamma_{th} = 5$dB based on Proposition 3. It is seen that the analytical results are in perfect agreement with the Monte Carlo simulation results, confirming the correctness of the analytical expressions. The outage probability decreases with increasing K but changes slowly when K is small. It is because the stronger line-of-sight will improve the performance of the system when considering the single input single output (SISO) scenario. When $[\boldsymbol{\Theta}]_{i,j} = \sqrt{\frac{K}{K+1}} \simeq 1$ as $K \to \infty$, the outage probability converges to $1 - Q_{\frac{1}{2}}\left(\frac{1}{\varepsilon}, \frac{\sqrt{\gamma_{th}/\rho}}{\varepsilon}\right)$, which is a lower bound when K tends to infinity. Moreover, the slope of the outage curves declines when K decreases in the low SNR regime which also fits our expectation when considering Rician fading channels. However, all the curves will have the same slope at high SNR regime, which means the LOS factor K does not affect the slopes of the outage curves when SNR is large.

Fig. 3 depicts the ergodic sum rate of multi-way relaying system with different numbers of users based on Proposition 4 and high-SNR approximation of ergodic sum rate in Proposition 5. The ergodic sum rate increases when more users are in this system, but the decoding becomes more complex. However, the effect of M on ergodic sum rate reduces when M increases. The ergodic sum rate increases sharply when both of SNR and M are large. In addition, the high SNR approximation works quite well when SNR is large, especially the computation complexity is reduced significantly that provides significant computational advantage. Moreover, the slopes of the ergodic sum rate curves can be derived by the high SNR approximations.

Fig. 4 shows the performance comparison between relays selection based on semiorthogonal, random relays selection methods and exhaustive search[5]. The special case is presented when $M = 3$ and $\gamma_{th} = 10$dB. One of the straightforward strategies for maximizing the sum rate is to carry out exhaustive search, whereas our semiorthogonal approach yields less computational

[5]Here, we define the outage probability of the whole system as: the system is in outage if and only if the maximum channel gain (the relay with the best channel) is in outage.

complexity. However, it shows that the exhaustive search performs better when SNR increases, but there is no obvious advantage at low SNR regime compared to semiorthogonal selection. Meanwhile, the performance is improved by the semiorthogonal relays selection method compared to random selection which is because the semiorthogonal method selects the relays with the channels which between the relays and users are as orthogonal as possible. This result confirms that the users can decode their messages correctly and improve the outage probability performance when semiorthogonal selection is applied. It is obvious that the semiorthogonal selection method will be more effective when there are more candidate relays to choose from. Meanwhile, Fig. 5 presents the comparison of ergodic rate between general single relay system (only one relay in the system) and the selected multi-relay scheme during the same time slots.

It is worth to notice that Fig. 2 and Fig. 4 are based on different scenarios. Fig. 2 presents the outage probability of average channel gain for unbiased randomly selected relays. It means that we consider the outage probability of single average channel gain of the system. Also, the outage probability shown in Fig. 2 is independent of relay selection which means it has the same properties as single relay system. On the other hand, Fig. 4 is the outage probability when the whole group of selected relays are considered, according to the semiorthogonal relays selection, which means the curves shown in Fig. 4 are the outage probability of the whole system.

Considering the multiple relay scenario as in Fig. 4, the outage probability is presented for different values of parameter K with $M = 5$, $N = 10$ and threshold $\gamma_{th} = 10$dB in Fig. 6. It is shown that the outage probability increases with K, but the effect of K reduces when K is large. When the whole system with multiple sources and relays is considered, the increasing K will degrade the performance of the system. In this case, it is same as the multiple input multiple output (MIMO) system where the Rician factor K represents the ratio between the deterministic (specular) and the random (scattered) energies. The performance will decrease with K, because the increase in K emphasizes the deterministic part of the channels but the deterministic channels are of rank 1.

The upper and lower bound of the maximum channel gain $\gamma_{i,j}, j = 1, 2, \ldots, N$ based on the formula (32) are presented in Fig. 7. The difference between the lower bound and upper bound is less than 3 when $N = 200$, which means the two bounds which have been derived by extreme value theory are tight. Also, from the two bounds, it can be noticed that the $\max_j \gamma_{i,j}, j = 1, 2, \ldots, N$ increases quickly when N is less than 60, but will converge when N goes to infinity.

From the simulation result, It shows the maximum channel gain close to the upper bound when N is more, and converge to the lower bound when N is large.

