

A Simple Design of IRS-NOMA Transmission

Zhiguo Ding and H. Vincent Poor,

Abstract—This letter proposes a simple design of intelligent reflecting surface (IRS) assisted non-orthogonal multiple access (NOMA) transmission, which can ensure that more users are served on each orthogonal spatial direction than spatial division multiple access (SDMA). In particular, by employing IRS, the directions of users' channel vectors can be effectively aligned, which facilitates the implementation of NOMA. Both analytical and simulation results are provided to demonstrate the performance of the proposed IRS-NOMA scheme and also study the impact of hardware impairments on IRS-NOMA.

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) has been recognized as a promising multiple access candidate for future mobile networks [1]. The key idea of NOMA is to serve multiple users on each orthogonal bandwidth resource block. In scenarios with multiple-antenna nodes, orthogonal spatial directions can be viewed as a type of resource blocks. Conventional orthogonal multiple access (OMA), such as spatial division multiple access (SDMA), is to serve a single user on each spatial direction, whereas the use of NOMA is to ensure that multiple users are served simultaneously on each spatial direction and hence improves spectral efficiency. However, it is important to point out that the use of NOMA is not always preferable [2]. For example, if users' channel vectors are orthogonal to each other, SDMA is more preferable than NOMA, whereas the situation, in which the directions of the users' channel vectors are the same, is the ideal case for the implementation of NOMA.

Therefore, an important question to broaden the applications of NOMA is whether the directions of users' channel vectors can be manipulated, i.e., aligning one user's channel with the others'. This is difficult in conventional wireless systems, since the users' channels are fixed and determined by propagation environments. Motivated by this difficulty, this letter is to propose a new type of NOMA transmission by employing the intelligent reflecting surface (IRS) which can be viewed as a low-cost antenna array consisting of a large number of reconfigurable reflecting elements [3], [4]. By applying IRS, the direction of a user's channel vector can be effectively tuned, which facilitates the implementation of NOMA. In particular, the spectral efficiency and connectivity can be improved by IRS-NOMA since a single spatial direction can be used to serve multiple users, even if their original channels are not aligned. Due to the low-cost feature of IRS, e.g., finite-resolution phase shifters, a user's channel vector cannot be accurately aligned to a target direction. The impact of this hardware impairment on IRS-NOMA is investigated and the performance of the developed practical IRS-NOMA transmission scheme is characterized in this letter.

Z. Ding and H. V. Poor are with the Department of Electrical Engineering, Princeton University, Princeton, NJ 08544, USA. Z. Ding is also with the School of Electrical and Electronic Engineering, the University of Manchester, Manchester, UK (email: zhiguo.ding@manchester.ac.uk, poor@princeton.edu).

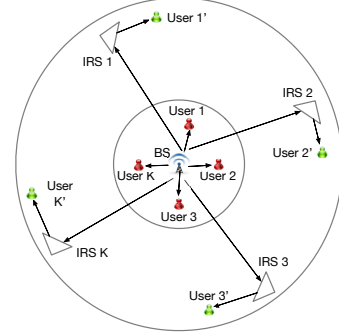


Fig. 1. A system diagram for IRS-NOMA with $2K$ users and K IRS's.

II. SYSTEM MODEL

Consider a multi-user downlink scenario as shown in Fig. 1. There are two types of users, namely near users and cell-edge users, where it is assumed that there is no direct link between the base station and the cell-edge users. SDMA is used as a benchmark for IRS-NOMA, where K near users are served by the base station which is equipped with M ($M \geq K$) antennas and performs zero forcing beamforming. To better illustrate the benefit of IRS-NOMA, we further assume that the K near users are scheduled because their channel vectors are orthogonal to each other, which means that the beamforming vectors, denoted by \mathbf{w}_k , $1 \leq k \leq K$, are orthonormal vectors.

