

(Practice Paper 3)

MATH19832

Two hours

To be provided by the examinations office: Mathematical formula tables.

**THE UNIVERSITY OF MANCHESTER**

**MATH19832 (MATHEMATICS 0C2)**

25th May 2016

Time: 14:00 – 16:00

Answer **ALL EIGHT** questions (80 marks in total).

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University approved calculators may be used.

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1.

- (a) It is known that the twelfth term of an arithmetic progression is  $a_{12} = -3$ , whilst the sum of the first seven terms is  $S_7 = -77$ . Find the 31st term of the arithmetic progression.

[6 marks]

- (b) Find the sum of the first ten terms of the geometric progression with first term 1024 and common ratio  $1/2$ . What is the sum of the infinite series?

[4 marks]

2.

- (a) Use the binomial theorem to expand the brackets in the expression  $(x - 1)^6$ .

[5 marks]

- (b) Hence write  $\frac{(x - 1)^6}{x^4 - 1}$  as the sum of a polynomial and a proper rational fraction.

[5 marks]

3.

- (a) Split the proper rational function  $\frac{7x - 1}{2x^2 + x - 10}$  into partial fractions.

[6 marks]

- (b) Hence find the indefinite integral  $\int \frac{7x - 1}{2x^2 + x - 10} dx$ .

[4 marks]

4.

- (a) Use the addition formulas and other identities for sine and cosine to express  $\cos^4(x)$  in the form  $\cos^4(x) = A + B \cos(x) + C \cos(2x) + D \cos(3x) + E \cos(4x)$ , where  $A, B, C, D$  and  $E$  are constants to be determined.

[6 marks]

- (b) Hence find  $\int \cos^4(x) dx$ .

[5 marks]

5.

- (a) Find the angles of the right-angled triangle with sides of length 2,  $2\sqrt{3}$  and 4. Make a sketch of the triangle, labelling the angles you have found.

[3 marks]

- (b) Find all solutions of the equation  $\sin^2(5x) = \frac{1}{2}$ .

(It is assumed that angles are measured in radians. Your answer should contain exact numbers like  $\pi$ , not approximate values.)

[6 marks]

6.

- (a) Find the differential  $dy$  of  $y = \tan(x) \cos(x^2 - 2x + 1)$ .

[4 marks]

- (b) Calculate the derivative  $dy/dx$  of the function  $y = x^{3x-1}$ , for  $x > 0$ .

[6 marks]

7.

- (a) Find a Taylor expansion to degree 3 of the function  $f(x) = \cos(x)$  at  $x = \frac{\pi}{2}$ .

[8 marks]

- (b) Hence evaluate  $\lim_{x \rightarrow \frac{\pi}{2}} \left( \frac{\cos(x)}{2x - \pi} \right)$ .

[2 marks]

8.

- (a) By integration by parts (or otherwise) evaluate  $\int_1^e \frac{1}{x} \ln(x) dx$ .

[5 marks]

- (b) Use a suitable substitution to evaluate  $\int_0^\pi \frac{2x + \cos(x)}{x^2 + \sin(x) + 1} dx$ .

[5 marks]

END OF EXAMINATION PAPER

