Topology Feedback Quiz, week 2: bases, continuous functions

Open books. 10–15 minutes. Not for credit. To be marked in class.

We will consider three topological spaces:

- $\mathbb{R}_{\text{antidiscrete}}$, the real line with antidiscrete topology
- $\mathbb{R}$, the real line with Euclidean topology **(RECALL: open set = union of open intervals)**
- $\mathbb{R}_{\text{discrete}}$, the real line with discrete topology **(RECALL: <u>all</u> subsets of $\mathbb{R}$ are open)**

Question 1 ♣ Is the collection

$$\{(a,b) : a, b \in \mathbb{R}\}$$

of all intervals a base, or at least an open cover, for each of the three spaces?

- ◯ base for $\mathbb{R}_{\text{antidiscrete}}$
- ◯ open cover for $\mathbb{R}_{\text{antidiscrete}}$
- ◯ base for $\mathbb{R}$
- ◯ open cover for $\mathbb{R}$
- ◯ base for $\mathbb{R}_{\text{discrete}}$
- ◯ open cover for $\mathbb{R}_{\text{discrete}}$

Question 2 ♣ Is the collection

$$\{\{p\} : p \in \mathbb{R}\}$$

of all **singletons** a base, or at least an open cover, for each of the three spaces?

- ◯ base for $\mathbb{R}_{\text{antidiscrete}}$
- ◯ open cover for $\mathbb{R}_{\text{antidiscrete}}$
- ◯ base for $\mathbb{R}$
- ◯ open cover for $\mathbb{R}$
- ◯ base for $\mathbb{R}_{\text{discrete}}$
- ◯ open cover for $\mathbb{R}_{\text{discrete}}$

Question 3 ♣ Which functions on $\mathbb{R}^2$ are continuous? Here $\mathbb{R}^2$ has Euclidean topology, and (x, y) denotes a point in $\mathbb{R}^2$.

- ◯ $\mathbb{R}^2 \to \mathbb{R}_{\text{antidiscrete}}$, $(x,y) \mapsto x$
- ◯ $\mathbb{R}^2 \to \mathbb{R}$, $(x,y) \mapsto x$
- ◯ $\mathbb{R}^2 \to \mathbb{R}_{\text{discrete}}$, $(x,y) \mapsto x$

Question 4 Write down an example of a **continuous** function $f\colon \mathbb{R}^2 \to \mathbb{R}$ (Euclidean topology on both) and sets $A, B \subset \mathbb{R}^2$ such that:

- A is open but $f(A)$ is not open;
- B is closed but $f(B)$ is not closed.