The main benefit for employing IRS-NOMA is to ensure that more users are connected than conventional SDMA. For illustration purposes, we assume that on each beam, \mathbf{w}_k , one additional user, denoted by user k' , is served with the help of an IRS which is equipped with N antennas, as shown in Fig. 1. In addition, we assume that only user k' can hear IRS k since the IRS's are deployed close to the cell-edge users¹

The base station broadcasts $\sum_{k=1}^K \mathbf{w}_k (\alpha_1 s_k + \alpha_2 s_{k'})$, where $s_{k'}$ denotes the signal to be sent to user k' , s_k is the signal to be sent to user k , α_i denotes the power allocation coefficient, and $\alpha_1^2 + \alpha_2^2 = 1$. Because it is assumed that there is no direct link between the IRS's and the near users, the performance analysis for user k is exactly the same as those in conventional NOMA systems, and hence in this letter we focus on the performance at user k' only.

Therefore, the signal received by user k' is given by

$$y_{k'} = \mathbf{h}_{k'}^H \mathbf{\Theta}_k \mathbf{G}_k \sum_{k=1}^K \mathbf{w}_k (\alpha_1 s_k + \alpha_2 s_{k'}) + w_{k'}, \quad (1)$$

where \mathbf{G}_k denotes the $N \times M$ complex Gaussian channel matrix from the base station to the IRS associated with user k' , $\mathbf{h}_{k'}$ denotes the complex Gaussian channel vector from the IRS to user k' , and $w_{k'}$ denotes the noise. $\mathbf{\Theta}_k$ is a diagonal matrix, and each of its main diagonal elements is denoted by $\beta_{k,i} e^{-j\theta_{k,i}}$, where $\theta_{k,i}$ denotes the reflection phase shift and $\beta_{k,i}$ denotes the amplitude reflection coefficient [3], [4].

¹These assumptions facilitate the illustration for the benefit of IRS-NOMA. How to extend the obtained analytical results to more general scenarios is an important topic for future research but beyond the scope of this letter.

As in conventional NOMA, it is assumed that $\alpha_1 \leq \alpha_2$, and hence the signal-interference-plus-noise (SINR) for user k' to decode its message is given by

$$\begin{aligned} \text{SINR}_{k'} &= \frac{|\mathbf{h}_{k'}^H \Theta_k \mathbf{G}_k \mathbf{w}_k|^2 \alpha_1^2}{|\mathbf{h}_{k'}^H \Theta_k \mathbf{G}_k \mathbf{w}_k|^2 \alpha_2^2 + \sum_{i=1, i \neq k}^K |\mathbf{h}_{k'}^H \Theta_k \mathbf{G}_k \mathbf{w}_i|^2 + \frac{1}{\rho}} \\ &= \frac{|\boldsymbol{\theta}_k^H \mathbf{D}_{k'} \mathbf{h}_k|^2 \alpha_1^2}{|\boldsymbol{\theta}_k^H \mathbf{D}_{k'} \mathbf{h}_k|^2 \alpha_2^2 + \sum_{i=1, i \neq k}^K |\boldsymbol{\theta}_k^H \mathbf{D}_{k'} \mathbf{h}_i|^2 + \frac{1}{\rho}}, \end{aligned} \quad (2)$$

where ρ denotes the transmit signal-to-noise ratio (SNR), $\mathbf{h}_i = \mathbf{G}_k \mathbf{w}_i$, $\boldsymbol{\theta}_k$ is an $N \times 1$ vector containing the elements on the main diagonal of Θ_k^H , and $\mathbf{D}_{k'}$ is a diagonal matrix with its diagonal elements obtained from $\mathbf{h}_{k'}^H$.

