

Two hours

To be provided by the examinations office: Mathematical formula tables.

**THE UNIVERSITY OF MANCHESTER**

**MATH19832 (MATHEMATICS 0C2)**

21 May 2018

Time: 09:45 – 11:45

Answer **ALL EIGHT** questions (80 marks in total).

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University approved calculators may be used.

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1.

- (a) Find the indefinite integral  $\int f(x) dx$  of  $f(x) = x^4 + 5x + 1$ . Check your answer by differentiating. Find the antiderivative  $F(x)$  of  $f(x)$  satisfying  $F(1) = 2$ .

[6 marks]

- (b) Find the area under the curve  $y = \frac{1}{x+1}$  between  $x = 0$  and  $x = 1$ . Make a sketch of the function indicating the area.

[4 marks]

2.

- (a) Use the identities for  $\cos(A + B)$ ,  $\cos(2A)$  and  $\sin(2A)$  to prove  $\cos(3x) = 4\cos^3(x) - 3\cos(x)$ .

[5 marks]

- (b) Use the formula  $\cos^3(\theta) = \frac{1}{4}\cos(3\theta) + \frac{3}{4}\cos(\theta)$  and the substitution  $u = x^2$  to find

$$\int x \cos^3(x^2) dx.$$

[5 marks]

3.

- (a) Find all the solutions  $x$  to the equation  $\cos(2x) = \frac{\sqrt{3}}{2}$  expressing your answer in terms of exact numbers, not approximate values.

[5 marks]

- (b) Find all the solutions  $x$  to the equation  $\sin(x) = \cos(x)$  within the range  $0 \leq x < 2\pi$ .

[5 marks]

4.

- (a) The first term of an arithmetic progression  $a_1, a_2, a_3, \dots$  is  $a_1 = 5$  and common difference is 7. Find the 10th term  $a_{10}$  and the sum of the first 10 terms  $a_1 + a_2 + \dots + a_{10}$ .

[5 marks]

- (b) Find the sum of the first 6 terms of the following geometric series:

$$1 + 3 + 9 + 27 + 81 + \dots$$

After how many terms is the sum greater than 20000?

[5 marks]

- 5.
- (a) Use the binomial theorem to expand the brackets in the expression  $(x + 3)^5$ .  
[3 marks]
- (b) Reduce  $\frac{x^2 + x + 1}{x^2 + 3x + 2}$  to proper fractions and then express as partial fractions. Use this to find the indefinite integral  $\int \frac{x^2 + x + 1}{x^2 + 3x + 2} dx$ .  
[7 marks]
- 6.
- (a) Find the differential  $dy$  of  $y = x^2 \cos(\ln(x))$  for  $x > 0$ .  
[5 marks]
- (b) Suppose real numbers  $x, y > 0$  satisfy the equation  $x^2 + y^2 = 3$ . Use implicit differentiation to find  $\frac{dy}{dx}$  as a function of  $x$ .  
[5 marks]
- 7.
- (a) Find a Taylor expansion to degree 3 of the function  $f(x) = \cos(x)$  at  $x = 0$ .  
[7 marks]
- (b) Use this to find the limit  $\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x}$ .  
[3 marks]
- 8.
- (a) Use integration by parts to evaluate  $\int_0^1 e^x(x + 2) dx$ .  
[5 marks]
- (b) Find the indefinite integral  $\int x^2 e^x dx$ .  
[5 marks]