Math 309 Assignment 3

Due on February 28

Problem 1(6 pt). Consider the inconsistent system of linea equations

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 0 \\ x_1 + x_2 &= 2 \\ x_1 - x_2 &= 0. \end{aligned}$$

(1) Write down the formulation for the least square solution of this problem, i.e., find A and b in the minimization problem $\min \|Ax - b\|^2$. (2) Find the equation that the minimizer $x^* = \begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix}$ satisfies and find the minimizer $\begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix}$. (3) Calculate $r = Ax - b$, verify that r is orthogonal to the columns of A.

Problem 2(4 pt) If $\|a\|_2 = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2} = A$ and $\|b\|_2 = \sqrt{b_1^2 + b_2^2 + \cdots + b_n^2} = B$ for some vectors $a, b \in \mathbb{R}^n$ with nonnegative components ($a_i, b_i \geq 0$), show that

$$\max\ a^t b = \max\ (a_1 b_1 + \cdots + a_n b_n) \leq AB.$$

Problem 3(5 pt). Find all the values of the parameters a and b such that $(0, 0)^t$ minimizes or maximizes the following function

$$f(x_1, x_2) = (a + 2)x_1 + 4x_2$$

subject to the constraint

$$a(x_1 + e^{x_1}) + b(x_2 + e^{x_2}) = 1.$$

(Just find a and b, no need to find any second order conditions to determine whther it is a maximizer or a minimizer. you may get nonlinear equations to solve; just eliminate the variables and you will solve a quadratic equation in the end).

Problem 4(5 pt). Consider the problem of finding the minimum distance from a point $r \in \mathbb{R}^n$ to a set $\{x : a^t x = b\}$. The problem can be written as

$$\min \qquad f(x) = \frac{1}{2}\|x - r\|_2^2 = \frac{1}{2}(x - r)^t(x - r)$$

subject to

$$a^t x = b.$$

Here $a \in \mathbb{R}^n$ and b is a scalar. Find the minimizer x^*.