

# Optimal Design of Non-Orthogonal Multiple Access with Wireless Power Transfer

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**Abstract**—In this work, we study a non-orthogonal multiple access scheme used for the uplink of a wireless-powered communication system. We focus on data rates optimization and fairness increase. We show that the formulated optimization problems can be optimally and efficiently solved by either linear programming methods or convex optimization, which means that the proposed scheme can be easily implemented in practical applications. Simulation results illustrate that the proposed scheme outperforms the baseline orthogonal scheme, while they reveal the dependence between sum-throughput, minimum data rate, and wireless power transfer.

## I. INTRODUCTION

Energy harvesting is a promising solution for energy-sustainability of wireless nodes in communication networks, enabling them to overcome the constraint of fixed energy supplies [1]. An alternative to traditional energy harvesting, such as solar power, is the wireless power transfer [2], [3]. In this framework, nodes use the power by the received signal to charge their batteries [4], or to transmit the information to a base station (BS) [5]. However, in practice, nodes cannot harvest energy and transmit information simultaneously [5]–[7]. In order to overcome this difficulty, a *harvest-then-transmit* protocol has been proposed in [5], where the users first harvest energy, and then they transmit their independent messages to the BS. More specifically, the authors in [5] assume that the users utilize time division multiple access (TDMA) for information transmission, using solely the energy they harvested. Although using the harvested energy for transmission has many benefits, the dependence of the nodes on the harvested energy has a negative impact on the data rates that they can achieve. Consequently, methods which increase power-bandwidth efficiency should be carefully explored [8], [9]. Toward this direction, the utilization of orthogonal multiple access schemes, such as TDMA, might not be the most appropriate option.

On the other hand, non-orthogonal multiple access (NOMA) proves to increase spectral efficiency [10]. For this reason, it has been recently proposed for LTE Advanced, as it can be verified by [11], in which it is cited as multi-user superposition transmission. Furthermore, it has been recognized as a promising multiple access technique for fifth generation (5G) networks [12]–[14]. NOMA is substantially different from orthogonal multiple access schemes, i.e. time/frequency/code division multiple access (TDMA/FDMA/CDMA), since its

basic principle is that the users can achieve multiple access by using the power domain. For this reason, the decoder needs to implement a joint processing technique, such as successive interference cancellation (SIC). The performance of a downlink NOMA scheme with randomly deployed users has been investigated in [13], while the application of NOMA for the downlink of cooperative communication networks in [15], among others. Finally, in [14], the authors study NOMA for the case of the uplink of a communication system, consisting of traditional nodes with fixed energy supplies.

Unlike recent literature, in this work, we study the application of NOMA for the uplink of a wireless-powered communication system, which consists of one BS and several energy harvesting users, in order to increase the data rates and fairness. Note that the implementation of NOMA in the uplink is not a burden for the users, since the encoding complexity is not affected, while their synchronization is simpler than the case of TDMA. To this end, we optimize the related variables, taking into account two different criteria: the sum-throughput and the equal rate maximization. Moreover, in the case of sum-throughput maximization, further improvement of the minimum data rate among users is also discussed. In the formulated problems, we take into account the time used for energy harvesting and the time-sharing variables related to SIC. Moreover, we show that all formulated problems can be optimally solved by either linear programming or convex optimization, which is important for the practical implementation of the proposed scheme. Extended simulation results illustrate that the application of the proposed NOMA scheme, when comparing to the case of TDMA, has the following advantages: i) it leads to a notable increase of the minimum data rate, and/or, ii) it improves fairness. Finally, an interesting trade-off between the time used for energy harvesting and information transmission is revealed, as well as the dependence between sum-throughput, minimum data rate, and wireless power transfer.

The rest of the paper is organized as follows. Section II describes the considered communication and energy harvesting model, while it formulates the users data rates and system's sum-throughput. The optimization problems of throughput maximization and equal rates maximization are formulated and solved in sections III and IV, respectively. Section IV presents and discusses the simulation results and finally, section V summarizes our conclusions.

## II. SYSTEM MODEL

We consider a wireless network consisting of  $N$  users and one BS, which are all equipped with a single antenna. The path loss factor from the BS to user  $n$  is denoted by  $\mathcal{L}_{0n}$ , while the channel coefficient is given by the complex random variables  $h_{0n} \sim \mathcal{CN}(0, 1)$ . The communication is divided into time frames of unitary duration. It is assumed that the channel conditions remain constant during a time frame, and their exact values are known by the BS.

