List of Suggested Dissertation Titles By Supervisor

MSc Applied Mathematics

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Contents

1 Simon Cotter

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1.1 Bayesian inverse problems in applied mathematics

Much of science is reliant on observations to understand the world around us, for instance observations of our atmosphere in weather forecasting. However these observations are often noisy, which leads to uncertainty about what has been observed. There is also the significant problem of assimilating that data into complex models which describe the observed system. The Bayesian framework is a statistical/probabilistic approach which allows for us to combine prior beliefs, complex dynamical models, and data, to arrive at probability distributions which describe our updated beliefs about important parameters in the system, conditioned on the observations. In this project, we will implement Markov chain Monte Carlo methods to sample from these probability distributions, in order to identify not only likely values of the parameters, e.g. the value of the initial condition of an ODE, or a rate constant, but also to assess the uncertainty in that estimation.

[1] Stuart, A. M. (2010) Inverse problems: A Bayesian perspective. Acta Numerica, 19 . pp. 451-559. ISSN 0962-4929.

1.2 Stochastic models of chemical reactions and their approximations

Systems of chemical reactions are traditionally modelled using ODEs, which give repeatable, deterministic outputs. However, in reality, many important chemical reaction systems are not well represented by these types of models, for instance biochemical reactions occurring in the cell. This is because in small volumes, the number of copies of each chemical species is low, and as such the discrete, sporadic and random nature of the reactions can play an important part in the dynamics. In this project, we will investigate stochastic models of chemical reactions, including continuous time Markov chains, Itô stochastic differential equations. We may consider different approximations of these models, including Tau- leaping, Euler-Maruyama approximations of SDEs, or even multiscale approximations.

[1] Gillespie, Daniel T. (1977). "Exact Stochastic Simulation of Coupled Chemical Reactions". The Journal of Physical Chemistry. 81 (25): 2340–2361.

[3] Gillespie, Daniel T. "The chemical Langevin equation." The Journal of Chemical Physics 113.1 (2000): 297-306.

2 Raphael Assier

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I am interested in problems related to wave propagation. Two of my areas of research are the mathematical theory of diffraction (see first project) and the interaction between acoustic waves and flames (see second project).

2.1 Laplace-Beltrami eigenvalues on a sphere with a cut

In the context of wave diffraction, it is frequent to encounter the Laplace-Beltrami operator (that can be interpreted as the spherical Laplace operator restricted to the unit sphere) defined in the spherical coordinates (θ, φ) by

$$\tilde{\Delta} = \frac{1}{\sin(\theta)} \frac{\partial}{\partial \theta} \left(\sin(\theta) \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2(\theta)} \frac{\partial^2}{\partial \varphi^2}.$$

In the particular case of diffraction by a quarter-plane, one must consider the Laplace-Beltrami operator on a sphere with a cut (i.e. the surface of the unit sphere with a portion of the equator line being solid). The knowledge of the eigenvalues of the Laplace-Beltrami operator is essential. These eigenvalues differ according to which boundary conditions are applied at the cut. This project would have a theoretical (characterisation of the eigenvalues) and a numerical part (computation of the eigenvalues). Previous knowledge in spectral theory and finite element methods would be advantageous for this project. Experience with C++ and Linux would also be a plus.

References

[1] R. C. Assier and N. Peake. On the diffraction of acoustic waves by a quarter-plane. Wave Motion, 49(1):64-82, 2012.

[2] E. B. Davies. Spectral Theory and Differential Operators . Cambridge University Press, 1996.

2.2 Flame-acoustic interaction in premixed combustion

A premixed flame usually excites acoustic modes of a combustion chamber through unsteady heat release. The spontaneously emitted sound waves in turn modulate the flame. Such a coupling leads to different types of combustion instabilities, which may have catastrophic effects on the combustion chamber. Recently, a general first-principle flame-acoustic model has been developed. This project will focus on understanding how different simplification assumptions would affect this model.

References

[1] G. Searby. Acoustic instability in premixed flames. Combust. Sci. Technol., 81:64-82, 1992.

[2] Wu, X et al. Combustion instability due to non-linear interaction between sound and flame. J. Fluid. Mech. 497:23-53, 2003.

3 Joel Daou

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Several projects are available related to the mathematical theory of flame propagation or to travelling-waves in reaction-diffusion systems. These are fascinating multidisciplinary areas of applied mathematics involving ordinary and partial differential equations. The approach will typically adopt a combination of analytical techniques (asymptotic methods) and/or numerical techniques (solution of ODEs or PDEs, mostly parabolic and elliptic). Depending on the preference of the candidate, each of the projects can be tailored in its scope and the methodology of study.

3.1 Sample of suggested projects

Propagating Flames and their Stability: This involves the investigation of the various instabilities of flames using analytical and/or numerical approaches. The flames will be modelled as travelling wave solutions to reaction-diffusion-convection equations, which may, or may not, include full coupling with the hydrodynamics (the Navier-Stokes equation).