Fig. 8 presents the upper and lower bounds of ergodic rate based on the bounds of $\max_j \gamma_{i,j}, j = 1, 2, \ldots, N$ with different N. It should be noticed that the curves in Fig. 8 are the rates of the single maximum channel gain calculated using lower and upper bounds provided in formula (32). It is expected that the bounds of ergodic sum rate of the system is equal to $M-1$ times of the values in the figure, because the channel gain of each selected relay will be close to $\max_j \gamma_{i,j}, j = 1, 2, \ldots, N$ when N is large enough. The difference between the upper and lower bounds is less than 0.6. Moreover, the slopes of the bounds converge to zero when N increases.

VII. Conclusions

In this paper, multi-way relaying scenario was studied with multiple sources and relays. The new scenario presented in this paper reduces the transmit time significantly compared to the traditional single relay transmission. In order to reduce the transmit time, $M-1$ relays were selected to help M users to exchange their information. For random relays selection, the analytical expression of outage probability and ergodic sum rate were derived based on the statistical property of the average channel gain. Meanwhile, the approximation of ergodic sum rate was investigated at high SNR regime to gain better insight into this system and simplify the calculation. Based on our network coding scheme, the multi-way relaying scenario has achieved diversity order of L which increases with the total number of relays and is a lower bound of the diversity order based on semiorthogonal selection. Moreover, the semiorthogonal relays selection method was applied to select the relays to guarantee that all the users can decode others' information and improve the properties of the system. In addition, the performance of random and semiorthogonal relays selection methods were compared through outage probability. Furthermore, the maximum channel gain was studied by extreme value theory and tight upper and lower bounds were derived. Especially, the maximum channel gain is bounded by $\log N + \log\log N + \mathcal{O}(\log\log\log N)$, where N is the total number of relays. The simulation and analytical results show that the multi-way relaying protocol not only reduces the transmission time, but also improves system properties.

APPENDIX A
PROOF OF PROPOSITION 1

Suppose $\widetilde{\mathbf{G}_i}$ is the matrix $\mathbf{G}_i$ without jth column $\mathbf{g}_j$, we have

$$\gamma_{i,j}^{(\mathrm{R})} = \frac{\rho}{\left[\left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)^{-1}\right]_{jj}} = \rho \frac{\det\left(\mathbf{G}_i^\dagger \mathbf{G}_i\right)}{\det\left(\widetilde{\mathbf{G}_i}^\dagger \widetilde{\mathbf{G}_i}\right)}, \tag{34}$$

using the property of the block matrices determinant, we have

$$\begin{aligned}\gamma_{i,j}^{(\mathrm{R})} &= \rho\left[\mathbf{g}_j^\dagger \mathbf{g}_j - \mathbf{g}_j^\dagger \widetilde{\mathbf{G}_i}\left(\widetilde{\mathbf{G}_i}^\dagger \widetilde{\mathbf{G}_i}\right)^{-1} \widetilde{\mathbf{G}_i}^\dagger \mathbf{g}_j\right] \\ &= \rho \mathbf{g}_j^\dagger \left[\mathbf{I}_{M-1} - \mathbf{P}_{M-1}\right] \mathbf{g}_j, \end{aligned} \tag{35}$$

where

$$\mathbf{P}_{M-1} = \widetilde{\mathbf{G}_i}\left(\widetilde{\mathbf{G}_i}^\dagger \widetilde{\mathbf{G}_i}\right)^{-1} \widetilde{\mathbf{G}_i}^\dagger. \tag{36}$$

We note that matrix $(\mathbf{I}_{M-1} - \mathbf{P}_{M-1})$ is a Hermitian matrix, perpendicular to matrix $\widetilde{\mathbf{G}_i}^\dagger$ and independent of $\mathbf{g}_j$. Considering $\mathbf{G}_i \sim \mathcal{CN}\left(\mathbf{\Theta}, \varepsilon^2 \mathbf{I}\right)$, $\mathbf{g}_j^\dagger \left[\mathbf{I}_{M-1} - \mathbf{P}_{M-1}\right] \mathbf{g}_j$ is distributed as noncentral Wishart distribution $\mathrm{W}_1(1, \varepsilon^2 \mathbf{I}, \mathbf{\Omega})$, where $\mathbf{\Omega} = \mathbf{\Theta}^\dagger \mathbf{\Theta}$ is the noncentral parameter, i.e.,

