

Two hours

THE UNIVERSITY OF MANCHESTER

MATH19832 (MATHEMATICS 0C2/1C2)

Date: XX May 2013

Time: XX.00 – XX.00

Answer any **FOUR** of Questions 1–6.

If more than FOUR questions are attempted, then credit will be given for the best FOUR answers.

Formula Tables to be provided by the exams office. University-approved calculators may be used.

1.

(a) Calculate the area under the graph

$$y = x^{\alpha-1}$$

and above the x -axis, between $x = 1$ and $x = 2$. (Here α is a given real number.)

[6 marks]

(b) Find the angles of the right-angled triangle with sides 1, $\sqrt{3}$ and 2. Make a sketch to show the angles. (The angles should be expressed in radians.)

[6 marks]

(c) Use the Binomial Theorem to evaluate the expression

$$\frac{(1 + \sqrt{2})^5 - (1 - \sqrt{2})^5}{\sqrt{2}}$$

without using a calculator. (The answer should be a whole number.)

[8 marks]

2.

(a) Find the unique antiderivative $F(x)$ of the function

$$f(x) = \cos 11x$$

satisfying the condition $F(0) = \frac{1}{11}$.

[6 marks]

(b) Calculate the differential:

$$d \ln \left(\frac{\pi}{2} + \arctan x \right) = ?$$

[6 marks]

(c) Split the proper rational function

$$\frac{2x - 1}{(3x + 5)^2}$$

into partial fractions.

[8 marks]

3.

(a) Evaluate the definite integral:

$$\int_0^{\pi/6} (\cos^2 x - 1) dx.$$

(You should give exact numbers in the answer using radicals, π , etc., and not approximate values.)

[6 marks]

(b) For an arithmetic progression $\{a_k\}$ it is known that the sum of the first 9 terms is 153 and the sixth term is 22. Find the seventh term.

[6 marks]

(c) Use the Taylor expansion to evaluate the limit

$$\lim_{\alpha \rightarrow 0} \frac{2^\alpha - 1}{\alpha}.$$

(Hint: express 2^α using base e .)

[8 marks]

4.

(a) Find the indefinite integral by applying integration by parts:

$$\int x^2 \ln x \, dx .$$

[6 marks]

(b) By using long division of polynomials, or otherwise, write the improper rational function

$$\frac{x^4 + 7x^3 + 9x^2 - 5x - 16}{x^2 + x}$$

as the sum of a polynomial and a proper rational function.

[6 marks]

(c) Find all solutions of the equation

$$\cos^2 3x = \frac{3}{4} .$$

(It is assumed that angles are measured in radians. Your answer should contain exact numbers like π , not approximate values.)

[8 marks]

5.

(a) Use table integrals or a suitable substitution to evaluate

$$I = \int_0^1 \frac{dx}{\sqrt{1-x^2}}.$$

(You should give an exact number as the answer, not an approximate value.)

[6 marks]

(b) Find the sum of the first 12 terms of the following geometric progression:

$$1, 2, 4, 8, 16, \dots$$

[6 marks]

(c) Find the undetermined constants in the identity

$$\frac{x^3 + 12x^2 - 10x - 15}{(x-1)^2(x^2 + 5x + 6)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} + \frac{D}{x+3}.$$

[8 marks]

6.

(a) Find the indefinite integral

$$\int \frac{dx}{x(x^2 + 2x + 2)}$$

by applying expansion into partial fractions.

[6 marks]

(b) Use the addition formulas and other identities for sine and cosine to express

$$\cos^4 x$$

in the form $A + B \cos x + C \cos 2x + D \cos 3x + E \cos 4x$.

[6 marks]

(c) Calculate the derivative dy/dx of the function

$$y = x^{x^2},$$

where $x > 0$.

[8 marks]