Flame initiation and propagation in spatially non-uniform mixtures: This is a problem of considerable interest in combustion, whenever the reactants are spatially separated. The approach will be based on asymptotic and/or numerical methods. The Combustion basics needed for the projects will be provided and explained to the candidate.

Generalized Flame Balls and their Stability: Flame balls are balls of burnt gas in a reactive mixture, which constitute stationary solutions to non-linear Poisson's equations. These were first described by the famous Russian physicist Zeldovich (the father of Combustion Theory) about 70 years ago. The fact that these solutions are typically unstable provides a powerful fundamental criterion for successful ignition, i.e. determines the minimum energy (of the spark) required to generate propagating flames. Several projects are available to extend the study of these fascinating flames (mainly their existence and stability) to take into account realistic effects such as the presence of flow-field, non-uniformity of the reactive mixture, proximity of walls, etc.

Taylor dispersion in premixed combustion/reaction-diffusion systems: In 1953, the British physicist G.I. Taylor published an influential paper describing the enhancement of diffusion processes by a (shear) flow, a phenomenon later termed Taylor dispersion. This has generated to date thousands of publications in various areas involving transport phenomena, none of which, until very recently, in the field of combustion, or dedicated to reaction-diffusion front propagation. The project consists of pioneering investigations linking Taylor dispersion and front propagation, and is expected to provide interesting results regarding such propagating fronts and their stability.

4 Jitesh Gajjar

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I can offer projects on topics in hydrodynamic stability theory, computational methods, depending on what students are interested in. They can be either purely numerical, theoretical, or a combination of both. Feel free to contact me (j.gajjar@manchester.ac.uk) if would like to discuss a topic.

4.1 Spin coating of thin films

A number of important applications involve rotating liquid layers to generate uniform solid films. Typically solvents mixed with the material evaporate leaving behind a film on the surface. However, the process does not always generate the desired uniform film and non-uniformities can arise because of various factors. The main objective of this project is to review previous work on the instability of spin coating of a viscoelastic fluid, and investigate the instability problem using asymptotic methods. In the short time available for the project it may not be possible to look at all aspects and it may be necessary to focus on just a few things which can affect the stability of the flow such as hydromagnetic effects, see Chen & Lin (2016). The project will require some knowledge of fluid mechanics, and may involve doing some numerical work using Matlab.

References

[1] Chen, C.K. & Lin, M.C. (2016) Hydromagnetic stability of a thin viscoelastic magnetic fluid on coating flow using Landau equation. J. Mech., 32, 643-651.

5 Paul Glendinning

Email: paul.glendinning@manchester.ac.uk

I am interested in most areas of dynamical systems, particularly bifurcation theory for maps and flows. My current research is in four main areas: Bifurcation theory (particularly global bifurcations), synchronization and blowout bifurcations, low dimensional maps and quasi- periodically forced systems.

5.1 Nonsmooth Bifurcation Theory

Review the essential differences between nonsmooth and smooth bifurcations and then concentrate on some aspect (e.g. border collision normal forms, grazing-sliding bifurcations, chatter).

5.2 Pivot instabilities for pendulums

Very little is written about what happens at the pivot point of a pendulum. The project could explore a recent model using a perpendicularly jointed double pendulum (see Glendinning and Murphy from my webpage) or the use of stabilisers such as counterbalances, or develop new mechanical models.

6 Nico Gray

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I am interested in problems related to flows of granular materials, which are the second most common material on Earth after fluids. There are important applications to a range of hazardous geophysical flows, such as snow avalanches, debris flows and pyroclastic flows, as well as industrial problems.

6.1 Granular avalanches

Avalanches have a shallow aspect ratio, which allows depth-averaged models to be formulated. The governing equations for the thickness h and depth-averaged velocity $\bar{\boldsymbol{u}}$, are similar to the shallow water equations of fluid dynamics

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\bar{\boldsymbol{u}}) = 0,$$

$$\frac{\partial}{\partial t}(h\bar{\boldsymbol{u}}) + \nabla \cdot (h\bar{\boldsymbol{u}} \otimes \bar{\boldsymbol{u}}) + \nabla \left(\frac{1}{2} g h^2 \cos\zeta\right) = gh\boldsymbol{S} + \nabla \cdot \left(\nu h^{\frac{3}{2}} \bar{\boldsymbol{D}}\right),$$

where $\cdot$, $\otimes$ and $\nabla = (\partial/\partial x, \partial/\partial y)$ are the two-dimensional dot product, dyadic product and gradient operators, respectively, g is the constant of gravitational acceleration, $\boldsymbol{S}$ are the gravitational and frictional source terms and $\bar{\boldsymbol{D}}$ is the depth-averaged strain-rate tensor. They differ through the inclusion of depth-averaged in-plane viscous terms on the right-hand side of the second equation. This project will look at the underlying theory, as well as applications to a rich variety of problems with erosion and deposition.

