0C2 Class test 1, 19.2.2019
ANSWER ALL FOUR QUESTIONS

You may use electronic calculators and one A4 sheet of revision notes.

You must hand your revision sheet in with your test paper.

Duration: 40 minutes.

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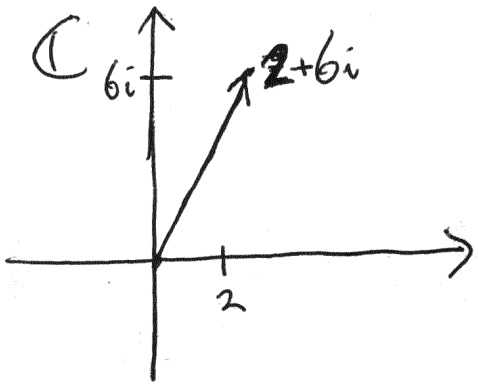
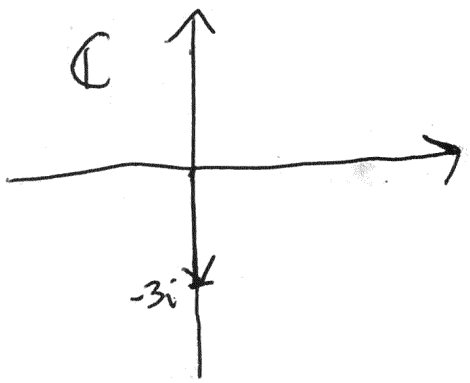
1. Sketch the following complex numbers in the complex plane

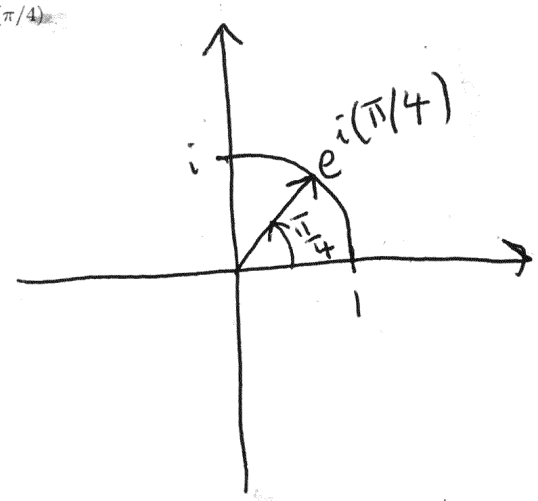
(i) $2 + 6i$ [3 marks]



(ii) $-3i$ [3 marks]



(iii) $e^{i(\pi/4)}$ [4 marks]



2. Let $z_1 = 2 + i \in \mathbb{C}$ and $z_2 = -3 + 2i \in \mathbb{C}$. Compute the product $z_1 z_2$ and find the real and imaginary parts of $z_1 z_2$. [5 marks]

$$z_1 z_2 = (2+i)(-3+2i)$$
$$= 2\cdot(-3) + 2\cdot 2i + i\cdot(-3) + i\cdot(2i)$$
$$= -6 + 4i - 3i + 2i^2$$
$$= -6 + 4i - 3i - 2 \quad (i^2 = -1)$$
$$= \underline{-8 + i}$$

Thus

$$\underline{\mathrm{Re}(z_1 z_2) = -8} \quad \& \quad \underline{\mathrm{Im}(z_1 z_2) = 1}$$

3. Find all the complex solutions to the quadratic equation $x^2 + 2x + 5 = 0$. What are their imaginary parts? [5 marks]

Use quadratic formula the solutions are

$$x = \frac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot 5}}{2\cdot 1}$$
$$= \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$$

Hence the two solutions are

$$x_1 = -1 + 2i \quad \& \quad x_2 = -1 - 2i$$

Their imaginary parts are 2 & -2

$\| \qquad \|$

$\mathrm{Im}(x_1) \quad \mathrm{Im}(x_2)$

4. Find the exponential form of $z = 1 + i$. You may use the fact that $\tan(\pi/4) = 1$.
Use the exponential form of z to compute the argument $\text{Arg}(z^{10})$. [10 marks]

- Modulus of z: $|z| = \sqrt{1^2+1^2} = \sqrt{2}$
- Argument of z: Find a solution in $[0, 2\pi)$

for $\tan\theta = \frac{1}{1} = 1$

This is $\theta = \frac{\pi}{4}$ as $\tan\left(\frac{\pi}{4}\right) = 1$.

Hence the argument is

$$\text{Arg}(z) = \frac{\pi}{4}$$

- Exponential form:

$$z = |z| e^{i\,\text{Arg}(z)} = \underline{\underline{\sqrt{2}\, e^{i\frac{\pi}{4}}}}$$

Power 10:

$$z^{10} = \sqrt{2}^{10}\left(e^{i\frac{\pi}{4}}\right)^{10} = \sqrt{2}^{10} e^{i\frac{10\pi}{4}}$$

$$= \sqrt{2}^{10} e^{i\frac{5\pi}{2}}$$

Note that $2\pi < \frac{5\pi}{2} < 4\pi$

so we need to substract 2π to get $\text{Arg}(z^{10})$: $\text{Arg}(z^{10}) = \frac{5\pi}{2} - 2\pi = \frac{5\pi}{2} - \frac{4\pi}{2}$

$$= \underline{\underline{\frac{\pi}{2}}}$$