

(Practice Paper 1)

MATH19832

Two hours

To be Provided by the Examinations Office: Mathematical formula tables.

THE UNIVERSITY OF MANCHESTER

MATHEMATICS 0C2

28 May 2015

14:00 - 16:00

Answer **ALL EIGHT** questions.

Electronic calculators may be used, provided they cannot store text.

1.

- (a) For an arithmetic progression $\{a_k\}$ it is known that the sum of the first eight terms is 86, whilst the twelfth term is 13. Find the 22nd term.

[6 marks]

- (b) Find the sum of the first six terms of the geometric progression with first term 1 and common ratio 0.2. What is the sum of the infinite series?

[4 marks]

2.

- (a) Use the binomial theorem to expand the brackets in the expression $(2 - x)^5$.

[5 marks]

- (b) Hence write $\frac{(2 - x)^5}{x^2 + 2}$ as the sum of a polynomial and a proper rational fraction.

[5 marks]

3.

- (a) Split the proper rational function $\frac{x + 3}{2x^2 + 3x - 2}$ into partial fractions.

[6 marks]

- (b) Hence find the indefinite integral $\int \frac{x + 3}{2x^2 + 3x - 2} dx$.

[4 marks]

4.

- (a) Use the addition formulas and other identities for sine and cosine to express $\sin^4(x)$ in the form $\sin^4(x) = A + B \cos(x) + C \cos(2x) + D \cos(3x) + E \cos(4x)$, where A, B, C, D and E are constants to be determined.

[6 marks]

- (b) Hence find $\int \sin^4(x) dx$.

[5 marks]

5.

- (a) A right-angled triangle has two sides of length 5. Find the remaining angles and side length.

[3 marks]

- (b) Find all solutions of the equation $\sin^2(3x) + 2\sin(3x) + 1 = 0$.

(It is assumed that angles are measured in radians. Your answer should contain exact numbers like π , not approximate values.)

[6 marks]

6.

- (a) Find the differential dy of $y = x^3 \cos(\sin(x))$.

[4 marks]

- (b) Calculate the derivative dy/dx of the function $y = x^{x^2+1}$, where $x > 0$.

[6 marks]

7.

- (a) Find a Taylor expansion to degree 3 of the function $f(x) = \tan(x)$ as $x \rightarrow 0$.

[8 marks]

- (b) Hence evaluate $\lim_{x \rightarrow 0} \frac{\tan(x)}{x}$.

[2 marks]

8.

- (a) Apply integration by parts to evaluate the definite integral $\int_{\pi/2}^{\pi} x \cos(x) dx$.

(You should give exact numbers in the answer using radicals, π , etc., and not approximate values.)

[5 marks]

- (b) Use a suitable substitution to evaluate $\int_1^2 \frac{4x^3 + 1}{x^4 + x} dx$.

[5 marks]

END OF EXAMINATION PAPER