References

[1] Rocha, Johnson & Gray (2019) Self-channelisation and levee formation in monodisperse granular flows. J. Fluid Mech. 876, 591–641.

[2] Viroulet, Baker, Edwards, Johnson, Gjaltema, Clavel & Gray (2017) Multiple solutions for granular flow over a smooth two-dimensional bump. J. Fluid Mech. 815, 77–116.

[3] Gray & Edwards (2014) A depth-averaged $\mu(I)$-rheology for shallow granular free-surface flows. J. Fluid Mech. 755, 503-534.

6.2 Particle segregation in dense granular flows

Granular materials composed of particles with differing grain sizes, densities, shapes, or surface properties may experience unexpected segregation during flow. A bi-disperse mixture of large and small particles satisfies a particle-size segregation equation of the form

$$\frac{\partial \phi^s}{\partial t} + \nabla \cdot (\phi^s \boldsymbol{u}) + \nabla \cdot \left(f_{sl} \phi^s \phi^l \frac{\boldsymbol{g}}{|\boldsymbol{g}|}\right) = \nabla \cdot (\mathcal{D}_{sl} \nabla \phi^s),$$

where ϕ^s is the concentration of small particles, $\boldsymbol{u}$ is the bulk flow field, $\boldsymbol{g}$ is the gravitational acceleration vector, f_{sl} is the segregation velocity magnitude and $\mathcal{D}_{sl}$ is the diffusivity. This project will look at basic theory and solutions, as well as generalizations to polydisperse mixtures.

References

[1] Gray (2018) Particle segregation in dense granular flows. Ann. Rev. Fluid Mech. 50, 407-433.

[2] Gray & Ancey (2011) Multi-component particle-size segregation in shallow granular avalanches. J. Fluid Mech. 678, 535-588.

[3] Barker, Rauter, Maguire, Johnson & Gray (2021) Coupling rheology and segregation in granular flows. J. Fluid Mech. 909, A22-1–46.

7 Stefan Güttel

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My main research area is numerical linear algebra, in particular, fast algorithms for the solution for large sparse eigenvalue problems and the approximation of matrix functions. I'm particularly interested in rational Krylov methods and related rational approximation and parameter optimisation problems. I'm interested in parallel algorithms for solving ordinary differential equations and applications in scientific computing.

For 2020/21 the following project is proposed:

7.1 The paraexp algorithm for linear initial-value problems and its parallel implementation

Paraexp is an algorithm for solving linear initial-value problems on a parallel computer. Apart from discussing the mathematical ingredients of this algorithm, one aim of this thesis is the parallel implementation of paraexp, and a numerical performance comparison with other existing parallel algorithms for linear IVPs. It is necessary that the interested student is familiar with Matlab and optionally has some experience with the parallel computing toolbox. A possible starting point is M. J. Gander and S. Guettel: PARAEXP: A parallel integrator for linear initial-value problems, submitted to SISC, 2012.

8 Andrew Hazel

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8.1 Chemical Induced Growth

Chemical cues often drive growth in biological systems. The aim of this project is to develop the appropriate continuum mechanical framework to incorporate the coupling between chemical signals and the mechanics of growth. The growth will be modelled using similar ideas to those reviewed by Ambrosi et al. (2012) [1], but with close coupling to a system of reaction-diffusion equations for the chemistry. In this project we shall concentrate on the growth of cell walls and initially, a one-dimensional line in two-dimensional space will be studied, before extending the work to two-dimensional surfaces in three-dimensional space. The project will involve an element of computer programming, particularly for the two-dimensional simulations. If desired, finding efficient methods for solving the resulting discrete system may form the end of the project.

References

[1] D. Ambrosi, G. A. Ateshian, E. M. Arruda, S. C. Cowin, J. Dumais, A. Goriely, G. A. Holzapfel, J. D. Humphrey, R. Kemkemer, E. Kuhl, J. E. Olberding, L. A. Taber, and K. Garikipati 2011 Perspectives on biological growth and remodeling J Mech Phys Solids; 59(4): 863-883. doi: 10.1016/j.jmps.2010.12.011

8.2 Morphologies of Sliding Drops

Have you ever wondered why rain drops sliding down a pane of glass adopt particular shapes and how quickly they travel? This project will investigate such questions by using a numerical models and, where appropriate, asymptotic techniques. The work is motivated by a recent paper by Engelnkemper et al (2016) (https://arxiv.org/abs/1607.05482) in which different steady solutions for finite drops of fixed volume are found under the lubrication approximation. The first part of the project will be to recompute these solutions using the advanced numerical methods implemented in our open source, finite-element library oomph-lib (written in C++). On completion of the first part of the project (validation), the robustness of the solutions to changes in the model assumptions will be investigated. Specifically, we shall investigate whether the solution structure persists in the full Stokes equations and, if not, whether a recently developed “improved” reduced model is a better approximation to the behaviour of the Stokes system.

The project will require background knowledge (or willingness to learn) in fluid mechanics, scientific computing and dynamical systems.