III. DESIGNS OF IRS-NOMA

A. IRS-NOMA with Ideal Beamforming

As shown in (2), the performance of IRS-NOMA is determined by the choice of Θ_k . Ideal designs of IRS-NOMA typically require beamforming with infinite resolution, i.e., the hardware circuit can support arbitrary choices for the phase shift and amplitude coefficient, $\theta_{k,i}$ and $\beta_{k,i}$. Take the zero-forcing design as an example. In order to suppress inter-pair interference, the use of zero-forcing beamforming implies that $\boldsymbol{\theta}_k$ should satisfy the following constraints:

$$\boldsymbol{\theta}_k^H \mathbf{D}_{k'} \mathbf{h}_i = 0, \quad (3)$$

for $i \neq k$. Denote \mathbf{V}_k by an $N \times (N - K + 1)$ matrix collecting the basis vectors of the null space of $[\mathbf{D}_{k'} \mathbf{h}_1 \cdots \mathbf{D}_{k'} \mathbf{h}_{i-1} \mathbf{D}_{k'} \mathbf{h}_{i+1} \cdots \mathbf{D}_{k'} \mathbf{h}_K]$. Therefore, $\boldsymbol{\theta}_k$ can be obtained as $\mathbf{V}_k \mathbf{x}$, and the optimal \mathbf{x} can be obtained as follows:

$$\max. \quad \frac{|\mathbf{x}^H \mathbf{V}_k^H \mathbf{D}_{k'} \mathbf{h}_k|^2 \alpha_1^2}{|\mathbf{x}^H \mathbf{V}_k^H \mathbf{D}_{k'} \mathbf{h}_k|^2 \alpha_2^2 + \frac{1}{\rho}} \quad (4)$$

$$\text{s.t.} \quad |\mathbf{x}|^2 \leq 1. \quad (5)$$

By using the fact that $\frac{\alpha_1^2 y}{\alpha_2^2 y + \frac{1}{\rho}}$ is a mono-increasing function of y and also applying Cauchy-Schwarz inequality, the maximum of the SINR is achieved by $\boldsymbol{\theta}_k^* = \mathbf{V}_k \frac{\mathbf{V}_k^H \mathbf{D}_{k'} \mathbf{h}_k}{|\mathbf{V}_k^H \mathbf{D}_{k'} \mathbf{h}_k|}$. Evidently, such an ideal design requires that the number of possible choices for the phase shift and the amplitude, $\theta_{k,i}$ and $\beta_{k,i}$, is infinite. We note that the other ideal designs, e.g., directly maximizing the SINR in (2), lead to the same conclusion.

B. IRS-NOMA With Finite Resolution Beamforming

In practice, the choices for $\theta_{k,i}$ and $\beta_{k,i}$ cannot be arbitrary due to the hardware limitations. A straightforward design for IRS-NOMA with finite resolution beamforming is inspired by lens antenna arrays in millimeter-wave networks [5]. In particular, denote an $N \times N$ discrete Fourier transform (DFT) matrix by \mathbf{F}_n , and the optimal $\boldsymbol{\theta}_k$ to maximize the SINR in (2) can be found by an exhaustive search among the columns of \mathbf{F}_N , which can be realized by finite-resolution phase shifters.

An alternative low-cost implementation is to apply on-off control to IRS-NOMA, i.e., each diagonal element of Θ is either 0 (off) or 1 (on). Without loss of generality, assume that $N = PQ$, where P and Q are integers. Define $\mathbf{V} =$

$\frac{1}{\sqrt{Q}} \mathbf{I}_P \otimes \mathbf{1}_Q$, where \mathbf{I}_P is a $P \times P$ identity matrix, $\mathbf{1}_Q$ is a $Q \times 1$ all-ones vector, and \otimes denotes the Kronecker product. Denote \mathbf{v}_p by the p -th column of \mathbf{V} , where it is easy to show that $\mathbf{v}_p^H \mathbf{v}_l = 0$ for $p \neq l$, and $\mathbf{v}_p^H \mathbf{v}_p = 1$. $\boldsymbol{\theta}_k$ is selected based on the following criterion:

$$\max_{\mathbf{v}_p} \frac{|\mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k|^2 \alpha_1^2}{|\mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k|^2 \alpha_2^2 + \sum_{i=1, i \neq k}^K |\mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_i|^2 + \frac{1}{\rho}}. \quad (6)$$

As shown in the remaining of the letter, the use of on-off control not only yields better performance than the DFT-based design, but also ensures that insightful analytical results can be developed.

