0C2 Class test 2, 2.4.2019
ANSWER ALL FIVE QUESTIONS

You may use electronic calculators and one A4 sheet of revision notes.

You must hand your revision sheet in with your test paper.

Duration: 40 minutes.

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1. In the following geometric progression $a_1, a_2, a_3, \ldots$ given by

$$1, 2, 4, 8, 16, 32, 64, \ldots$$

after how many terms n is the **sum** $\sum_{k=1}^{n} a_k$ greater than 10000? [5 marks]

First term $a_1 = 1$

Ratio $r = 2$

$\Rightarrow$ sum $\sum_{k=1}^{n} a_k = a_1 \frac{1-r^n}{1-r}$

$= 1 \cdot \frac{1-2^n}{1-2} = 2^n - 1$

Hence $\sum_{k=1}^{n} a_k > 10000 \iff 2^n - 1 > 10000$

$\iff 2^n > 10001$

$\iff n > \log_2(10001)$

≈ 13.287

Hence after $n \geq 14$ terms

2. Compute the **series**

$$\sum_{k=1}^{\infty} \frac{1}{3^k}$$

[5 marks]

$\sum_{k=1}^{\infty} \frac{1}{3^k} = \frac{a_1}{1-r} = \frac{1/3}{1-1/3} = \frac{3}{3-1} = \frac{3}{2}$

First term $a_1 = \frac{1}{3}$

Ratio $r = \frac{1}{3}$

3. Use the **binomial theorem** to expand the brackets in the expression $(x+2)^5$. [5 marks]

Pascal's triangle up to 5th degree!

0th 1

1st 1 1

2nd 1 2 1

3rd 1 3 3 1

4th 1 4 6 4 1

5th 1 5 10 10 5 1

Binomial thm

$\Rightarrow (x+2)^5 = \sum_{r=0}^{5} \binom{5}{r} x^r 2^{5-r}$

$= 1 \cdot x^0 2^{5-0} + 5 \cdot x^1 2^4 + 10x^2 2^3 + 10x^3 2^2 + 5x^4 2^1 + 1x^5 2^0$

$= \underline{32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5}$

4. Apply **logarithmic differentiation** to find the differential dy of $y = 3^x$. [5 marks]

$y = 3^x \quad || \ln(\ )$

$\Rightarrow \ln y = \ln(3^x)$

$\Rightarrow \ln y = x \ln 3 \quad || \frac{d}{dx}(\ )$

chain rule $\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \ln 3 \quad || \cdot y$

$\Rightarrow \frac{dy}{dx} = y \ln 3$

$y=3^x \Rightarrow \frac{dy}{dx} = \underline{\underline{3^x \ln 3}}$ so $\underline{\underline{dy = 3^x \ln 3 \, dx}}$

5. Find the **Taylor series** of e^x as $x \to 1$. ~~Apply this to prove the series~~

$$\cancel{\sum_{k=0}^{\infty} \frac{1}{k!} = e}$$

~~where we use the convention $0! = 1$.~~

[10 marks]

Set $f(x) = e^x \Rightarrow f(1) = e$

Then $f^{(1)}(x) = e^x \Rightarrow f^{(1)}(1) = e$

$f^{(2)}(x) = e^x \Rightarrow f^{(2)}(1) = e$

$f^{(3)}(x) = e^x \Rightarrow f^{(3)}(1) = e$

Thus as $x \to 1$

$$e^x = f(1) + \frac{f^{(1)}(1)}{1!}(x-1)^1 + \frac{f^{(2)}(1)}{2!}(x-1)^2 + \frac{f^{(3)}(1)}{3!}(x-1)^3 + \dots$$

$$= e + e(x-1) + \frac{e}{2}(x-1)^2 + \frac{e}{6}(x-1)^3 + \dots$$