Other Projects

I am happy to discuss other possible projects in mechanics if none of these appeal.

9 Matthias Heil

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oomph-lib is the Object-Oriented Multi-PHysics finite-element LIBrary, jointly developed by M. Heil, A. L. Hazel and co-workers. The library provides a general framework for the discretisation and the robust, adaptive solution of a wide range of multi- (and single-)physics problems. The library allows the solution of many "classical" partial differential equations (the Poisson, Advection-Diffusion, and the Navier-Stokes equations; the Principle of Virtual Displacements for solid mechanics; etc.) and it is easy to formulate other, more "exotic" problems. Furthermore, it is straightforward to combine existing single-physics elements to hybrid elements that can be used in multi-physics simulations, such as fluid-structure-interaction and other free-boundary problems.

oomph-lib is open-source software and is freely available at http://www. oomph-lib.org. Have a look around the webpage to see what's currently available.

I usually have a number of (constantly changing) open problems, typically involving fluid and/or solid mechanics. For students who are more interested in numerical methods/ scientific computing, there are also projects that focus on the methodology rather than any specific applications. All projects require either good knowledge of C++ and object-oriented programming or a willingness to acquire such (eminently useful!) skills during the course of the project. Please come and see me if any of this sounds interesting (in principle!) so we can discuss potential projects in more detail.

10 Richard Hewitt

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Prof. Hewitt's research interests include rotating flowss, particle-laden fluids and hydrodynamic stability.

11 Sean Holman

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My general areas of research are in inverse problems, partial differential equations, and geometry, and my work often has a more "pure" flavour. In addition to the three potential projects mentioned below, I am open to discuss other possibilities so if these topics interest you, but don't seem quite right, please don't hesitate to contact me.

11.1 Microlocal analysis and the characterisation of artefacts in CT images

Computerised tomography is a well established technique to construct an image of the density inside an object based on X-ray projections from many different angles. CT images often contain artefacts arising from some aspect of the reconstruction procedure, and these artefacts may in many cases be characterised using the mathematical technique of microlocal analysis. The project will look into this applicaton of microlocal analysis.

11.2 Mathematics of imaging from seismic reflections

This project will look at the mathematics underpinning the exploration of the interior structure of the earth from seismic experiments, and in particular the determination of reflectors within the earth, corresponding to discontinuities or rapid changes in material properties, from reflected seismic waves. This problem is complicated by the fact that the velocities of seismic waves are not constant, and so there is refraction. The mathematics involved are highly nontrivial, and very interesting!

11.3 Numerical investigation of the geodesic X-ray transform.

This project would look at the problem of determining a function from its integrals along the geodesics of a Riemannian metric. The map which takes a function to its integrals over such curves is called the geodesic X-ray transform. It arises in applications as the linearisation of the mapping from a wave speed to the travel time of waves between points on the boundary of a manifold. For example in seismic imaging the "manifold" is the interior of the earth and the metric "distance" is the amount of time it takes for a seismic wave created by a source at one location to reach a receiver at another location. There are quite a few partial results on this problem, but still a precise characterisation of when the transform is injective is not known. The goal of this project would be to investigate numerically the two, and possibly three dimensional cases when the geometry is complicated. When there are conjugate points (i.e. geodesics starting at the same point cross) very little is known, and some insight may be gained by conducting numerical experiments.

12 Yanghong Huang

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My research interest is on applied analysis and numerical computation of partial differential equations and integral equations, usually arising from mathematical biology. Other projects are available from my webpage at

http://personalpages.manchester.ac.uk/staff/yanghong.huang/Projects/

and feel free to meet with me about more details.

12.1 Travelling wave solutions for nonlocal bistable reaction-diffusion equation

In this project, travelling waves solutions for the following non-local bistable reaction-diffusion equation

$$\frac{\partial}{\partial t}u = J * u - u + f(u)$$

is studied. Here J is a non-negative, continuously differentiable function such that $\int_{\mathbb{R}} J(x)dx = 1$, $\int_{\mathbb{R}} |x|J(x)dx < \infty$ with the typical examples $J(x) = \max(0, 1-|x|)$ or $J(x) = \exp(-\pi x^2)$. Moreover, $J * u(x) = \int_{\mathbb{R}} J(x-y)u(y)dy$ is the convolution between the kernel J and the solution u and $f(u) = F'(u)$ is the derivative of a double-well potential $F(u)$. Both numerical solutions and qualitative properties of the travelling waves will be investigated.

References:

[1] X. Chen, "Generation, propagation, and annihilation of metastable patterns", J. Differential Equations 206, 399-437 (2004).

[2] Bates, Peter W., Paul C. Fife, Xiaofeng Ren, and Xuefeng Wang. "Traveling waves in a convolution model for phase transitions." Archive for Rational Mechanics and Analysis 138, no. 2 (1997): 105-136.

