

0C2 Exercise Sheet 3

Further differentiation

1. Calculate the differential dy when $y = \sin(x) \ln(\ln(x))$
2. Use implicit differentiation to find derivative $\frac{dy}{dx}$ as a function of x when $y = \cos^{-1}(x)$.
3. Use implicit differentiation to find derivative $\frac{dy}{dx}$ as a function of x when $x^2 + y^2 = 3$ with $x, y \geq 0$.
4. Use logarithmic differentiation to find derivative $\frac{dy}{dx}$ as a function of x when $y = \frac{\ln(x)}{\sin(x)}$.
5. Use logarithmic differentiation to find derivative $\frac{dy}{dx}$ as a function of x when $y = 2^{2^x}$.
6. Use parametric differentiation to find derivative $\frac{dy}{dx}$ as a function of t when $x = t^2$ and $y = \cos(t)$.
7. Use parametric differentiation to find derivative $\frac{dy}{dx}$ as a function of t when $x = \cos(t)$ and $y = \sin(t)$.
8. Find the Taylor expansion of $\tan(x)$ near $x = 0$ up to degree 3.
9. Using the Taylor expansion for $\frac{1}{1+x^2}$, find the Taylor expansion for $\tan^{-1}(x)$.
10. Calculate the following limits by using Taylor expansions:
 - (i) $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x^2} - 1}{x^2}$
 - (ii) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x} - 1}{\sqrt[5]{1-x} - 1}$