Math 309 **Spring 2012**

Continuous Optimization

Midterm Exam 2 March 22, 2012

Name (print): ______________________

Computing ID: ______________________

Signature: ______________________

Question	Grade
1	
2	
3	
Total	

Theorem 0.1 (First-order necessary (KKT) conditions)**.** *Suppose x^* is a local solution, f and c_i are continuously differentiable and the* **LICQ** *holds at x^*. Then there exist a Lagrange multiplier λ^*, $i \in \mathcal{E} \bigcup \mathcal{I}$, such that*

(1) $c_i(x^*) = 0$, *for all* $i \in \mathcal{E}$ *(Feasible condition for equality constraints)*

(2) $c_i(x^*) \geq 0$, *for all* $i \in \mathcal{I}$ *(Feasible condition for inequality constraints)*

(3) $\lambda_i^* \geq 0$, *for all* $i \in \mathcal{I}$

(4) $\lambda_i^* c_i(x^*) = 0$, *for all* $i \in \mathcal{E} \bigcup \mathcal{I}$ *(Complementarity)*

(5) $\nabla_x L(x^*, \lambda^*) = 0$

Theorem 0.2 (Second-order necessary conditions)**.**

$$w^T \nabla_{xx} L(x^*, \lambda^*) w \geq 0, \qquad \forall \ w \in \mathcal{C}(x^*, \lambda^*).$$

Theorem 0.3 (Second-order sufficient conditions)**.**

$$w^T \nabla_{xx} L(x^*, \lambda^*) w > 0, \qquad \forall \ w \in \mathcal{C}(x^*, \lambda^*), \ w \neq 0.$$

1. (4+8+8 pt) Consider the problem

$$\begin{array}{ll} \min & f(x) = x_1^2 + (x_2 - 3)^2 \\ \text{subject to} & c_1(x) = x_1^2 - 2x_2 \geq 0, \\ & c_2(x) = x_1 \geq 0. \end{array}$$

(a) Plot the feasible region and three contours of the objective function (No need to find the minimizer, because it depends on how accurately you draw them).

(b) Given that c_2 is NOT active at the global minimizer x^*, find x^*.

(c) Show that the **second order sufficient condition** is satisfied at x^* (no need to check the first order conditions).

2.(15 pt) Consider the problem

$$\begin{array}{ll} \min & f(x) = x_1^2 + (x_2 - 1)^2 \\ \text{subject to} & c(x) = x_1^2 - \kappa x_2 \geq 0. \end{array}$$

Here κ is a positive constant. Find the critical κ_c, such that $(0,0)$ is a local minimizer for any $\kappa > \kappa_c$ (and $(0,0)$ is not a local minimizer when $\kappa < \kappa_c$).

3. (15 pt) Write down the dual problem of the following linear programming

$$\begin{array}{ll} \min & f(x) = x_1 \\ \text{subject to} & x_1 + x_2 = 1, \\ & x_2 \leq 0, \\ & x_2 \geq -1. \end{array}$$