<u>**Two hours**</u>

To be provided by the examinations office: Mathematical formula tables.

THE UNIVERSITY OF MANCHESTER

MATH19832 (MATHEMATICS 0C2)

March 2019

MOCK EXAM

Answer **ALL EIGHT** questions (80 marks in total).

University approved calculators may be used.

1.

(a) Sketch the complex numbers

$$z_1 = 2 + 2i, \quad z_2 = 1 - i, \quad z_3 = 3i$$

in the complex plane. Compute the real part of the product $z_1 z_2 z_3$.

[5 marks]

(b) Use the binomial theorem to find the imaginary part of $(1 + i)^4$.

[5 marks]

2.

(a) Find the exponential form of the complex number $z = 3 + 3i$.

[5 marks]

(b) Use the exponential form of z to compute the argument of z^8 in $[0, 2\pi)$.

[5 marks]

3.

(a) Consider the arithmetic progression $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \ldots$. Find a formula for the sum S_n of the first n elements of this arithmetic progression.

[5 marks]

(b) After how many terms n is the sum $S_n \geq 100$?

[5 marks]

4.

(a) Find a Taylor expansion to degree 3 of the function $f(x) = \sin(x)$ at $x = 0$.

[5 marks]

(b) Use this to prove $\lim_{x \to 0} \dfrac{\sin(x)}{x} = 1$.

[5 marks]

5.

(a) Find the differential dy of $y = e^x \ln(\ln(x))$ for $x > 0$.

[5 marks]

(b) Suppose real numbers $x, y > 0$ satisfy the equation $x^2 y^3 = 1$. Use implicit differentiation to find $\frac{dy}{dx}$ as a function of x.

[5 marks]

6.

(a) For $x > 0$ let $y = x^x$. Use logarithmic differentiation to prove $\dfrac{dy}{dx} = x^x \ln x + x^x$.

[5 marks]

(b) Use parametric differentiation to find dy/dx when $x = t^2$ and $y = \ln(t)$ when $t > 0$.

[5 marks]

7.

(a) Use integration by parts to find the indefinite integral $\displaystyle\int x^2 e^x \; dx$.

[5 marks]

(b) Find the improper integral $\displaystyle\int_0^\infty \dfrac{dx}{(x+1)^5}$.

[5 marks]

8.

(a) Prove
$$\frac{x^4 + 2x^3 - 2x^2 - x + 4}{x^3 + 4x^2 + 5x + 2} = x - 2 + \frac{3}{x+1} + \frac{2}{(x+1)^2} - \frac{2}{x+2}$$
by reducing $\dfrac{x^4 + 2x^3 - 2x^2 - x + 4}{x^3 + 4x^2 + 5x + 2}$ to proper fractions and then expressing as partial fractions.

[5 marks]

(b) Use this to find the indefinite integral $\displaystyle\int \frac{x^4 + 2x^3 - 2x^2 - x + 4}{x^3 + 4x^2 + 5x + 2} \, dx$.

[5 marks]

END OF EXAMINATION PAPER