12.2 Numerical solution of second order ODEs on the complex plane

In this project, numerical solution of second order ODEs are extended from the real line (treated in standard textbooks) to the complex plane. Special methods based on Padé approximation may be preferred because of the possible poles in the solutions and other singularities like branch points. A family of ODEs called Painlevé transcendents will be studied. Working knowledge on complex variables and numerical methods for ODEs is essential.

References:

[1] Corliss GF. Integrating ODEs in the complex plane—pole vaulting. Mathematics of Computation. 1980;35(152):1181-9.

[2] Corliss G, Chang YF. Solving ordinary differential equations using Taylor series. ACM Transactions on Mathematical Software (TOMS). 1982 Jun 1;8(2):114-44.

[3] Fornberg B, Weideman JA. A numerical methodology for the Painlevé equations. Journal of Computational Physics. 2011 Jul 1;230(15):5957-73.

[4] Reeger JA, Fornberg B. Painlevé IV: A numerical study of the fundamental domain and beyond. Physica D: Nonlinear Phenomena. 2014 Jul 1;280:1-3.

12.3 Fredholm determinant for integral equations

In this project, Fredholm determinant of Fredholm integral equation of the type

$$f(x) = g(x) + \lambda \int_a^b K(x,t) f(t) dt$$

will be explored. When the kernel K is separable in the sense that

$$K(x,t) = \sum_{j=1}^{N} \alpha_j(x) \beta_j(t),$$

the solution is reduced to a linear equation of the form $\mathbf{g} = (I - \lambda \mathbf{K})\mathbf{f}$, where the Fredholm determinant is simply the determinant $\det(I - \lambda \mathbf{K})$. For more general kernel K, the Fredholm determinant arises naturally in the theory of solvability. In the project, numerical methods will be explored and will be applied to some practical equations.

References:

[1] Kanwal, R. P. (2013). Linear integral equations. Springer Science & Business Media.

[2] Bornemann, F. (2010). On the numerical evaluation of Fredholm determinants. Mathematics of Computation, 79(270), 871-915.

13 Ashleigh Hutchinson

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13.1 Models for confined viscous gravity currents

Confined viscous gravity currents have important applications in both nature and industry. In this project, we consider axisymmetric viscous gravity currents in which fluid is injected at a constant flux between two parallel plates. Vertical confinement results in two regions of flow: an inner region where the fluid fills the entire space between the two plates, and an outer annular region where the free surface of the fluid lies below that of the top plate and the fluid spreads along the lower plate. In the latter region, the flow is essentially unconfined. Simple models, based on conservation of mass principles, are used to predict the location of the grounding line, which is where the inner contact region intersects with the outer annular region, and the leading edge of the outer annular region. Comparison between theory and experiment shows that there is 'missing physics' at the grounding line. An investigation has taken place where surface tension is incorporated at the grounding line, leading to an improvement in the alignment between theory and experiment. However, this surface tension model is rather primitive, and other potential candidates for this 'missing physics' have not been investigated. The aim of this project is to reproduce the existing models, as well as provide improvements on these theoretical models.

References

[1] [1] A. J. Hutchinson, R. J. Gusinow and M. G. Worster. The evolution of a viscous gravity current in a confined geometry, Journal of Fluid Mechanics, 2023

[2] [2] A. J. Hutchinson. The effect of surface tension on axisymmetric confined viscous gravity currents. Partial Differential Equations in Applied Mathematics, 2024

14 Oliver Jensen

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14.1 Mechanics of multicellular tissues

Growth in developing plant and animal tissues takes place through a combination of cell division, cell expansion and cell rearrangements. Mathematical models of such processes are formulated either at the individual cell level, or at the continuum level. There are many open questions at the interface between these two descriptions, relating for example to inherently discrete features that may be lost through spatial averaging. This project offers an opportunity to explore some of these questions within the context of the so-called vertex-based model [1] of planar multicellular tissues.

References:

[1] Farhadifar, R., Röper, J. C., Aigouy, B., Eaton, S., & Jülicher, F. (2007). The influence of cell mechanics, cell-cell interactions, and proliferation on epithelial packing. Current Biology 17, 2095-2104.

15 Chris Johnson

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My research interests are mainly in fluid and granular flows, especially geophysical flows. A range of topics are possible, including:

15.1 Grain size segregation in debris flows

Debris flows are rapid flows of mixtures of sediment and water, which occur frequently in mountainous areas after heavy rain. The sediment ranges from boulders to micron-sized clay particles, and the interaction between these different sized particles leads to complex heterogeneity in the flow, with large particles clustered towards the front and edges of the flow. The aim of this computational project is to model this heterogeneity mathematically.

15.2 Numerical methods for the shallow water equations

The shallow water equations are a set of hyperbolic partial differential equations, that are the basis of mathematical models for a wide range of fluid flows, particularly those that occur naturally such as floods, avalanches and atmospheric motion. The purpose of this project is to apply a class of numerical methods called non-oscillatory central schemes to solve generalisations of the shallow water equations, and analyse their behaviour in detail. This project involves a significant component of computer programming.

