

0C2 Exercise Sheet 5 - Rational Functions

1. Use polynomial long division to divide the polynomial $P(x) = x^4 - 3x^3 + 3x - 1$ by

$$(i) D(x) = x - 2, \quad (ii) D(x) = x + 1, \quad (iii) D(x) = x^2 + 7x - 9$$

(In each case express your answer in the form $\frac{P}{D} = Q + \frac{M}{D}$.)

2. Express the following as partial fractions:

$$\frac{2x - 1}{(x - 1)(x + 5)}$$

3. Use your answer to question 2 to find the following indefinite integral:

$$\int \frac{2x - 1}{(x - 1)(x + 5)} dx$$

4. Express the following as partial fractions:

$$\frac{x^2 + x + 1}{x^2 + 3x + 2}$$

5. Use your answer to question 4 to find the following indefinite integral:

$$\int \frac{x^2 + x + 1}{x^2 + 3x + 2} dx$$

6. Prove

$$\frac{x^4 + x^3 - 2x^2 - x + 5}{x^3 + 4x^2 + 5x + 2} = x - 3 - \frac{2}{x + 1} + \frac{4}{(x + 1)^2} + \frac{7}{x + 2}$$

by reducing $\frac{x^4 + x^3 - 2x^2 - x + 5}{x^3 + 4x^2 + 5x + 2}$ to proper fractions and then expressing as partial fractions.

7. Use question 6 to find the indefinite integral $\int \frac{x^4 + x^3 - 2x^2 - x + 5}{x^3 + 4x^2 + 5x + 2} dx$.