$$\alpha = \mathbf{g}_j^\dagger \left[\mathbf{I}_{M-1} - \mathbf{P}_{M-1}\right] \mathbf{g}_j$$

is a noncentral Chi-squared variable distributed as

$$f(\alpha) = \frac{1}{2\varepsilon^2}\left(\frac{\alpha}{[\mathbf{\Theta}]_{i,j}^2}\right)^{-\frac{1}{4}} e^{-\frac{[\mathbf{\Theta}]_{i,j}^2 + \alpha}{2\varepsilon^2}} I_{-\frac{1}{2}}\left(\frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon^2}\sqrt{\alpha}\right) \tag{37}$$

applying the change of variable, $\gamma_{i,j}^{(\mathrm{R})} = \rho\alpha$, we derive the p.d.f. of effective channel gains shown in the Proposition 1.

APPENDIX B
PROOF OF PROPOSITION 2

By the definition of c.d.f., we have

$$F\left(\gamma_{i,j}^{(\mathrm{R})} \le x\right) = \int_0^x f_{\gamma_{i,j}^{(\mathrm{R})}}(\gamma) d\gamma_{i,j}^{(\mathrm{R})}. \tag{38}$$

Assuming $\gamma_{i,j}^{(\mathrm{R})} = \varepsilon^2 \rho y^2$, we have

$$F\left(\gamma_{i,j}^{(\mathrm{R})} \le x\right) = \int_0^{\frac{\sqrt{x/\rho}}{\varepsilon}} y \left(\frac{\varepsilon y}{[\mathbf{\Theta}]_{i,j}}\right)^{-\frac{1}{2}} e^{-\frac{\frac{[\mathbf{\Theta}]_{i,j}^2}{\varepsilon^2} + y^2}{2}} I_{-\frac{1}{2}}\left(\frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon} y\right) dy. \tag{39}$$

With the help of [37, Eq. (2.3-37)], we derive Proposition 2 directly.

Appendix C
Proof of Proposition 7

Using L'Hospital's rule, we have

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{1-F_{\gamma_{i,j}}(x)}{f_{\gamma_{i,j}}(x)}$$
$$= \lim_{x\to\infty} \frac{(1-F_{\gamma_{i,j}}(x))'}{f'_{\gamma_{i,j}}(x)}$$
$$= \lim_{x\to\infty} -\frac{f_{\gamma_{i,j}}(x)}{f'_{\gamma_{i,j}}(x)}$$
$$= \lim_{x\to\infty} -\frac{\frac{1}{2\rho\varepsilon^2}\left(\frac{x}{\rho[\mathbf{\Theta}]_{i,j}^2}\right)^{-\frac{1}{4}} e^{-\frac{[\mathbf{\Theta}]_{i,j}^2+\frac{x}{\rho}}{2\varepsilon^2}} I_{-\frac{1}{2}}\left(\frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon^2}\sqrt{\frac{x}{\rho}}\right)}{\left(\frac{1}{2\rho\varepsilon^2}\left(\frac{x}{\rho[\mathbf{\Theta}]_{i,j}^2}\right)^{-\frac{1}{4}} e^{-\frac{[\mathbf{\Theta}]_{i,j}^2+\frac{x}{\rho}}{2\varepsilon^2}} I_{-\frac{1}{2}}\left(\frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon^2}\sqrt{\frac{x}{\rho}}\right)\right)'}. \tag{40}$$

Using the following identity

$$I_{-\frac{1}{2}}\left(\frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon^2}\sqrt{\frac{x}{\rho}}\right) = \sqrt{\frac{2\varepsilon^2}{\pi[\mathbf{\Theta}]_{i,j}}\sqrt{\frac{\rho}{x}}}\cosh\left(\frac{[\mathbf{\Theta}]_{i,j}\sqrt{\frac{x}{\rho}}}{\varepsilon^2}\right), \tag{41}$$

we have

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} -\left(-\frac{3}{4}x^{-1} - \frac{1}{2\rho\varepsilon^2} + \frac{e^x+e^{-x}}{e^x-e^{-x}}\right)$$
$$= \frac{1-2\rho\varepsilon^2}{2\rho\varepsilon^2}. \tag{42}$$