Lemma 1. For the single user case ($K = 1$), the use of IRS-NOMA with on-off control can achieve the following outage probability at user k' :

$$P_{k'} = \frac{\xi^{\frac{N}{2}}}{(\Gamma(Q))^P} \left(\xi^{-\frac{Q}{2}} \Gamma(Q) - 2K_Q \left(2\xi^{\frac{1}{2}} \right) \right)^P, \quad (7)$$

if $\alpha_1^2 - \alpha_2^2 \epsilon_{k'} > 0$, otherwise $P_{k'} = 1$, where $\xi = \frac{Q \epsilon_{k'}}{\rho(\alpha_1^2 - \alpha_2^2 \epsilon_{k'})}$, $\epsilon_{k'} = 2^{R_{k'}} - 1$, $R_{k'}$ denotes the target rate of user k' , $K_n(\cdot)$ denotes the modified Bessel function of the second kind, and $\Gamma(\cdot)$ denotes the gamma function. At high SNR, the outage probability can be approximated as follows:

$$P_{k'} \approx \begin{cases} \xi^N (-\ln(\xi))^N, & Q = 1 \\ \frac{\xi^P}{(Q-1)^P}, & Q \geq 2 \end{cases}, \quad (8)$$

for $\alpha_1^2 - \alpha_2^2 \epsilon_{k'} > 0$.

Proof. For the case $K = 1$ and $\boldsymbol{\theta}_k = \mathbf{v}_p$, $\text{SINR}_{k'}$ in (2) can be simplified as follows:

$$\text{SINR}_{k',p} = \frac{|\mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k|^2 \alpha_1^2}{|\mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k|^2 \alpha_2^2 + \frac{1}{\rho}}. \quad (9)$$

Because of the structure of \mathbf{v}_p , $\sqrt{Q} \mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k$ is simply an inner product of two $Q \times 1$ complex Gaussian vectors. By first treating $\sqrt{Q} \mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k$ as a complex Gaussian random variable with zero mean and variance $|\mathbf{h}_k|^2$ and using the fact that $|\mathbf{h}_k|^2$ is chi-square distributed, the probability density function (pdf) of $\sqrt{Q} \mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k$ can be obtained as follows:

$$f_{Q|\mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k|^2}(x) = \frac{2x^{\frac{Q-1}{2}}}{\Gamma(Q)} K_{Q-1}(2\sqrt{x}), \quad (10)$$

where the details for the derivation can be found in [6], [7].

Therefore, for the case $\boldsymbol{\theta}_k = \mathbf{v}_p$, the outage probability can be expressed as follows:

$$P_{k',p} = \text{P}(\log(1 + \text{SINR}_{k',p}) < R_{k'}). \quad (11)$$

By applying the pdf shown in (10), the outage probability can be expressed as follows:

$$\begin{aligned} P_{k',p} &= \int_0^\xi f_{Q|\mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k|^2}(x) dx \\ &= \frac{2}{\Gamma(Q)} \int_0^\xi x^{\frac{Q-1}{2}} K_{Q-1}(2\sqrt{x}) dx \\ &= \frac{1}{\Gamma(Q)} \xi^{\frac{Q+1}{2}} \left(\xi^{-\frac{Q+1}{2}} \Gamma(Q) - 2\xi^{-\frac{1}{2}} K_Q \left(2\xi^{\frac{1}{2}} \right) \right), \end{aligned} \quad (12)$$

where the last step follows from Eq. (6.561.8) in [8].

For IRS-NOMA with on-off control, $\mathbf{V} = \frac{1}{\sqrt{Q}} \mathbf{I}_P \otimes \mathbf{1}_Q$, and hence one can easily verify that $\mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k$ and $\mathbf{v}_l^H \mathbf{D}_{k'} \mathbf{h}_k$ are independent and identically distributed (i.i.d.) for $p \neq l$. Therefore, the use of the selection criterion in (6) ensures that the outage probability at user k' can be expressed as follows:

$$P_{k'} = \frac{\xi^{\frac{P(Q+1)}{2}}}{(\Gamma(Q))^P} \left(\xi^{-\frac{Q+1}{2}} \Gamma(Q) - 2\xi^{-\frac{1}{2}} K_Q \left(2\xi^{\frac{1}{2}} \right) \right)^P. \quad (13)$$

With some algebraic manipulations, (7) in the lemma can be obtained.