### A. Harvest-then-Transmit Protocol

We consider that the network adopts a harvest-then-transmit protocol, i.e. at first, the amount of time  $1 - T, 0 \leq T \leq 1$  is assigned to the BS to broadcast wireless energy to all users [5]. The remaining amount of time,  $T$ , is assigned to users, which simultaneously transmit their independent information to the BS. In order to detect the users' signals, the BS implements a joint processing technique [13], [16]. For this purpose, it employs a non-orthogonal multiple access (NOMA) scheme [14]. We assume that the energy transmitted by each user  $n$  is limited by the amount of harvested energy, i.e. during time portion  $T$ , each user can only use the energy that was harvested during  $1 - T$ . The harvested energy by the  $n$ -th user is

$$E_n = \mathcal{G}_0 \mathcal{G}_n \eta_1 \mathcal{L}_{0n} |h_{0n}|^2 P_0 (1 - T), \quad (1)$$

where  $\mathcal{G}_0$  and  $\mathcal{G}_n$  are the directional antenna gains of the BS and the  $n$ -th user, respectively,  $0 < \eta_1 < 1$  is the energy harvesting efficiency,  $P_0$  is the transmit power of the BS. The transmit power of the  $n$ -th user is given by

$$P_n = \frac{E_n}{T}. \quad (2)$$

### B. User-Throughput and Sum-Throughput

In this subsection, the user data rate and the system sum-throughput are defined. Interestingly, the decoding order does not affect the sum-throughput, and any arbitrary decoding order can be assumed. Accordingly, in this section it is assumed that the users' messages are decoded in an increasing order of their indices. Therefore, for decoding the first user's message ( $n = 1$ ), interference is imposed due to all other users  $n = 2, \dots, N$ , while on the second user's message, interference is imposed due to users  $n = 3, \dots, N$ , and so on. Using Shannon's capacity formula, the throughput of the  $n$ -th user,  $1 \leq n \leq (N - 1)$ , is given by [14]

$$\begin{aligned} R_n &= T \log_2 \left( 1 + \frac{P_n g_n}{\sum_{j=n+1}^N (P_j g_j) + N_0} \right) \\ &= T \log_2 \left( 1 + \frac{\frac{\eta \rho (1-T) g_n}{T}}{\frac{\eta \rho (1-T) \sum_{j=n+1}^N g_j}{T} + 1} \right), \end{aligned} \quad (3)$$

while the throughput of the  $N$ -th user is

$$R_N = T \log_2 \left( 1 + \frac{\eta \rho (1-T) g_N}{T} \right). \quad (4)$$

In the above equations,  $\rho = \frac{P_0}{N_0}$ ,  $\eta = \eta_1 \eta_2$ , with  $\eta_2$  being the efficiency of the user's amplifier, and  $N_0$  the additive

white gaussian noise (AWGN) power. Also, assuming channel reciprocity,  $g_n = \mathcal{G}_0^2 \mathcal{G}_n^2 \mathcal{L}_{0n}^2 |h_{0n}|^4$ .

The sum-throughput achieved by NOMA is given by [14]

$$\begin{aligned} R_{sum} &= \sum_{n=1}^N R_n = T \sum_{n=1}^{N-1} \left( \log_2 \left( \eta \rho \sum_{i=n}^N g_i + \frac{T}{1-T} \right) \right. \\ &\quad \left. - \log_2 \left( \eta \rho \sum_{i=n+1}^N g_i + \frac{T}{1-T} \right) \right) \\ &\quad + T \left( \log_2 \left( \eta \rho g_N + \frac{T}{1-T} \right) - \log_2 \left( \frac{T}{1-T} \right) \right) \\ &= T \log_2 \left( 1 + \frac{\eta \rho \sum_{n=1}^N g_n}{\frac{T}{1-T}} \right). \end{aligned} \quad (5)$$

## III. SUM-THROUGHPUT MAXIMIZATION

It can be easily observed that, when  $T = 0$  or  $T = 1$ , no time or no energy, respectively, is available to the users in order to transmit, and thus the sum-throughput is zero. The optimization problem which aims at maximizing the sum-throughput can be written as

$$\begin{aligned} \max_T \quad & \mathcal{R}_{sum} \\ \text{C:} \quad & 0 < T < 1. \end{aligned} \quad (6)$$