16 Gareth Wyn Jones

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I am interested in elastic materials, and specifically thin materials like plates and shells. Apart from the specific projects below, I am happy to discuss other potential projects in elasticity and continuum mechanics more generally.

16.1 Residual strain in composite plates

Composite plates can be formed by sticking layers of laminates together. Each of these laminates can be composed of long stiff fibers surrounded by a resin. Since the resin and fibers are different materials, they will expand at different rates with temperature changes, and the layer will expand preferentially in one direction. This means that when layers with different orientations are stuck together, the mismatch will cause a stress field in the composite. The goal of this project is to investigate this effect and predict the stress field for a given configuration.

16.2 Why are leaves flat?

Most leaves that we see in nature are remarkably flat. The evolutionary reason for this is to ensure that as many of the leaf cells as possible are exposed to sunlight, which would be difficult to achieve if the leaf were curly (though some plants do have curved morphologies for different reasons: such as the defensive spikes of holly leaves).

What controls this flat morphology? Is there a master controller inside the plant that notices non-flat areas and sends extra signals to those areas to flatten them? This seems unlikely, so the control must be local. Can we uncover a simple rule that modifies the local growth profile of a leaf based on its current shape?

This project will be based on elasticity theory, and in particular the theory of plates and shells. Continuum Mechanics would be helpful but not essential. It is likely that the resulting equations may require some knowledge of Finite Elements to solve, though progress may potentially be made with more standard numerical methods.

17 Yoshihito Kazashi

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17.1 Spectral methods for density optimal control

Steering the behaviour of stochastic dynamical systems arises in many applications, including molecular dynamics, optics, and models of collective phenomena. In many situations the objective is not to control individual realisations of the system, but rather to influence the probability distribution of its state. When the stochastic dynamics are modelled as diffusion processes, the evolution of this distribution is governed by a deterministic equation known as the Fokker–Planck equation.

Introducing control into the stochastic dynamics leads to the problem of determining a control input that steers the density towards a desired target distribution while minimising a suitable cost functional. The resulting optimisation problem is posed in an infinite-dimensional space and must therefore be discretised carefully.

This project studies such a problem using methods based on polynomial approximations. The mathematical tools involved include polynomial approximation, numerical integration, approximation theory, elementary functional analysis, and elementary probability theory. Knowledge of stochastic differential equations is helpful but not required.

References

[1] Mohammadi, M. and Borzì, A. A Hermite spectral method for a Fokker–Planck optimal control problem in an unbounded domain. Int. J. Uncertainty Quantif., vol. 5, no. 3, pp. 233–254, 2015.

[2] Canuto, C., Hussaini, M. Y., Quarteroni, A., & Zang, T. A. (2006). Fundamentals in single domains. Springer

18 Jonas Latz

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18.1 Theory and computation for the Lasso

Background

In linear regression and, e.g., dictionary learning, the parameter estimation that delivers that minimises the squared loss (and, thus, delivers the best data fit) is not always the best model: an overly complex model fitting the data perfectly may also fit observational noise and then generalise suboptimally to unseen data. Regularisation helps to overcome this so-called overfitting. If also a particularly simple model (i.e., consisting of few terms only) shall be found, sparsity-inducing regularisation is necessary, such as the Lasso. The Lasso has interesting mathematical properties and is, due its non-smoothness, computational challenging.

Description

We explore mathematical properties of and computational algorithms (as well as their analysis) for the Lasso. We study simple test problems, such as polynomial regression, and possibly problems in image reconstruction.

Deliverables

- A basic exploration of convex analysis, necessary to derive optimality criteria for the Lasso
- Explanation and analysis of a basic optimisation method for the Lasso, such as forward-backward splitting
- Numerical experiments

Prerequisites and skills:

A good understanding of linear regression and (convex/numerical) optimisation. Knowledge of numerical analysis/numerical linear algebra is advantageous, as is basic knowledge of convex functions. Programming skills in Python, MATLAB, or C/C++ are necessary.

References

[1] Daubechies, Ingrid, Defrise, Michel, and De Mol, Christine (2004). An iterative thresholding algorithm for linear inverse problems with a sparsity constraint. Communications on Pure and Applied Mathematics 57(11):1413-1457.

[2] Goldstein, Tom, Studer, Christoph, and Baraniuk, Richard (2014). A Field Guide to Forward-Backward Splitting with a FASTA Implementation, arXiv:1411.3406.

[3] Tibshirani, Robert (1996). Regression Shrinkage and Selection via the Lasso, J.R.Statist.Soc. B, 58(1):267-288.

