

0C2 Exercise Sheet 5 - Rational Functions

1. Use polynomial long division to divide the polynomial $P(x) = x^4 - 3x^3 + 3x - 1$ by

(i) $D(x) = x - 2$

(ii) $D(x) = x + 1$

(In each case express your answer in the form $\frac{P}{D} = Q + \frac{M}{D}$.)

2. Check (by multiplying out) that

$$\frac{x^6 - 2x^4 + x^2 - 2}{x^2 - x - 2} = x^4 + x^3 + x^2 + 3x + 6 + \frac{12x + 10}{x^2 - x - 2}.$$

3. Factorise the following cubic polynomials

(i) $2x^3 + x^2 - 13x + 6$

(ii) $x^3 - 2x^2 - 5x + 6$

(iii) $2x^3 + x^2 - 8x - 4$

(iv) $2x^3 - x^2 + 2x - 1$

4. Express the following as partial fractions:

(i) $\frac{1}{(x+2)(x+3)}$

(ii) $\frac{2x-1}{(x-1)(x+5)}$

(iii) $\frac{x^2 - 11x - 6}{(2x-1)(x-2)(x+2)}$

5. Use your answer to question 4 to find each of the following indefinite integrals:

(i) $\int \frac{1}{(x+2)(x+3)} dx$

(ii) $\int \frac{2x-1}{(x-1)(x+5)} dx$

(iii) $\int \frac{x^2 - 11x - 6}{(2x-1)(x-2)(x+2)} dx$

6. Reduce the following to proper fractions and then express as partial fractions:

(i) $\frac{x^2 + x + 1}{x^2 + 3x + 2}$

(ii) $\frac{x^3}{x^2 - x - 6}$

7. Use your answer to question 6 to find each of the following indefinite integrals:

(i) $\int \frac{x^2 + x + 1}{x^2 + 3x + 2} dx$

(ii) $\int \frac{x^3}{x^2 - x - 6} dx$

8. Express the following as partial fractions:

(i) $\frac{x+3}{(x-1)^2}$

(ii) $\frac{2x-3}{x^2+8x+16}$

(iii) $\frac{x^2+x-1}{x^3-x^2}$

9. Use your answer to question 8 to find each of the following indefinite integrals:

(i) $\int \frac{x+3}{(x-1)^2} dx$

(ii) $\int \frac{2x-3}{x^2+8x+16} dx$

(iii) $\int \frac{x^2+x-1}{x^3-x^2} dx$

10. Express the following as partial fractions:

(i) $\frac{2}{(x^2+2)(x+1)}$

(ii) $\frac{3x+2}{(x^2+x+1)(x-1)}$

(iii) $\frac{x}{(x+2)^3}$

11. Use your answer to question 10 to find each of the following indefinite integrals:

(i) $\int \frac{2}{(x^2+2)(x+1)} dx$

(ii) $\int \frac{3x+2}{(x^2+x+1)(x-1)} dx$

(iii) $\int \frac{x}{(x+2)^3} dx$

12. Write down the partial fraction type of the following, but do not work out the constants.

$$\frac{1}{(x^2+2x+3)^2(x^2+2x-3)^2}$$

13. Use partial fractions to solve the following integrals:

(i) $\int \frac{dx}{(x+3)(2x+3)}$,

(ii) $\int \frac{4x^3}{x^2-1} dx$,

(iii) $\int \frac{4x+3}{(2x-1)^2} dx$,

(iv) $\int \frac{x+1}{x^3+x} dx$.