

Two hours

To be provided by the examinations office: Mathematical formula tables.

THE UNIVERSITY OF MANCHESTER

MATH19832 (MATHEMATICS 0C2)

20 May 2019

Time: 09:45 — 11:45

Answer **ALL EIGHT** questions (80 marks in total).

University approved calculators may be used.

1.

(a) Sketch the complex numbers

$$z_1 = 1 + i, \quad z_2 = -2 - i, \quad z_3 = i$$

in the complex plane. Compute the real part of the product $z_1 z_2 z_3$.

[5 marks]

(b) Use the binomial theorem to find the imaginary part of $(1 + i)^5$.

[5 marks]

2.

(a) Find the exponential form of the complex number $z = 2 + 2i$.

[5 marks]

(b) Use the exponential form of z to compute the argument of z^9 in $[0, 2\pi)$.

[5 marks]

3.

(a) Consider the arithmetic progression $1, 3, 5, 7, 9, 11, \dots$. Find a formula for the sum S_n of the first n elements of this arithmetic progression.

[5 marks]

(b) After how many terms n is the sum $S_n \geq 1000$?

[5 marks]

4.

(a) Find a Taylor expansion to degree 3 of the function $f(x) = \sin(x)$ at $x = 0$.

[5 marks]

(b) Use your answer to (a) to prove $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$.

[5 marks]

5.

(a) Find the differential dy of $y = e^x \sin(\sin(x))$ for $x > 0$.

[5 marks]

(b) Suppose real numbers $x, y > 0$ satisfy the equation $x^2 y^3 = 1$. Use implicit differentiation to find $\frac{dy}{dx}$ as a function of x .

[5 marks]

6.

(a) For $x > 0$ let $y = x^x$. Use logarithmic differentiation to prove $\frac{dy}{dx} = x^x \ln x + x^x$.

[5 marks]

(b) Use parametric differentiation to find dy/dx when $x = \cos t$ and $y = \sin t$ for those $0 \leq t \leq 2\pi$ with $\tan t \neq 0$.

[5 marks]

7.

(a) Use integration by parts to find the indefinite integral $\int \ln(x) dx$.

[5 marks]

(b) Find the improper integral $\int_1^{\infty} \frac{dx}{x^3}$.

[5 marks]

8.

(a) Prove

$$\frac{x^4 + 2x^3 - 2x^2 - x + 4}{x^3 + 4x^2 + 5x + 2} = x - 2 + \frac{3}{x + 1} + \frac{2}{(x + 1)^2} - \frac{2}{x + 2}$$

by reducing $\frac{x^4 + 2x^3 - 2x^2 - x + 4}{x^3 + 4x^2 + 5x + 2}$ to proper fractions and then expressing as partial fractions.

[5 marks]

(b) Use this to find the indefinite integral $\int \frac{x^4 + 2x^3 - 2x^2 - x + 4}{x^3 + 4x^2 + 5x + 2} dx$.

[5 marks]

END OF EXAMINATION PAPER