19 Bill Lionheart

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19.1 Viruses

Transmission electron microscope tomography is widely used in biology for the study of viruses. We have a team working on reconstruction algorithms in X-ray and neutron tomography and we hope to rapidly transfer our techniques to electron tomography and if we can improve imaging of viruses. We are collaborating with local experts in the current Covid-19 epidemic our aim is to the see the density and arrangement of the spike protein and other antigens on the virus surface. This will tell our collaborators if any of the other viral proteins displayed on the membrane surface are suitable vaccine targets. The work involves some analysis and geometry as well as numerical linear algebra and scientific computing. The student will be working as part of a team, although this can be done remotely of course. We hope there will be opportunity to work with real data.

19.2 Land Mines

We have a large team working on improved detection of land mines using magnetic induction and ground penetrating radar. We represent the metal object as a polarization tensor and have made considerable progress on the mathematics and experimental fronts. Mathematically the works involves partial differential equations, asymptotic analysis, integral operators and some algebra. We have a range of experimental, computational and analytic challenges the student could work on. The student will have the chance to work with our engineering team on the SEMIS III project (funded by the Sir Bobby Charlton Foundation) in the Alan Turing Building (when we are allowed to reopen) as well as our collaborators in Swansea.

19.3 Proton Therapy

Proton therapy is a cancer treatment that targets the tumour more accurately reducing damage to health tissue. During treatment neutrons and gamma rays are emitted and our collaborators in Bergen have developed a detector that can register that the emission came from a specific conical surface. To locate where the does is delivered with have to invert a transform that integrates a function of space over cones, a kind of conical Radon transform. There are some analytical reconstruction formulae for this problem and this project is to implement and test one on simulated data. Some analysis and scientific computing is involved. There is physics and medicine to read about but prior experience of this is not necessary.

20 Tom Shearer

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20.1 How Inequality Affects the Economy

At the time of writing, in 2025, the UK has the sixth largest national economy in the world as measured by gross domestic product (GDP), the tenth largest by purchasing power parity, and the twentieth largest by nominal GDP per capita. It also has the second most unequal income distribution in the G7. Wealth is even more unequally distributed than income. The wealthiest 20% of the population receive 36% of the UK's income but own 63% of the country's wealth. The least wealthy 20% receive 8% of the income and own only 0.5% of the country's wealth. How does this distribution of resources affect the country? In this project, you will use mathematical modelling to investigate how the distribution of quantities such as income amongst the population, and intellectual property (IP) and capital amongst firms that produce goods and services, affect the economy, and society more generally. Economics textbooks often model the market for a good by considering demand functions to describe how much of it is desired by people ("consumers") and supply functions to describe the willingness of firms in the economy to provide it. Where the supply and demand functions cross is a market equilibrium. However, the textbooks rarely consider how income inequality affects demand, and how unequal distributions of firm productivity (due to unequal concentrations of IP and capital) affect supply. Can you do a better job? The above is just one area you could consider investigating. Another option would be to model how access to goods and wealth throughout society affects the happiness and wellbeing of the population. Regardless of what direction you take, you will need to use a combination of calculus and statistics to produce mathematical models that can make meaningful predictions.

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21 David Silvester

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My main research area is the numerical analysis of incompressible flow problems (especially the Navier-Stokes equations). While I am interested in all aspects of finite element approximation, the development of high level software is especially important.

Research dissertation projects:

21.1 Assessing the stability of symmetric channel flow

This project is concerned with the stability of solutions of the Navier-Stokes equations. If the flow is slow-moving then there is a unique steady solution that is symmetric with respect to the centreline of the channel. If the velocity at the inflow is increased past a critical value then the symmetric solution is unstable. In this case there are typically two alternative stable configurations with either the top eddy or the bottom eddy being longer than the other (it is an example of a pitchfork bifurcation). This project will study the mathematics of hydrodynamic stability and describe how the channel flow instability can be characterised by an infinite-dimensional eigenvalue problem that has a zero eigenvalue when the symmetric solution becomes unstable. Computations will be performed using the IFISS software package (http://www.manchester.ac.uk/ifiss).

21.2 Exploring angles between finite element subspaces

Hierarchical finite elements play an important role in a posteriori error estimation. The quality of the error estimation is intrinsically connected to the angle between the original and the enhanced subspace. This project will study the mathematics of error estimation and will describe how the angle between subspaces can be characterised by a generalised eigenvalue problem associated with a discrete inf-sup condition. Computations will be performed using the recently released T-IFISS software package.

22 Alice Thompson

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22.1 Optimal control of PDE systems

The aim of this project is to explore methods for feedback control in linear and nonlinear PDE systems. The project will start with revision of optimal control for ODE systems, and then work through examples of control of PDEs such as in [1]. You will compare how problem formulation and numerical solutions for forward and adjoint problems differ in ODE and PDE systems, producing new code where possible. There is scope to apply optimal control methods to a variety of new PDE problems with physical applications, such as feedback control of radial viscous fingering, which can be approached through models of varying complexity and nonlinearity.