Appendix D
Proof of Proposition 8

The following Lemma has been used to proof the Theorem,

Lemma 2. *(Uzgoren, 1956) Let $x_1, x_2, \ldots, x_n$ be a sequence of i.i.d. positive random variables with continuous and strictly positive p.d.f. $f(x)$ for $x > 0$ and c.d.f. of $F(x)$. Also, assume that $g(x)$ be the growth function. Then if*

$$\lim_{x\to\infty} g(x) = c > 0, \tag{43}$$

then,

$$\begin{aligned} &\log\{-\log F^n(x^* + ug(x^*))\} \\ =& - u - \frac{u^2 g'(x^*)}{2!} - \cdots - \frac{u^m g^{(m)}(x^*)}{m!} + \mathcal{O}\left(\frac{e^{-u+\mathcal{O}(u^2 g'(x^*))}}{n}\right) \end{aligned} \tag{44}$$

where x^* *is defined before.*

Considering the scenario in this paper, such a unique root can be found by solving the equation

$$\frac{1}{N} = 1 - F_{\gamma_{i,j}}(x^*). \tag{45}$$

After submitting $F_{\gamma_{i,j}}(x^*)$ in this equation, we have

$$\frac{1}{N} = Q_{\frac{1}{2}}\left(\frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon}, \frac{\sqrt{x/\rho}}{\varepsilon}\right) \tag{46}$$

when x is large enough, we can approximate the equation as [41], [42]

$$\begin{aligned} \frac{1}{N} &= \left(\frac{\sqrt{x/\rho}}{[\mathbf{\Theta}]_{i,j}}\right)^{1/2-1/2} Q\left(\frac{\sqrt{x/\rho}}{\varepsilon} - \frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon}\right), \\ &= Q\left(\frac{\sqrt{x/\rho}}{\varepsilon} - \frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon}\right). \end{aligned} \tag{47}$$

For solving this equation, a pure exponential approximation is used which given by [43]

$$\begin{aligned} Q\left(\frac{\sqrt{x/\rho}}{\varepsilon} - \frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon}\right) =& \frac{1}{12}\exp\left(-\frac{\left(\frac{\sqrt{x/\rho}}{\varepsilon} - \frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon}\right)^2}{2}\right) \\ &+ \frac{1}{4}\exp\left(-\frac{2}{3}\left(\frac{\sqrt{x/\rho}}{\varepsilon} - \frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon}\right)^2\right) + \mathcal{O}\left(\frac{1}{x}\right). \end{aligned} \tag{48}$$

Using this approximation, the equation (47) can be approximate as

$$\begin{aligned} \frac{1}{N} &= Q\left(\frac{\sqrt{x^*/\rho}}{\varepsilon} - \frac{[\mathbf{\Theta}]_{i,j}}{\varepsilon}\right) \\ &\approx \exp(-x^*) + \mathcal{O}\left(\frac{1}{x^*}\right), \quad \text{when} \quad x^* \to \infty. \end{aligned} \tag{49}$$

Compared to the results in [39], the unique solution x^* to our above equation is given by

$$x^* = \log N + \mathcal{O}(\log\log\log N). \tag{50}$$

It is obvious that $g'(x^*) = \mathcal{O}(\frac{1}{x^*})$. Therefore, the maximum value of channel gains, $\gamma_{i,j}, j = 1,\ldots,N$ which are i.i.d. random variables satisfies Proposition 8.

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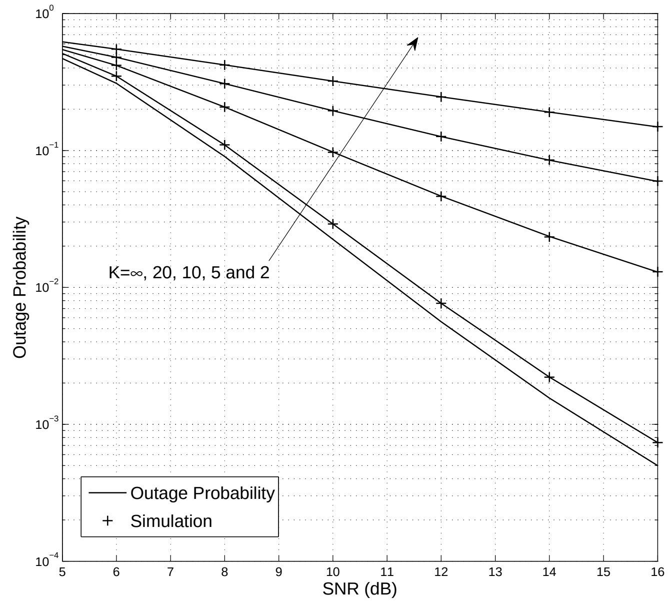

Fig. 2. Outage probability of average channel gain based on random selection for multi-way relaying system.

