

Two hours

THE UNIVERSITY OF MANCHESTER

MATHEMATICS 1C2

Date: XX May 2011

Time: XX.00 – XX.00

Answer any **FOUR** of Questions 1–6.

Formula Tables to be provided by the exams office. University-approved calculators may be used.

1.

(a) Use integration 'by inspection' to find the indefinite integral

$$\int (-3x + 2)^8 dx.$$

[6 marks]

(b) Calculate the area under the graph $y = \cos 2x$ and above the x -axis between the values $x = 0$ and $x = \frac{\pi}{4}$.

[6 marks]

(c) Find the unique primitive $F(x)$ of the function

$$f(x) = \frac{1}{7x + 5}$$

satisfying the condition $F(0) = 0$.

[8 marks]

2.

(a) Use polynomial long division, or otherwise, to write the improper rational function

$$\frac{x^3 + 6x^2 - 2x - 11}{x + 6}$$

as the sum of a polynomial and a proper rational function.

[6 marks]

(b) Split the rational function

$$\frac{13x - 12}{2x^2 + 9x - 5}$$

into partial fractions.

[6 marks]

(c) Find the undetermined constants in the identity

$$\frac{2x^2 + 3x + 6}{x(x^2 + 2x + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 2x + 3}.$$

[8 marks]

3.

- (a) For an arithmetic progression $\{a_k\}$ it is known that the sum of the first 17 terms is 136 and the fifth term is zero. Find the sixth term.

[6 marks]

- (b) Calculate the sum of the infinite geometric progression with the first term $a = 5$ and the common ratio $r = \frac{7}{8}$.

[6 marks]

- (c) Use the Binomial Theorem to evaluate exactly

$$(1 - \sqrt{2})^5 + (1 + \sqrt{2})^5$$

without using a calculator.

[8 marks]

4.

- (a) In a right-angled triangle the sides are 1, $\sqrt{3}$ and 2. Find the angle opposite to the side with length 1. (You should express the answer both in degrees and in radians.)

[6 marks]

- (b) Use addition formulas for trigonometric functions to express $\sin 3x$ as the product of $\sin x$ and a polynomial in $\cos x$:

$$\sin 3x = \sin x \cdot P(\cos x).$$

[6 marks]

- (c) Find all solutions of the equation

$$\cos^2 5x = \frac{1}{2}.$$

(You should give exact numbers in the answer.)

[8 marks]

5.

(a) Calculate the differential dy of the function $y = x^3e^{-2x}$.

[6 marks]

(b) Find the derivative dy/dx for the function

$$y = \arctan \sqrt{1+x}.$$

[6 marks]

(c) Use Taylor expansions to evaluate the limit

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - e^{3x}}{\ln(1-x)}.$$

[8 marks]

6.

(a) Apply integration by parts to find

$$\int x \sin x \, dx .$$

[6 marks]

(b) Use a suitable substitution to evaluate

$$I = \int_1^2 \frac{(4x^3 + 1) \, dx}{x^4 + x} .$$

(You should give an exact answer using known functions, not an approximate value.)

[6 marks]

(c) Find the indefinite integral

$$\int \frac{(x + 3) \, dx}{2x^2 + 3x - 2}$$

by applying expansion into partial fractions.

[8 marks]