0C2 Exercise Sheet 2
Sequences and series

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1. Complete the following sentences using the phrases:
sum, difference, first term, last term, $\mathbf{a_k}$, $\mathbf{a_k = a + (k-1)d}$, $\mathbf{d = a_{k+1} - a_k}$, $\mathbf{\frac{n}{2}}$

The kth term of the sequence $a_1, a_2, a_3, \ldots$ is denoted by ____________. We say that the sequence is an arithmetic progression if each pair of consecutive terms have a common ____________, d. That is $d =$ ____________, for all k. If the arithmetic progression has ____________ a, then the kth term can be expressed as ____________. The ____________ of the first n terms of an arithmetic progression can be found by adding together the ____________ and the ____________, and multiplying the result by ____________.

2. Find the 100^{th} term of the following arithmetic progression:

$$\tfrac{7}{2},\ \tfrac{19}{4},\ 6,\ \tfrac{29}{4},\ \ldots$$

What is the sum of the first 100 terms?

3. If the first term of an arithmetic progression is 1, and the sum of the first 5 terms is 37, what is the common difference?

4. Complete the following sentences using the phrases:
ratio, $\mathbf{a_k = ar^{k-1}}$, $\mathbf{r = \frac{a_{k+1}}{a_k}}$, **sum,** $\mathbf{\sum_{k=1}^{n} a_k}$, $\mathbf{a\frac{1-r^k}{1-r}}$, **first term,** $\mathbf{\frac{a}{1-r}}$

We say that the sequence $a_1, a_2, a_3, \ldots$ is a geometric progression if each pair of consecutive terms have a common ____________ r. That is ____________. If the geometric progression has first term a and common ____________ r, then the kth term can be expressed as ____________. The ____________ of the first n terms of a geometric progression is given by ____________. The sum of the first n terms of a geometric progression with ____________ a and common ____________ $r \neq 1$ is given by ____________. If $-1 < r < 1$, then the (infinite) sum of all terms in the geometric progression is ____________.

5. Find the sum of the first 10 terms of the following geometric progression:

$$1,\ \tfrac{2}{3},\ \tfrac{4}{9},\ \tfrac{8}{27},\ \tfrac{16}{81},\ \ldots$$

What is the sum of the infinite progression?

6. Find the sum of the first 8 terms of the following geometric series:

$$1 + 3 + 9 + 27 + 81 + \ldots$$

After how many terms is the sum greater than a million?

7. Use the Binomial Theorem to expand the following:

(i) $(a+b)^7$, (ii) $(\sqrt{3}-\sqrt{2})^4$.

(Note: for part (ii), give the exact answer using a radical; don't give approximate values.)

8. Expand $(1+x)^6$, and use this to approximate $(1.01)^6$ to four decimal places.