In (6),  $R_{sum}$  is strictly concave with respect to  $T$  in  $(0, 1)$ , since it holds that

$$\frac{d^2 R_{sum}}{dT^2} = - \frac{(\eta \rho \sum_{n=1}^N g_n)^2}{T^3 \ln(2) (1 - \eta \rho \sum_{n=1}^N g_n + \frac{\eta \rho \sum_{n=1}^N g_n}{T})^2} < 0. \quad (7)$$

Thus, the optimal value for  $T$  in  $(0, 1)$  that maximizes  $R_{sum}$  is unique and can be obtained by

$$\frac{dR_{sum}}{dT} = 0. \quad (8)$$

After some mathematical calculations, the optimal value can be expressed as

$$T^* = \frac{\eta \rho \sum_{n=1}^N g_n}{\eta \rho \sum_{n=1}^N g_n + \frac{\eta \rho \sum_{n=1}^N g_n - 1}{W(\frac{\eta \rho \sum_{n=1}^N g_n - 1}{e}) - 1}}, \quad (9)$$

where  $(\cdot)^*$  denotes a solution value and  $W(x)$  is the Lambert W function, also called omega function or product logarithm, which gives the principal solution for  $W(x)$  in  $x = W(x)e^{W(x)}$ . Note that  $W(x)$  is a built-in function in most well-known mathematical software packages [1]. In the following, we describe two decoding order methods.

### A. Descending Decoding Order

The simplest case of decoding is to adopt a fixed decoding order among users. For fairness, the users' indices are assigned in a way that the values  $g_n \forall N$  are sorted in descending order, i.e.  $g_1 \geq \dots \geq g_N$ , since this allows decoding the weakest user's message without interference. Therefore, this scheme increases fairness compared to other schemes with fixed decoding order, such as ascending decoding order.

### B. Time-Sharing Configuration for minimum-throughput Improvement

In this subsection, the indices of the users are ordered according to the relations  $g_1 \geq g_2 \geq \dots \geq g_N$ , however, the order of decoding depends on the time-sharing [16]. The basic principle of this technique is that the order of decoding for the users can change for specific fractions of the duration of  $T$ . Next, we present a simple method to optimize the time-sharing variables. In general, there are  $N!$  configurations with different decoding order, which will be called permutations. Let  $\tau_l, \sum_l \tau_l = 1$  denote the portion of time  $T$  for which the BS decodes, according to the  $l$ -th permutation. Hereinafter,  $\tau$  denotes the set of values of  $\tau_l \forall l$ . For mathematical clarity, let  $A$  be the matrix which represents all the permutations, with elements  $A(l, j_{l,n})$ , corresponding to the indices of the users, i.e.  $A(l, j_{l,n}) = n$ . The decoding order of the users during the  $l$ -th permutation is determined by the indices of the columns,  $j_{l,n}, \forall n$ , for the  $l$ -th row of matrix  $A$ , i.e. if  $j_{l,n} < j_{l,m}$ , the message of the  $n$ -th user will be decoded before the message of the  $m$ -th. For example, if  $A(2, 4) = 3$ , it means that, when the 2-nd permutation is applied, the message of the 3-rd user will be decoded in the 4-th order. Thus, taking into account the time-sharing configuration, the throughput of the  $n$ -th user, denoted by  $\tilde{R}_n$ , can be written as

$$\tilde{R}_n(T^*) = \sum_{l=1}^{N!} \tau_l T^* \log_2 \left( 1 + \frac{\frac{\eta\rho(1-T^*)g_n}{T^*}}{\frac{\eta\rho(1-T^*) \sum_{j_{l,m} > j_{l,n}} g_{A(l, j_{l,m})}}{T^*} + 1} \right). \quad (10)$$

Please note that NOMA is appropriate for a small number of users, thus the number of permutations is not a barrier for the determination of the time-sharing configuration. Moreover, taking into account all possible permutations can be considered as a benchmark to other less complex schemes, which possibly exclude some permutations at the expense of a suboptimal configuration. However, further discussion of these schemes is out of the scope of this paper, since it focuses only on optimal solutions.

