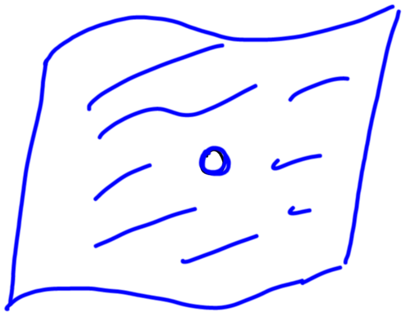
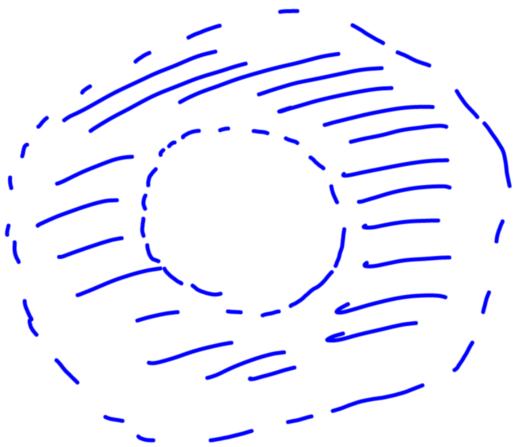
Week Ø3 tutorial

SEAtS 394617

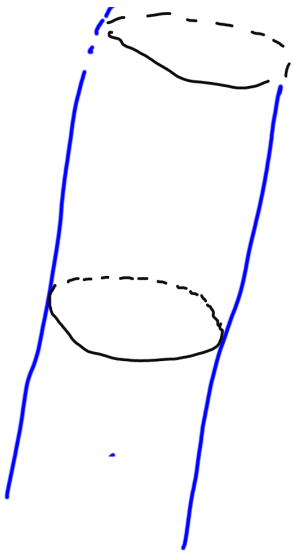
Determine which pairs of the following topological spaces are homeomorphic. Give a convincing description, or an explicit formula, for the homeomorphism where necessary; give reasons if not homeomorphic.



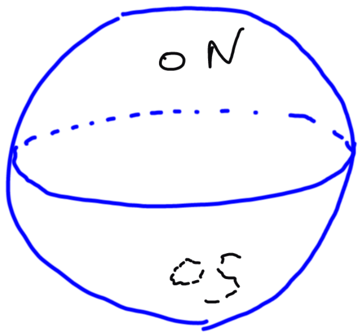
The punctured Euclidean plane $\mathbb{R}^2 \setminus \{(0,0)\}$



The open annulus $\{(x,y) \in \mathbb{R}^2 : 1 < \sqrt{x^2+y^2} < 2\}$

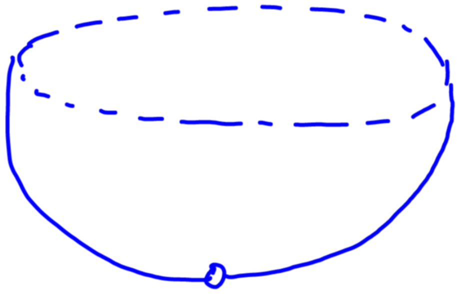


The cylinder $\{(x,y,z) \in \mathbb{R}^3 : x^2+y^2=1\}$



The twice-punctured sphere $S^2 \setminus \{(0,0,1), (0,0,-1)\}$

The punctured open hemisphere



The set $\mathbb{R}^2 \setminus \{(0,0)\}$ with the antidiscrete topology

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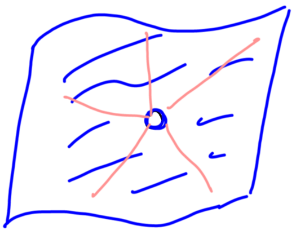
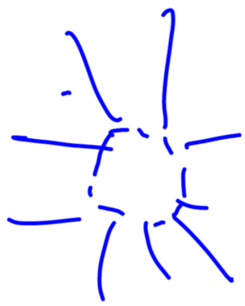
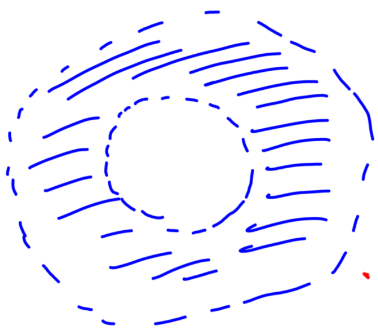


