Analysis, Random Walks and Groups
Spring 2019

Week 6 tutorial

1. Let $X_1, X_2, \dots,$ be the random walk driven by $\mu_\alpha = \frac{1}{2}\delta_1 + \frac{1}{2}\delta_{-1}$. Compute the probabilities

(1) $\mathbb{P}(X_1 = 1, X_2 = 2)$.
(2) $\mathbb{P}(X_1 = 0, X_2 = 2)$.

Compare these to the probabilities

(a) $\mathbb{P}(1 \oplus t_2 = 2)$.
(b) $\mathbb{P}(0 \oplus t_2 = 2)$.

2. Let $\mu_\alpha = \alpha\delta_0 + (1-\alpha)\delta_1$ on $\mathbb{Z}_5$ for some $0 < \alpha \leq 1$. For which α is μ_α ergodic? Explain your answer. Compute $d(\mu_\alpha * \mu_\alpha, \lambda)$ as a function of α.

3. Let μ be a probability distribution on $\mathbb{Z}_4$, which is not a Dirac mass, and assume that the support

$$\mathrm{spt}(\mu) = \{t \in \mathbb{Z}_4 : \mu(t) > 0\}$$

is a coset of a proper non-trivial subgroup of $\mathbb{Z}_4$. Is there a limit

$$\mu_\infty = \lim_{n\to\infty} \mu^{*n}?$$

What is it? No proofs necessary, just have a think how to maybe prove this.