Now, while the sum-throughput is maximized by setting  $T = T^*$ , where  $T^*$  is given by (9), we aim to boost the minimum-throughput among users,  $\mathcal{R}_{min}$ , by applying the time-sharing technique. The corresponding optimization problem can be written as

$$\begin{aligned} & \max_{\tau, \mathcal{R}_{min}} \mathcal{R}_{min} \\ & \text{s.t.} \quad C_n : \tilde{R}_n(T^*) \geq \mathcal{R}_{min}, \forall n \in \mathcal{N}, \end{aligned} \quad (11)$$

where  $\mathcal{N} = \{1, 2, \dots, N\}$  is the set of all users.

The optimization in (11) is a *linear programming* problem and can be efficiently solved by well-known methods in the literature, such as simplex or interior-point method [17]. As it has already been mentioned, the sum-throughput is independent of the decoding order of the messages, and thus it not affected by the solution of the optimization in (11).

### IV. TIME ALLOCATION ALGORITHM DESIGN FOR COMMON-THROUGHPUT MAXIMIZATION

In this section, we aim to maximize the common-throughput, i.e. the throughput in the case where all users aim to transmit with equal rate,  $\mathcal{R}_{eq}$ . For this purpose,  $T$  as well as the time-sharing configuration need to be optimized. In contrast to the time-sharing configuration of the previous section for minimum rate improvement, the solution provided in this section does not necessarily maximize the sum-throughput. Consequently,  $T$  can be adjusted accordingly, in order to maximize the common-throughput. Next, the indices of the users are ordered according to  $g_1 \geq g_2 \geq \dots \geq g_N$ , however, the order of decoding depends on the time-sharing.

#### A. Problem Formulation and Solving Process Discussion

Taking into account the above considerations, the problem of common-throughput maximization can be formulated as

$$\begin{aligned} & \max_{\tau, T, \mathcal{R}_{eq}} \mathcal{R}_{eq} \\ & \text{s.t.} \quad C_n : \tilde{R}_n \geq \mathcal{R}_{eq}, \forall n \in \mathcal{N}, \\ & \quad C_{N+1} : 0 < T < 1. \end{aligned} \quad (12)$$

A selection for  $T$  corresponds to a specific achievable rate region for the set of users  $\mathcal{N}$ , where the time-sharing technique can also be used. Thus, for a given time  $T$ , a set of time-sharing fractions,  $\tau$ , can be found that achieves any point of the achievable rate region defined by the inequalities

$$\begin{aligned} \tilde{R}_n(T) & \leq T \log_2 \left( 1 + \frac{\eta\rho g_n(1-T)}{T} \right), \forall n \in \mathcal{N} \\ \sum_{n \in \mathcal{M}_k} \tilde{R}_n(T) & \leq T \log_2 \left( 1 + \frac{\eta\rho(1-T) \sum_{n \in \mathcal{M}_k} g_n}{T} \right), \forall k : \mathcal{M}_k \subseteq \mathcal{N}, \end{aligned} \quad (13)$$

where the second inequality holds for any sum set,  $\mathcal{M}_k \subseteq \mathcal{N}$ . Now, suppose that the BS cancels all other users' messages, for the user with the weakest link. In this case it is desired that its throughput is at least equal to the final achievable  $\mathcal{R}_{eq}$ , i.e.

$$\underbrace{T \log_2 \left( 1 + \frac{\eta\rho(1-T)g_N}{T} \right)}_{R_N} \geq \mathcal{R}_{eq}. \quad (14)$$

Accordingly, for the two weakest users, that is for  $n = N$  and  $n = N - 1$ , the sum of their throughput is maximized when the BS cancels out all other users' messages, while for one of them it also cancels the other user's message. Since they can allow time-sharing for the time that each user's message will be canceled, for the sum of the throughput of these two users it must hold that

$$\underbrace{T \log_2 \left( 1 + \frac{\eta\rho(1-T) \sum_{n=N-1}^N g_n}{T} \right)}_{R_{N-1} + R_N} \geq 2\mathcal{R}_{eq}. \quad (15)$$

Following the same strategy for all other users, it yields that  $R_{eq}$  is bounded by the following set of inequalities

$$R_{eq} \leq \frac{T \log_2 \left( 1 + \frac{\eta\rho \sum_{j=n}^N g_i}{1-T} \right)}{(N+1-n)}, \forall n \in \mathcal{N}, \quad (16)$$

in which  $\tau$  does not appear. Consequently, the optimization in (12) can be optimally solved by reducing it into two disjoint optimization problems, minimizing the initial search space. The two optimization problems are:

*Problem 1: Optimization of  $T$  and  $\mathcal{R}_{\text{eq}}$*

$$\begin{aligned} & \mathbf{max}_{T, \mathcal{R}_{\text{eq}}} \mathcal{R}_{\text{eq}} \\ & \mathbf{s.t.} \quad C_n : T \log_2 \left( 1 + \frac{\eta\rho \sum_{i=n}^N g_i}{1-T} \right) \geq \\ & \quad \quad \quad (N+1-n)\mathcal{R}_{\text{eq}}, \forall n \in \mathcal{N}, \\ & \quad C_{N+1} : 0 < T < 1, \end{aligned} \quad (17)$$

*Problem 2: Calculation of the time-sharing vector  $\tau$*

$$\begin{aligned} & \mathbf{find}_{\tau} \tau \\ & \mathbf{s.t.} \quad C_n : \tilde{R}_n(T^*) \geq \mathcal{R}_{\text{eq}}^*, \forall n \in \mathcal{N}. \end{aligned} \quad (18)$$

In the above,  $\mathcal{R}_{\text{eq}}^*$  denotes the optimal solution for  $\mathcal{R}_{\text{eq}}$ , which is calculated by solving Problem 1. The solution of Problem 2 is calculated after the solution of Problem 1, while both of them are solved only once. Please note that, when solving Problem 2, since  $T^*$  and  $\mathcal{R}_{\text{eq}}^*$  have already been fixed, this is a linear optimization problem, with similar structure to (11). Thus, it can be solved by utilizing the same linear programming methods. On the other hand, Problem 1 is jointly concave with respect to  $T$  and  $\mathcal{R}_{\text{eq}}$ , and satisfies Slater's constraint qualification. Thus, it is a convex optimization problem, which can be solved by standard numerical methods such as a combination of interior point methods and bisection method<sup>1</sup>. However, we use dual-decomposition, which proves to be extremely efficient, since, given the Lagrange multipliers (LMs), the optimal  $T$  can be found by the solution of only one simple equation. More importantly, using the adopted method, it is guaranteed that the optimal solution can be obtained in polynomial time [17].

### B. Dual-Problem Formulation and Solution of Problem 1

In this subsection, the optimization problem (17), i.e. Problem 1, is solved by *Lagrange dual decomposition*. Since the primal problem is convex and satisfies the Slater's condition qualifications, strong duality holds, and thus, solving the dual problem is equivalent to solving the primal problem [17]. In order to formulate the dual problem, the Lagrangian of Problem 1 is needed, which is given by

$$\begin{aligned} \mathcal{L}(\boldsymbol{\lambda}, T, \mathcal{R}_{\text{eq}}) = & \mathcal{R}_{\text{eq}} + \sum_{n=1}^N \lambda_n \times \\ & \left( \frac{T \log_2 \left( 1 + \frac{\eta\rho \sum_{i=n}^N g_i}{1-T} \right)}{N+1-n} - \mathcal{R}_{\text{eq}} \right), \end{aligned} \quad (19)$$

where  $\lambda_n \geq 0$  is the Lagrange multiplier, which corresponds to the constraint  $C_n$  and  $\boldsymbol{\lambda}$  is the Lagrange multiplier vector with elements  $\lambda_n$ . The constraint  $C_{N+1}$  is absorbed into the

<sup>1</sup>The bisection method is used in [5], in a problem of similar structure in order to handle the linear objective function. In this paper, we do not use the bisection method, in order to avoid the corresponding extra complexity.

Karush-Kuhn-Tucker (KKT) conditions, and is presented in detail in the next subsection.

The dual problem is given by

$$\min_{\boldsymbol{\lambda}} \max_{T, \mathcal{R}_{\text{eq}}} \mathcal{L}(\boldsymbol{\lambda}, T, \mathcal{R}_{\text{eq}}). \quad (20)$$

Considering the parts of the Lagrangian related to  $\mathcal{R}_{\text{eq}}$ , it holds that

$$\max_{\mathcal{R}_{\text{eq}}} (1 - \sum_{n=1}^N \lambda_n) \mathcal{R}_{\text{eq}} = \begin{cases} 0 & \text{if } \sum_{n=1}^N \lambda_n = 1, \\ \infty & \text{otherwise.} \end{cases} \quad (21)$$