The punctured Euclidean plane $\mathbb{R}^2 \setminus \{(0,0)\}$

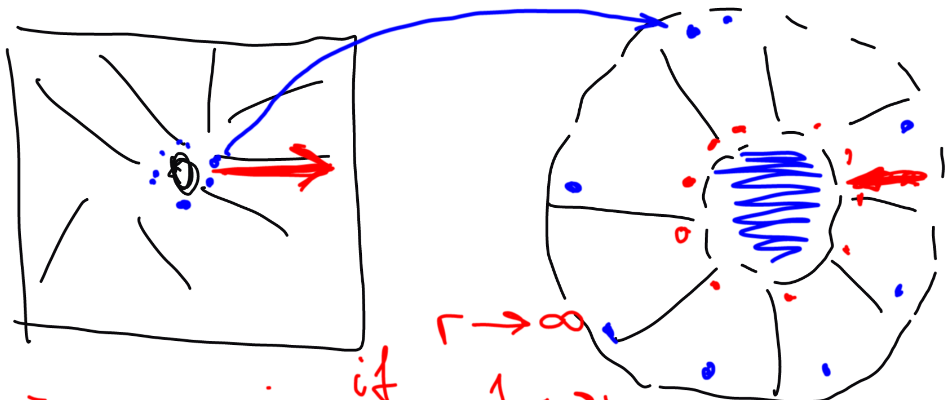


$$f(r,\theta) = \left(1 + \frac{1}{r+1}, \theta\right)$$



The open annulus

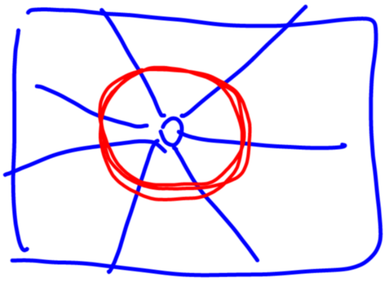
$$\{ (x,y) \in \mathbb{R}^2 : 1 < \sqrt{x^2+y^2} < 2 \}$$



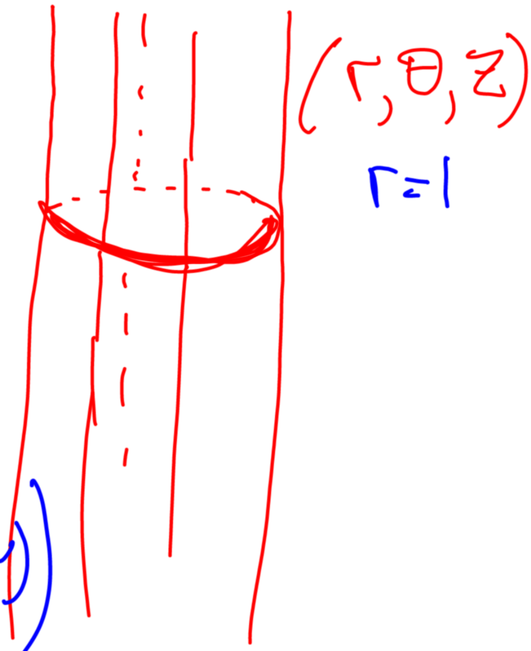
r very small

$$1 + \frac{1}{r+1} \lesssim 2$$

if $r \to \infty$ then $\frac{1}{r+1} \to 1$



$\xrightarrow[(r,\theta)]{g}$



$\mathbb{R}^2 \setminus \{(0,0)\}$

$\downarrow$

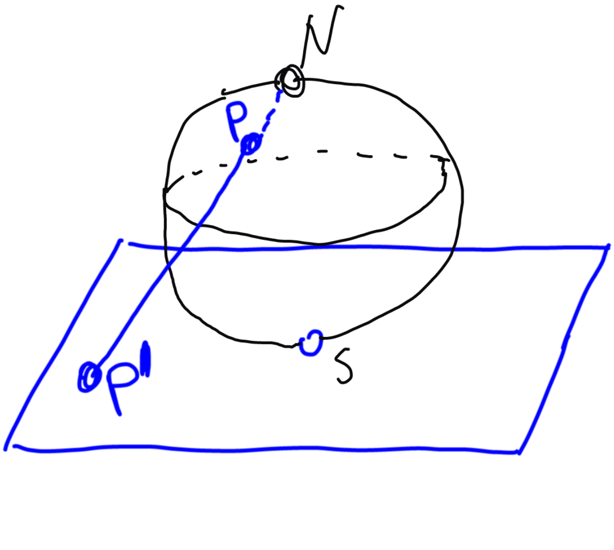
$(1, \theta, \ln(r))$

$(e^z, \theta) \longleftarrow (1, \theta, z)$

$\in \mathbb{R}^2 \setminus \{(0,0)\}$ $\quad$ $\in Cyl$

cylinder

$\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}$



$P \longmapsto P'$

$\mathbb{R}^2 \setminus \{(0,0)\}$ antidiscrete

$\uparrow f$

$$\left.\begin{aligned} f(B_1((0,0))) &\neq \mathbb{R}^2 \\ &\neq \emptyset \end{aligned}\right\}$$ the only open set

NOT HOMEOMORPHIC

$\underbrace{B_1((0,0))}_{\text{open}} \subseteq \mathbb{R}^2 \setminus \{(0,0)\}$ Euclidean