# Topology Feedback Quiz, week 1: sets and functions

Open books. 10–15 minutes. Not for credit. To be marked in class.

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We work with functions $f\colon \mathbb{R} \to \mathbb{R}$.

Recall that the image of a set $A \subseteq R$ under $f$ is $f(A) = \{f(a) : a \in A\}$, and the preimage of a set $B \subseteq \mathbb{R}$ is $f^{-1}(B) = \{x \in \mathbb{R} : f(x) \in B\}$.

"Is it always true that..." below asks whether the statement is true for **all** functions $f\colon \mathbb{R} \to \mathbb{R}$ and **all** subsets $A, B \subseteq \mathbb{R}$.

**Question 1** Is it always true that $f(A \cup B) = f(A) \cup f(B)$?

◯ Yes ◯ No, give a counterexample ______________________

**Question 2** Is it always true that $f(A \cap B) = f(A) \cap f(B)$?

◯ Yes ◯ No, give a counterexample ______________________

**Question 3** Is it always true that $f^{-1}(A \cup B) = f(A) \cup f(B)$? corrected: $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$?

◯ Yes ◯ No, give a counterexample ______________________

**Question 4** Is it always true that $f^{-1}(A \cap B) = f(A) \cap f(B)$? corrected: $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$?

◯ Yes ◯ No, give a counterexample ______________________

**Question 5** Is it always true that $f(\mathbb{R} \setminus A) = \mathbb{R} \setminus f(A)$?

◯ Yes ◯ No, give a counterexample ______________________

**Question 6** Is it always true that $f^{-1}(\mathbb{R} \setminus A) = \mathbb{R} \setminus f^{-1}(A)$?

◯ Yes ◯ No, give a counterexample ______________________

**Question 7** Is it always true that if $A$ is finite, then $f(A)$ is finite?

◯ Yes ◯ No, give a counterexample ______________________

**Question 8** Is it always true that if $A$ is finite, then $f^{-1}(A)$ is finite?

◯ Yes ◯ No, give a counterexample ______________________

**Question 9** Is it always true that if $A$ is infinite, then $f(A)$ is infinite?

◯ Yes ◯ No, give a counterexample ______________________

**Question 10** Is it always true that if $A$ is infinite, then $f^{-1}(A)$ is infinite?

◯ Yes ◯ No, give a counterexample ______________________

**Exercise.** Call a subset $A$ of $\mathbb{R}$ "cocountable" if $A = \emptyset$ or $\mathbb{R} \setminus A$ is finite or countably infinite.
(a) Show that the collection of all cocountable subsets of $\mathbb{R}$ is a topology on $\mathbb{R}$.
(b) Is this topology the same as discrete topology? Antidiscrete topology? Cofinite topology?
This exercise will be discussed in the week 2 tutorial.

**Exercise (harder).** Let

$$A = \bigcap_{n \in \mathbb{N}} \bigcup_{p \in \mathbb{Z},\ q \in \mathbb{N}} \left( \frac{p}{q} - \frac{1}{nq}, \frac{p}{q} + \frac{1}{nq} \right).$$

Denote by $\mathbb{Q}$ the set of all rationals.
Is $\mathbb{Q} \subseteq A$? Is $\mathbb{Q} = A$? Is $A = \mathbb{R}$?
This exercise will not be discussed in class, but a solution will be made available later.