Thus, the dual problem in (20) is bounded if and only if  $\sum_{n=1}^N \lambda_n = 1$ . By setting

$$\lambda_N = 1 - \sum_{n=1}^{N-1} \lambda_n \quad (22)$$

in (19), the variable  $\mathcal{R}_{\text{eq}}$  vanishes and the dual problem in (20) is simplified to

$$\min_{\boldsymbol{\lambda}} \max_T \tilde{\mathcal{L}}(\boldsymbol{\lambda}, T). \quad (23)$$

where  $\tilde{\mathcal{L}}(\boldsymbol{\lambda}, T) = \mathcal{L}(\boldsymbol{\lambda}, T, \mathcal{R}_{\text{eq}})|_{(22)}$ .

The simplified dual problem in (23) can be iteratively solved in two consecutive layers, namely *Layer 1* and *Layer 2*, which are explained below. In each iteration, the optimal  $T$  is calculated for a fixed LM vector, which is then updated in *Layer 2* using the gradient method.

1) *Layer 1*: According to the KKT conditions, the optimal value of  $T$  is given by

$$T^* = \left[ T \in \mathbb{R} : \sum_{n=1}^N \frac{\lambda_n}{N+1-n} \left( \ln \left( 1 + \frac{\eta\rho \sum_{i=n}^N g_i}{T} - \frac{\eta\rho \sum_{i=n}^N g_i}{T(1 - \eta\rho \sum_{i=n}^N g_i) + \eta\rho \sum_{i=n}^N g_i} \right) = 0 \right) \right]_{\epsilon}^{\epsilon} \quad (24)$$

where  $[\cdot]_x^y = \min(\max(\cdot, x), y)$ ,  $\epsilon \rightarrow 0^+$  and  $\epsilon \rightarrow 1^-$ .

2) *Layer 2*: Since the dual function is differentiable, using the gradient method the LMs can be updated as follows

$$\begin{aligned} \lambda_n[t+1] = & \left[ \lambda_n[t] - \hat{\lambda}_n[t] \left( \frac{T \log_2 \left( 1 + \frac{\eta\rho \sum_{i=n}^N g_i}{1-T} \right)}{N+1-n} \right. \right. \\ & \left. \left. - T \log_2 \left( 1 + \frac{\eta\rho g_N}{1-T} \right) \right) \right]_{\mathbb{U}_n}^+, \forall n \in \{1, \dots, N-1\}, \end{aligned} \quad (25)$$

where  $t$  is the iteration index,  $\hat{\lambda}_n$ ,  $n \in \{1, \dots, N-1\}$  are positive step sizes,  $[\cdot]^+ = \max(\cdot, 0)$ , and  $\mathbb{U}_n$  denotes the projection operator on the feasible set  $\mathbb{U}_n = \{\lambda_n | \sum_{n=1}^N \lambda_n = 1\}$ . The projection can be simply implemented by a clipping function  $[\lambda_n[t+1]]_0^{1 - \sum_{i=1}^{n-1} \lambda_i}$  and  $\lambda_N$  can be obtained from (22). Since Problem 1 is concave, it is guaranteed that the iterations between the two layers converge to the optimal solution if the

size of the chosen step satisfies the infinite travel condition [18]

$$\sum_{t=1}^{\infty} \hat{\lambda}_n[t] = \infty, n \in \{1, \dots, N-1\}. \quad (26)$$

3) *Calculation of  $\mathcal{R}_{\text{eq}}^*$* : After the two layers convergence, the optimal  $\mathcal{R}_{\text{eq}}$  can be evaluated by

$$\mathcal{R}_{\text{eq}}^* = \min_{n \in \mathcal{N}} \left( \frac{T^* \log_2 \left( 1 + \frac{\eta \rho \sum_{i=n}^N g_i}{T^*} \right)}{N+1-n} \right), \quad (27)$$

where  $T^*$  is given by (24). This is because  $\mathcal{R}_{\text{eq}}^*$  is actually limited by the most stringent constraint  $C_n$ .