In order to find the high SNR approximation for (7), we first note that at high SNR, $\rho \rightarrow \infty$, which means that $\xi \rightarrow 0$. Recall that $K_n(z)$ can be approximated as follows: [8]

$$K_n(z) \approx \frac{1}{2} \left(\frac{(n-1)!}{\left(\frac{z}{2}\right)^n} - \frac{(n-2)!}{\left(\frac{z}{2}\right)^{n-2}} \right), \quad (14)$$

for $n \geq 2$ and $z \rightarrow 0$. Therefore, the outage probability $P_{k'}$ can be approximated as follows:

$$\begin{aligned} P_{k'} &= \frac{1}{(\Gamma(Q))^P} \left(\Gamma(Q) - 2\xi^{\frac{Q}{2}} K_Q \left(2\xi^{\frac{1}{2}} \right) \right)^P \\ &\approx \frac{1}{(\Gamma(Q))^P} \left(\Gamma(Q) - \xi^{\frac{Q}{2}} \left(\frac{(Q-1)!}{\xi^{\frac{Q}{2}}} - \frac{(Q-2)!}{\xi^{\frac{Q-2}{2}}} \right) \right)^P \\ &\approx \frac{\xi^P}{(Q-1)^P}, \end{aligned} \quad (15)$$

for the cases with $Q \geq 2$.

For the case with $Q = 1$, unlike (14), a different approximation for the Bessel function will be used as shown in the following:

$$K_1(z) \approx \frac{1}{2} \frac{1}{\left(\frac{z}{2}\right)} + \left(\frac{z}{2}\right) \ln \left(\frac{z}{2}\right), \quad (16)$$

for $z \rightarrow 0$. Therefore, for the case with $Q = 1$, the outage probability can be approximated as follows:

$$\begin{aligned} P_{k'} &= \left(1 - 2\xi^{\frac{1}{2}} K_1 \left(2\xi^{\frac{1}{2}} \right) \right)^N \\ &\approx \left(1 - \xi^{\frac{1}{2}} \left(\frac{1}{\xi^{\frac{1}{2}}} + \xi^{\frac{1}{2}} \ln(\xi) \right) \right)^N \approx \xi^N (-\ln(\xi))^N. \end{aligned} \quad (17)$$

By combining (15) with (17), (8) in the lemma can be obtained, and the proof for the lemma is complete. \square

Remark 1: The diversity gain for the case with $Q = 1$ can be found as follows:

$$\begin{aligned} -\lim_{\rho \rightarrow \infty} \frac{\log P_{k'}}{\log \rho} &= \lim_{\xi \rightarrow 0} \frac{\log[\xi^N (-\ln(\xi))^N]}{\log \xi} \\ &= N + N \lim_{\xi \rightarrow 0} \frac{\log[(-\ln \xi)]}{\log \xi} = N, \end{aligned} \quad (18)$$

where the last step follows by applying L'Hospital's rule. It is straightforward to show that the diversity gain for the case $Q \geq 2$ is P . Therefore, the choice of $Q = 1$ is diversity optimal to IRS-NOMA with on-off control.

Remark 2: Lemma 1 is only applicable to IRS-NOMA with on-off control. The analytical results for IRS-NOMA with DFT are difficult to obtain, mainly because of the correlation

between $|\mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k|^2$ and $|\mathbf{v}_i^H \mathbf{D}_{k'} \mathbf{h}_k|^2$, for $i \neq p$. We note that simulation results indicate that this correlation is very weak, which results in an observation that the diversity order achieved by the DFT case is similar to that of the scheme with on-off control.

