MATH31010 TOPOLOGY AND ANALYSIS

Week 01 lecture A

SEAtS PIN 374880

* A year-long course unit worth 20 credits
* Assessment:
  - Class Test 1 (S1 week 07) 15% of final mark
  - Class Test 2 (S2 week 05) 15% of final mark
  - Final exam (May/June 2025) 70% of final mark
* Lecturers: (email = Name.Surname@manchester.ac.uk)

Yuri Bazlov	Intro. topology	w 1-9
Yotam Smilansky	Intro. linear analysis	w 10-12, (S2) 1-5
Donald Robertson	Advanced topics in topology & analysis	(S2) 6-11

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What to do to learn Intro. topology well and to do well in $\underbrace{\text{Class Test 1}}_{\text{Testing w1-5 material}}$?

- Attend lectures
- Do lecture review quizzes (online)
- Prepare for tutorials by doing HW exercises
- In the tutorial:
  * Do unseen quiz
  * Discuss the quiz
  * Discuss a HW exercise
- In week 6, revise the w1-5 material
- Important: ASK QUESTIONS!

Topology - origins & motivation

Goals : - define the notion of a continuous function (= map) between sets without a metric.

Year 1 $\quad f : X \to Y$ continuous

$X, Y \subseteq \mathbb{R}$

Year 2 $\quad X, Y$ metric spaces

Year 3 $\quad X, Y$ topological spaces

Fundamentals of sets

A a set (capital letter)

$a \in A$ an element of the set A (small letter)

$\mathcal{F}$ a collection of sets (script letter)

Examples

$$\mathcal{F} = \{A, A\}$$

$$\mathcal{F} = \emptyset \quad \text{(empty collection)}$$

$$\mathcal{F} = \{ (0, \tfrac{1}{n}) : n \in \mathbb{N} \}$$ a countable collection of subsets of $\mathbb{R}$

$$\mathcal{G} = \{ (0, x) : x \in \mathbb{R}, x > 0 \}$$ uncountable collection

$B \subseteq A$ B is a subset of A

$(\forall x \in B,\ x \in A)$

If $\mathcal{F}$ is a collection of subsets of X,
we can construct:

$$\bigcup \mathcal{F} \overset{\text{def}}{=} \{ x \in X \mid \exists A \in \mathcal{F},\ x \in A \}$$

$$\bigcap \mathcal{F} \overset{\text{def}}{=} \{ x \in X \mid \forall A \in \mathcal{F},\ x \in A \}$$

e.g. $\mathcal{F} = \{A_1, A_2\} \Rightarrow \bigcup \mathcal{F} = A_1 \cup A_2,$

$\bigcap \mathcal{F} = A_1 \cap A_2$

$\mathcal{F} = \{ A_\alpha \mid \alpha \in I \}$ (I = index set) $\bigcap \mathcal{F} = \bigcap_{\alpha \in I} A_\alpha$

Examples.

1. $$\bigcup_{\substack{x \in \mathbb{R} \\ x > 0}} (0, x) = \mathbb{R}_{>0} =: (0, +\infty)$$

2. $$\bigcap_{\substack{x \in \mathbb{R} \\ x > 0}} (0, x) = \emptyset$$

3. $\mathcal{F} = \emptyset$, an empty collection of subsets of $\mathbb{R}$. What is $\bigcup \mathcal{F}$? $\bigcup \mathcal{F} = \emptyset$

What is $\bigcap \mathcal{F}$? $\bigcap \mathcal{F} = \mathbb{R}$.

$$\bigcap \mathcal{F} = \{x \in \mathbb{R} : \forall A \in \emptyset, x \in A\} = \{x \in \mathbb{R}\}$$

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Functions

$f : X \to Y$
set set

Notation: $x \in X \mapsto f(x) \in Y$

image of x under the function (= map) (= mapping) f

image under f^{-1}

preimage

$A \subseteq X$ $\quad f(A) = \{ f(x) : x \in A \} \subseteq Y$

$B \subseteq Y$ $\quad f^{-1}(B) = \{ x \in X : f(x) \in B \}$

the preimage of the set B under f

Rem If f is bijective and has inverse function $f^{-1} : Y \to X$, then $f^{-1}(B) = f^{-1}(B)$

DEF (Topological space)

Let X be a set. Suppose $\mathcal{T}$ is a collection of subsets of X such that:

(i) $X \in \mathcal{T}$

(ii) for any subcollection $\mathcal{T}_1$ of $\mathcal{T}$,

$\bigcup \mathcal{T}_1$ is in $\mathcal{T}$

(iii) for any $A, B \in \mathcal{T}$, $A \cap B \in \mathcal{T}$

Then $\mathcal{T}$ is called a **topology** on (the set) X, and $(X, \mathcal{T})$ is a **topological space.**