## V. SIMULATION RESULTS AND DISCUSSION

For the simulations, we assume that the users are uniformly distributed in a ring-shaped surface, where  $r_{c1} = 5$  m is the radius of the inner circle and  $r_{c2} = 20$  m is the radius of the outer circle, while the BS is located at the center of the circles. We assume a carrier center frequency of 470 MHz which will be used in the standard IEEE 802.11 for the next generation of Wi-Fi systems [4], [19]. Furthermore, the TGn path loss model for indoor communication is adopted [4], [20]. All statistical results are averaged over  $10^5$  random channel realizations. The receiver of the BS is assumed to have a white power spectral density of  $-174$  dBm/Hz, while all directional antenna gains are assumed to be equal to 7.5 dB. Furthermore, the energy harvesting efficiency of each user is assumed to be  $\eta_1 = 0.5$ , while the amplifier's efficiency is  $\eta_2 = 0.38$ . Finally, the available bandwidth is considered to be 1 MHz.

In Fig. 1, the throughput that is achieved by all methods discussed in this paper is illustrated and compared for the case of  $N = 3$ . More specifically, Fig. 1 includes: i) the minimum-throughput that NOMA with fixed decoding order and TDMA achieve when maximizing the sum-throughput, ii) the normalized sum-throughput that is achieved in this case (i.e., the sum throughput divided by the number of users), iii) the equal rate among users that NOMA and TDMA can achieve, and iv) the minimum rate that NOMA achieves without reducing the sum-throughput, employing time-sharing. It is evident that both NOMA and TDMA achieve the same normalized sum-throughput, however in this case, the application of the proposed NOMA scheme results in a notable increase of the minimum-throughput, for the whole range of  $P_0$ , even when time-sharing is not used. When the time-sharing technique is applied, the minimum-throughput that NOMA achieves is greater, even compared to the common-throughput achieved by TDMA, for the medium and high  $P_0$  region. Moreover, when maximizing the common-throughput, NOMA clearly outperforms TDMA, increasing the common achievable rate for the whole range of the transmit power values of the BS, and especially for the high  $P_0$  region.

Fig. 2 depicts the effect of the number of users on the system's performance. It can be easily observed that, as the number of users increases, both the common-throughput and

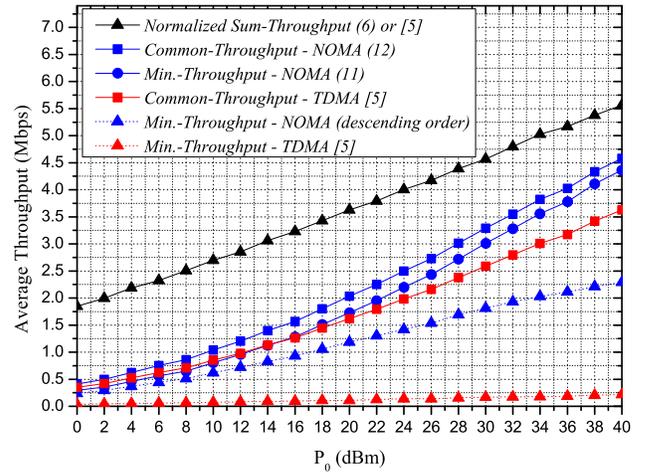


Fig. 1. Comparison of average throughput when  $N = 3$  and  $R_{c2} = 20m$ .

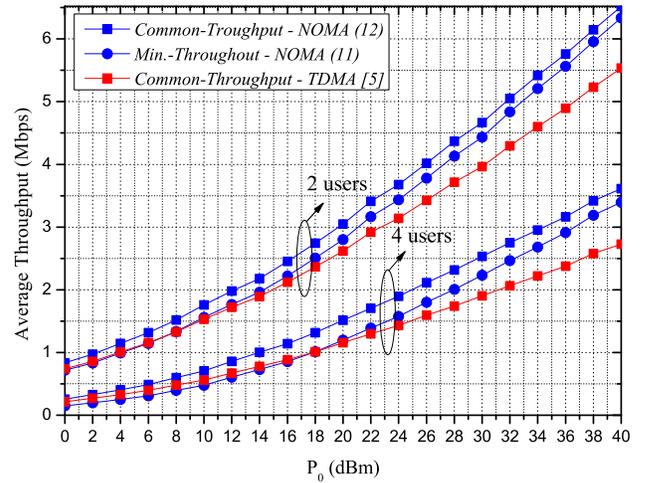


Fig. 2. Comparison of (11), (12) and [5] in terms of minimum-throughput when  $N = 2$  and  $N = 4$ .

the minimum-throughput that NOMA achieves decrease. However, the common-throughput that NOMA achieves is always higher than the common-throughput that TDMA achieves. Furthermore, as the number of users increases, the difference between common-throughput and minimum-throughput that NOMA achieves when maximizing the common-throughput and the sum-throughput, respectively, also increases. Thus, when  $N = 2$ , maximizing the sum-throughput has a lower impact on the minimum data rate, compared to the case when  $N = 4$ .