Note that for the multi-user case ($K \geq 2$), the outage probability achieved by IRS-NOMA with on-off control is difficult to analyze, due to the correlation between $|\mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k|^2$ and $|\mathbf{v}_i^H \mathbf{D}_{k'} \mathbf{h}_k|^2$, for $i \neq p$. Consistent to the single-user case, simulation results show that $Q = 1$ is also optimal in the high SNR regime for the multi-user case. Therefore, the choice of $Q = 1$ is focused in the following, where a closed-form expression for the outage probability can be obtained, as shown in the the following lemma.

Lemma 2. For the multi-user case $K \geq 2$, the outage probability achieved by IRS-NOMA with on-off control ($Q = 1$) is given by

$$P_{k'} = \left(1 - \frac{2\sqrt{\frac{\epsilon_{k'}}{\rho\tau}} K_1 \left(2\sqrt{\frac{\epsilon_{k'}}{\rho\tau}} \right)}{\left(1 + \frac{\epsilon_{k'}}{\tau} \right)^{K-1}} \right)^N, \quad (19)$$

where $\tau = \alpha_1^2 - \epsilon_{k'} \alpha_2^2$. At high SNR, the outage probability can be approximated as follows:

$$P_{k'} \approx \left(1 - \frac{1}{\left(1 + \frac{\epsilon_{k'}}{\tau} \right)^{K-1}} \right)^N. \quad (20)$$

Proof. For the special case $Q = 1$, \mathbf{v}_p is an $N \times 1$ vector with all of its elements being zero except its p -th element being one. Therefore, $\mathbf{v}_p^H \mathbf{D}_{k'} \mathbf{h}_k$ becomes $d_{k',p} h_{k,p}$, where $d_{k',p}$ is the p -th element on the diagonal of $\mathbf{D}_{k'}$ and $h_{k,p}$ is the p -th element of \mathbf{h}_k . Therefore, with $\boldsymbol{\theta}_k = \mathbf{v}_p$, $\text{SINR}_{k'}$ can be simplified as follows:

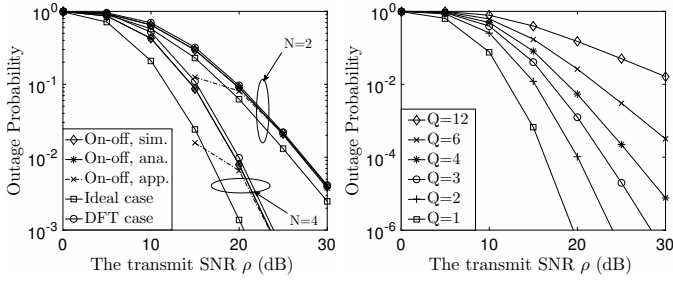
$$\text{SINR}_{k',p} = \frac{|d_{k',p}|^2 |h_{k,p}|^2 \alpha_1^2}{|d_{k',p}|^2 |h_{k,p}|^2 \alpha_2^2 + |d_{k',p}|^2 \sum_{i=1, i \neq k}^K |h_{i,p}|^2 + \frac{1}{\rho}}.$$

We first note that $\mathbf{h}_k = \mathbf{G}_k \mathbf{w}_k$ is still a complex Gaussian vector, since \mathbf{w}_k is normalized. We further note that \mathbf{h}_k and \mathbf{h}_i , $k \neq i$, are independent since \mathbf{w}_k and \mathbf{w}_i are assumed to be orthonormal vectors. Therefore, $h_{k,p}$ and $h_{i,p}$ are independent and complex Gaussian distributed. Hence, the SINR can be further simplified as follows:

$$\text{SINR}_{1',p} = \frac{xy\alpha_1^2}{xy\alpha_2^2 + xz + \frac{1}{\rho}}, \quad (21)$$

where $x = |d_{k',p}|^2$ and $y = |h_{k,p}|^2$ are two independent and exponentially distributed random variables, and $z = \sum_{i=1, i \neq k}^K |h_{i,p}|^2$ is chi-square distributed with $2(K-1)$ degrees of freedom. Therefore, the outage probability can be expressed as follows:

$$\begin{aligned} P_{k',p} &= \int_0^\infty \int_0^\infty \left(1 - e^{-\frac{\epsilon_{k'} x z + \frac{\epsilon_{k'}}{\rho}}{x(\alpha_1^2 - \epsilon_{k'} \alpha_2^2)}} \right) \frac{e^{-x} dx}{(K-2)!} z^{K-2} e^{-z} dz \\ &= 1 - \frac{1}{(K-2)!} 2\sqrt{\frac{\epsilon_{k'}}{\rho\tau}} K_1 \left(2\sqrt{\frac{\epsilon_{k'}}{\rho\tau}} \right) \\ &\quad \times \int_0^\infty e^{-\frac{\epsilon_{k'}}{\tau} z} z^{K-2} e^{-z} dz, \end{aligned}$$



(a) Comparison between different IRS-NOMA ($Q = 1$) (b) Impact of the choice of Q on IRS-NOMA ($N = 12$)

Fig. 2. Impact of IRS-NOMA on the downlink outage probability for the single-user case ($K = 1$). $M = 4$. $R_{k'} = 2$ bits per channel use (BPCU).

where the last step follows from Eq. (3.324.1) in [8].

By applying Eq. (3.381.4) in [8], the outage probability when $\theta_k = \mathbf{v}_p$ can be obtained as follows:

$$P_{k',p} = 1 - 2\sqrt{\frac{\epsilon_{k'}}{\rho\tau}} K_1 \left(2\sqrt{\frac{\epsilon_{k'}}{\rho\tau}} \right) \frac{1}{\left(1 + \frac{\epsilon_{k'}}{\tau}\right)^{K-1}}. \quad (22)$$

Because of the structure of \mathbf{V} , the SINRs for different \mathbf{v}_p are i.i.d., and therefore, the outage probability $P_{k'}$ can be obtained as shown in (19) in the lemma.

At high SNR, by using the fact that $K_1(x) \approx \frac{1}{x}$ for $x \rightarrow 0$, the outage probability can be approximated as follows:

$$P_{k'} \approx \left(1 - \frac{2\sqrt{\frac{\epsilon_{k'}}{\rho\tau}} \left(2\sqrt{\frac{\epsilon_{k'}}{\rho\tau}} \right)^{-1}}{\left(1 + \frac{\epsilon_{k'}}{\tau}\right)^{K-1}} \right)^N.$$

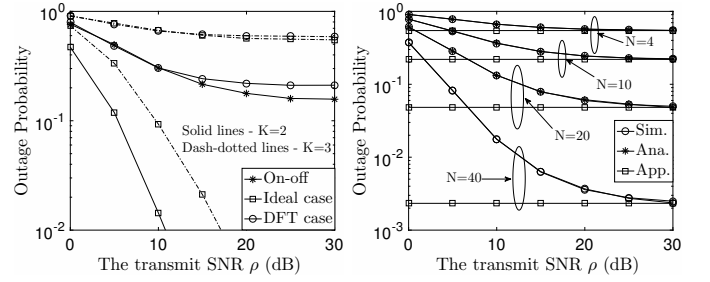
With some algebraic manipulations, the approximation shown in (20) can be obtained, and the lemma is proved. \square

Remark 3: The high SNR approximation shown in (20) indicates the existence of an error floor for the outage probability, i.e., the outage probability does not go to zero by simply increasing the transmission power. However, the outage probability can be reduced by increasing N , as shown in (20).

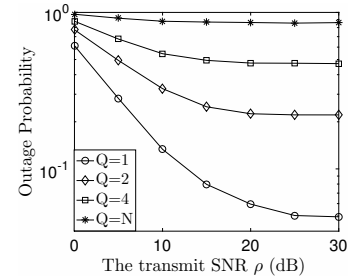
IV. NUMERICAL RESULTS

In this section, computer simulation results are presented to demonstrate the performance of IRS-NOMA, where we use $\alpha_1^2 = \frac{4}{5}$ and $\alpha_2^2 = \frac{1}{5}$. In Fig. 2, the performance of IRS-NOMA is studied by focusing on the single-user case ($K = 1$). Fig. 2(a) shows that the slope of the outage probability curves for the three schemes is the same, which indicates that they achieve the same diversity order. Among the three schemes, the one with ideal beamforming yields the best performance, but it might not be supported by a practical antenna array. Among the two practical IRS-NOMA schemes, the one with on-off control yields better performance. Fig. 2(a) also confirms the accuracy of the developed analytical results shown in Lemma 1. Remark 1 indicates that increasing Q decreases the achieved diversity gain, which is confirmed by Fig. 2(b).