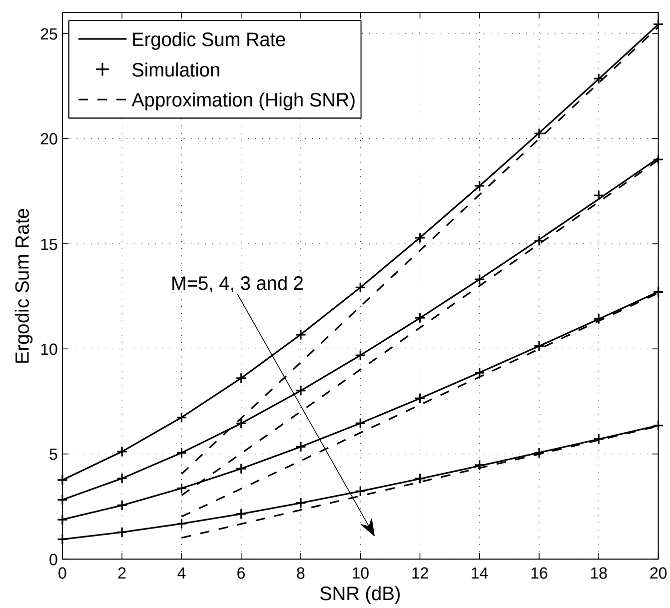

Fig. 3. Ergodic sum rate of multi-way relaying system based on random selection.

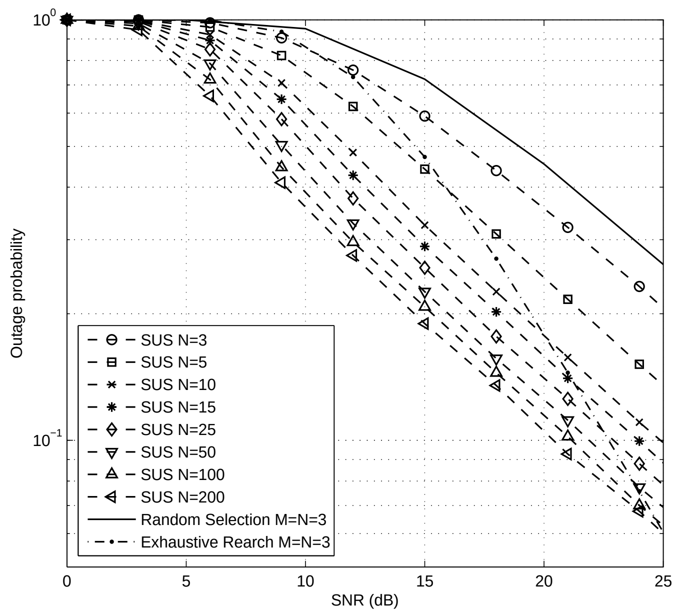

Fig. 4. Outage probability based on different relays selections.

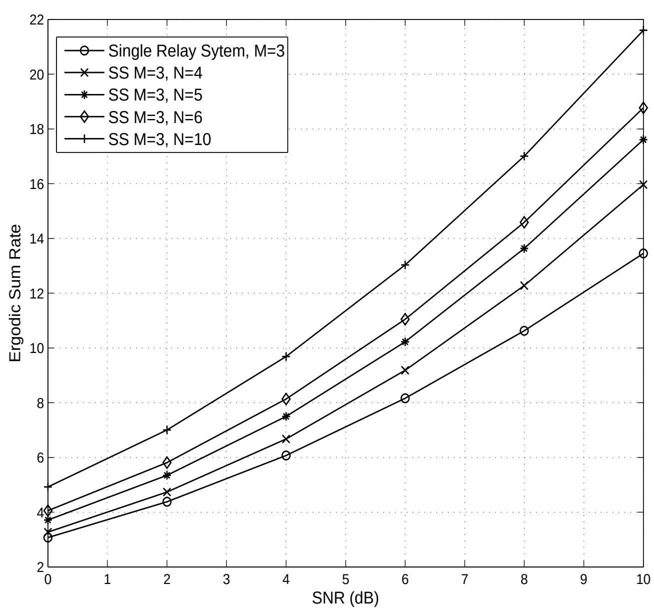

Fig. 5. Ergodic rate comparison between single relay system and multi-relay scheme.