References

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23 Anthony Thornton

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23.1 Numerical Coupling Schemes for Fluid-Particle Interactions in Granular Systems

This project focuses on the mathematical and numerical challenges of coupling discrete and continuum models. Motivated by the fluid-solid interactions inherent in complex industrial processes (such as sustainable packaging), the research will explore how to robustly couple the discrete element method (DEM, solving large systems of ODEs via MercuryDPM) with continuum fluid mechanics (solving PDEs via oomph-lib). A key objective is to investigate the stability, accuracy, and efficiency of different numerical coupling schemes. The student will formulate the mathematical interface between the particulate phase and the surrounding complex fluid, testing their algorithms on simplified baseline models. This project is ideal for a student interested in numerical analysis, PDE/ODE integration, and scientific computing.

23.2 Reduced-Order Modelling (ROM) for Cohesive Granular Dynamics

Simulating wet and cohesive powders involves tracking millions of particle interactions governed by highly nonlinear contact laws, making high-fidelity simulations computationally expensive. This project tackles this challenge through the lens of model reduction. Working with baseline high-fidelity data generated in MercuryDPM, the student will apply projection-based reduced-order modelling techniques (using tools like pyMOR) to project the high-dimensional dynamical system onto a lower-dimensional subspace. The mathematical focus will be on techniques such as Proper Orthogonal Decomposition (POD) or reduced basis methods, analysing the trade-off between computational speed and the preservation of essential physical structures (such as energy dissipation and agglomeration rates). This project suits a candidate with a strong background in linear algebra, dynamical systems, and numerical methods.

23.3 Inverse Problems and Bayesian Inference in Particulate Systems

This project frames the "calibration problem" in granular mechanics as a rigorous mathematical inverse problem. Granular simulations rely heavily on phenomenological contact models, and determining the correct micro-scale parameters from macro-scale experimental data is an ill-posed challenge. The research will focus on developing and implementing robust parameter-estimation frameworks using Bayesian inference and uncertainty quantification (UQ). To manage the high computational cost of the forward model, the student will construct and evaluate surrogate models. The goal is to formulate a data-driven, mathematically sound approach to parameter optimisation, providing a probability distribution of contact parameters rather than a single deterministic fit. This project is highly suitable for students interested in optimisation, applied probability, and machine learning mathematics.

24 Françoise Tisseur

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My research is in numerical linear algebra, and spans the full range from developing fundamental theory to deriving algorithms and implementing them in software. I am currently interested in the numerical solution of nonlinear eigenvalue problems, and in particular quadratic eigenvalue problems (QEPs), which have the form

$$\lambda^2\mathbf{Mx} + \lambda\mathbf{Dx} + \mathbf{Kx} = 0$$

where $\mathbf{M}$, $\mathbf{D}$, $\mathbf{K}$ are $n \times n$ matrices. If $\mathbf{x}$ is nonzero then λ is an eigenvalue and $\mathbf{x}$ is the corresponding right eigenvector. A major application area for QEPs is in finite element modelling of structures such as bridges and automobiles, wherein the eigenvalues determine the stability of the structure.

Research dissertation title:

24.1 The quadratic Arnoldi method for the solution of quadratic eigenvalue problems

The quadratic Arnoldi method is a Krylov method recently developed by Karl Meerbergen for the computation of a few eigenvalues and eigenvectors of large and sparse QEPs (see [1]). This project aims at understanding numerical techniques for large sparse QEPs and in particular Meerbergen's new method, at implementing the latter and at testing it on the collection of QEPs available in the MATLAB toolbox NLEVP.

References

[1] K. Meerbergen. The Quadratic Arnoldi method for the solution of the quadratic eigenvalue problem. SIAM J. Matrix Anal. Appl., 30(4):1463- 1482, 2008.

25 Marcus Webb

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25.1 Randomized Numerical Linear Algebra

This project is about using random numbers in numerical linear algebra computations to get algorithms which are faster than their deterministic counterparts, with the caveat that there is a tiny probability that the final result will be less accurate. In ideal circumstances, this probability of failure can be made arbitrarily small, backed up by rigorous mathematics. Numerical linear algebra is important in many scientific applications, notably in the data sciences, where matrices can be huge. The project could focus on one of the following two areas.

The first area is randomized low rank approximation. Matrices in the real world are approximately low rank in a surprising number of contexts, particularly when they are large matrices coming from real world data. It is advantageous to write a matrix A which is low rank in the form $A \approx BC$, where B and C are much smaller matrices than A. Computing such a decomposition can be very slow, but in recent years extremely fast randomized methods have been developed.

The second area is randomized least squares fitting. In least squares fitting we compute a vector x which minimizes $\|Ax = b\|$ where b is some given data and A is a tall, skinny, problem-dependent matrix. It is possible to take a random sample of rows (or a random mixture) and solve a modified but smaller least squares problem, which is much faster.

References

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25.2 Mathematics of Chebfun

Chebfun is a MATLAB software package for computing with functions in an intuitive way, and under the hood it uses a lot of special properties of Chebyshev polynomials. Go to www.chebfun.org for much more information. This project will look at some of the maths behind it, which passes through approximation theory, complex analysis and linear algebra.

References

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