Analysis, Random Walks and Groups
Spring 2019

Week 9 tutorial

1. Let $\mu = \frac{1}{2}\delta_0 + \frac{1}{2}\delta_1$ in $\mathbb{Z}_4$. Find an upper bound for the mixing time $n_{\text{mix}}(1/100)$ for μ, that is, after how many convolutions μ^{*n} is the total variation distance

$$d(\mu^{*n}, \lambda) < \frac{1}{100}?$$

2. Define the subgroup $\Gamma := \{0, 2\} \subset \mathbb{Z}_4$. Let μ be any probability distribution on $\mathbb{Z}_4$ with support $\text{spt}(\mu) = \Gamma$. Define the uniform measure on Γ by

$$\nu_\Gamma(t) = \frac{1}{2}\delta_0 + \frac{1}{2}\delta_2.$$

Prove the following version of the Upper Bound Lemma:

$$d(\mu^{*n}, \nu_\Gamma) \leq \frac{1}{2}\sqrt{\sum_{k \in \mathbb{Z}_4 \setminus \Gamma} |\widehat{\mu}(k)|^{2n}}$$

3. In the previous Exercise 2., after how many convolutions μ^{*n} is the total variation distance

$$d(\mu^{*n}, \nu_\Gamma) < \frac{1}{100}?$$