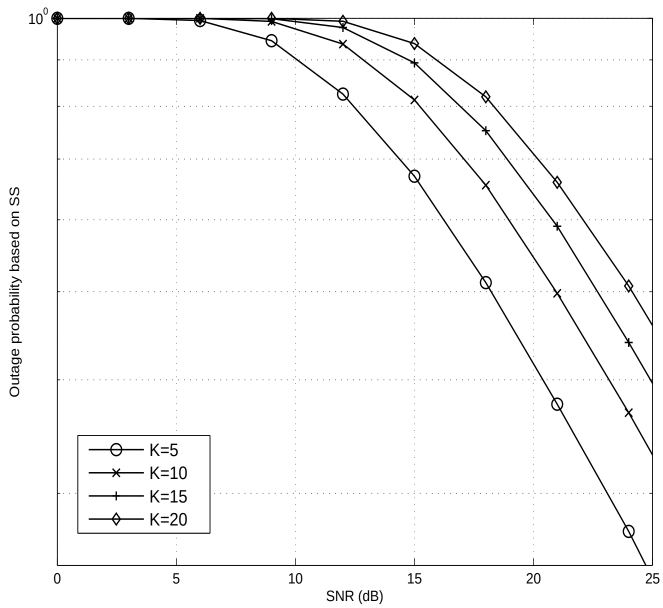

Fig. 6. Outage probability based on different value of parameter K for semiorthogonal selection.

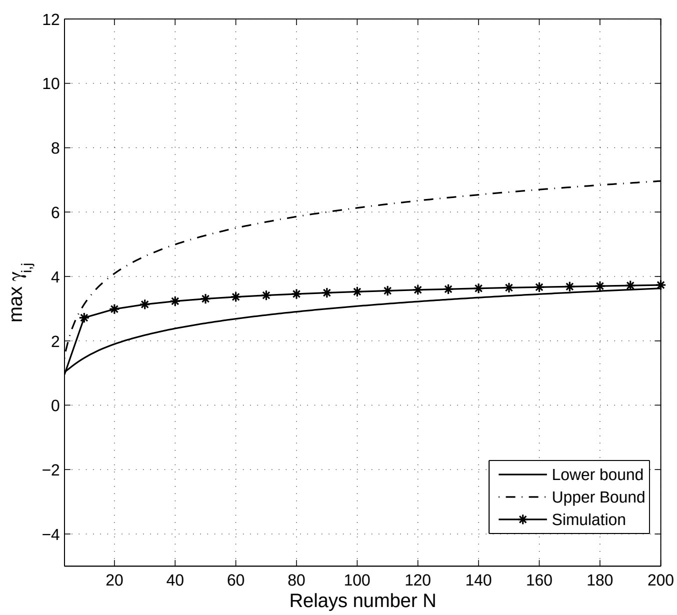

Fig. 7. Lower and upper bounds of channel gain $\max_j \gamma_{i,j}$.

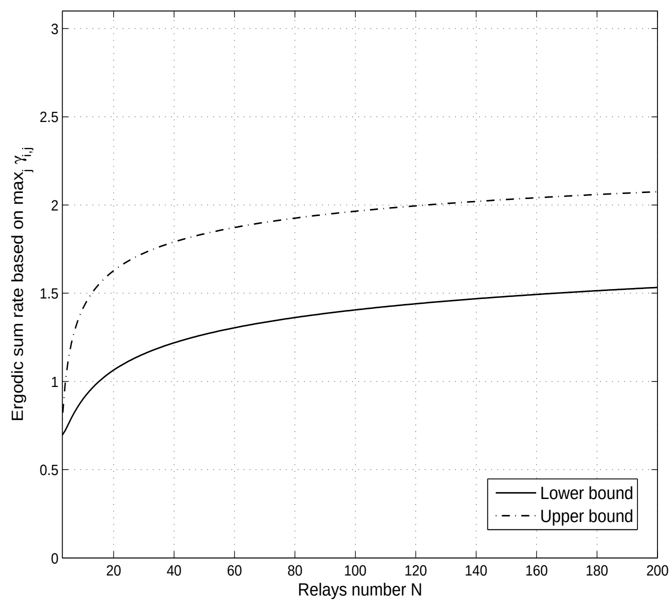

Fig. 8. Lower and upper bounds of ergodic rate based on the maximum channel gain $\max_j \gamma_{i,j}$.