In Fig. 3, the time dedicated to charging is depicted for the cases that the sum-throughput and the common-throughput are maximized, for  $N = 2$  and  $N = 4$ . As it can be observed, when the aim is to maximize the sum-throughput and the number of users increases, the portion of time dedicated to energy transfer is reduced. This happens because the number of users with good channel conditions increases. Since more users have more energy to transmit, the time that is dedicated to information transmission increases. However, when the common-throughput is maximized, the user with the worst

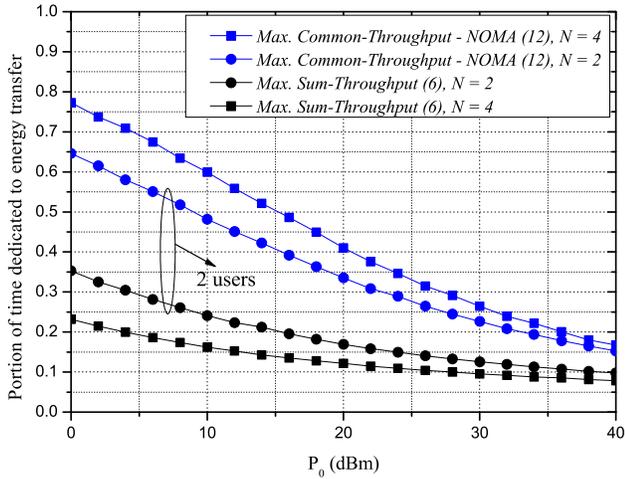


Fig. 3. Comparison of (6) and (12) in terms of portion of time dedicated to energy transfer when  $N = 2$  and  $N = 4$ .

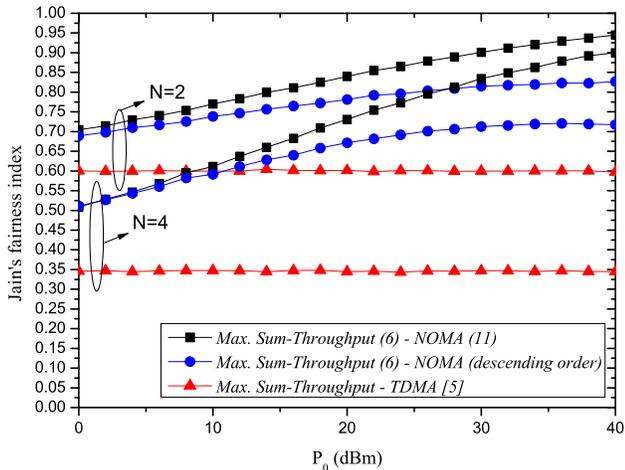


Fig. 4. Jain's fairness index comparison when  $N = 2$  and  $N = 4$ .

channel conditions must have enough energy supply to achieve the common rate. Thus, in this case, as the number of users increases, the portion of time dedicated to energy transfer also increases.

In order to fairly compare the two schemes (NOMA and TDMA) in Fig. 4, we use the Jain's fairness index,  $\mathcal{J}$ , which is given by [14]

$$\mathcal{J} = \frac{(\sum_{n=1}^N R_n)^2}{N \sum_{n=1}^N R_n^2}. \quad (28)$$

Note that Jain's fairness index is bounded between 0 and 1, with unitary value indicating equal users' rates. It is seen in Fig. 4 that NOMA provides more fairness compared to TDMA, for the whole range of  $P_0$ . Also note that the three illustrated schemes achieve the same sum-throughput, for the same number of users.

## VI. CONCLUSIONS

In this paper, we have studied time-allocation methods in order to maximize the data rates and improve fairness in wireless powered communication systems with NOMA. All

formulated optimization problems can be solved by using linear programming methods and convex optimization tools. Also we have compared the proposed scheme with the case that the energy harvesting nodes utilize TDMA, which is used as a baseline. Extensive simulation results have shown that the proposed scheme outperforms the baseline, in terms of throughput and fairness. Finally, they reveal an interesting dependence between sum-throughput, minimum data rate, and wireless power transfer.

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