Fig. 3 shows the performance of the IRS-NOMA schemes when there are multiple users ($K \geq 2$). Fig. 3(a) shows that for IRS-NOMA with ideal beamforming, the outage probability can be reduced to zero by increasing the transmission power. However, there are error floors for the two practical IRS-NOMA schemes. The reason for these error floors is due to the fact that the use of finite-resolution beamforming cannot



(a) Comparison between different IRS-NOMA ($N = 4$ and $Q = 1$) (b) Effects of N ($Q = 1$)



(c) Effects of Q ($N = 20$)

Fig. 3. Impact of IRS-NOMA on the downlink outage probability for the multi-user case ($K \geq 2$). $M = 4$. $R_{k'} = 1$ BPCU.

eliminate inter-pair interference completely. Fig. 3(a) also shows that the on-off scheme outperforms the DFT one, which is consistent to Fig. 2(a). The accuracy of the analytical results shown in Lemma 2 is confirmed by Fig. 3(b). In addition, Fig. 3(b) also shows that increasing N can effectively reduce the outage probability, as indicated in (20) in Lemma 2. Fig. 3(c) confirms the optimality of the choice of $Q = 1$ in the multi-user scenario, which is consistent to Fig. 2(b).

V. CONCLUSIONS

In this letter, IRS-NOMA transmission has been proposed to ensure that more users can be served on each orthogonal spatial direction, compared to SDMA. In addition, the impact of hardware impairments on the design of IRS-NOMA has been investigated and the performance of practical IRS-NOMA transmission has also been characterized.

REFERENCES

- [1] Z. Ding, Y. Liu, J. Choi, Q. Sun, M. Elkashlan, C.-L. I, and H. V. Poor, "Application of non-orthogonal multiple access in LTE and 5G networks," *IEEE Commun. Mag.*, vol. 55, no. 2, pp. 185–191, Feb. 2017.
- [2] Z. Chen, Z. Ding, X. Dai, and G. K. Karagiannidis, "On the application of quasi-degradation to MISO-NOMA downlink," *IEEE Trans. Signal Process.*, vol. 64, no. 23, pp. 6174–6189, Dec. 2016.
- [3] M. D. R. and Merouane Debbah, D.-T. Phan-Huy, A. Zappone, M.-S. Alouini, C. Yuen, V. Sciancalepore, G. C. Alexandropoulos, J. Hoydis, H. Gacanin, J. de Rosny, A. Bounceu, G. Lerosey, and M. Fink, "Smart radio environments empowered by AI reconfigurable meta-surfaces: An idea whose time has come," Available on-line at arXiv:1903.08925.
- [4] Q. Wu and R. Zhang, "Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming," Available on-line at arXiv:1810.03961.
- [5] Y. Zeng and R. Zhang, "Millimeter wave MIMO with lens antenna array: A new path division multiplexing paradigm," *IEEE Trans. Commun.*, vol. 64, no. 4, pp. 1557–1571, Apr. 2016.
- [6] N. C. Sagias, "On the ASEF of decode-and-forward dual-hop networks with pilot-symbol assisted M-PSK," *IEEE Trans. Commun.*, vol. 62, no. 2, pp. 510–521, Feb. 2014.
- [7] H. Liu, H. Ding, L. Xiang, J. Yuana, and L. Zhenga, "Outage and ber performance analysis of cascade channel in relay networks," in *Proc. the 9th International Conference on Future Networks and Communications (FNC-2014)*, India, Feb. 2014, pp. 1–8.
- [8] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 6th ed. New York: Academic Press, 2000.