0C2 Exercise Sheet 3
Sequences, series and summations

---

1. Complete the following sentences using the phrases:
sum, difference, first term, last term, $\mathbf{a_k}$, $\mathbf{a_k = a + (k-1)d}$, $\mathbf{d = a_{k+1} - a_k}$, $\mathbf{\frac{n}{2}}$

The kth term of the sequence $a_1, a_2, a_3, \ldots$ is denoted by _____. We say that the sequence is an arithmetic progression if each pair of consecutive terms have a common _____, d. That is $d =$ _____, for all k. If the arithmetic progression has _____ a, then the kth term can be expressed as _____. The _____ of the first n terms of an arithmetic progression can be found by adding together the _____ and the _____, and multiplying the result by _____.

2. Find the 100^{th} term of the following arithmetic progression:

$\frac{7}{2}$, $\frac{19}{4}$, 6, $\frac{29}{4}$, ...

What is the sum of the first 100 terms?

3. If the first term of an arithmetic progression is 1, and the sum of the first 5 terms is 37, what is the common difference?

4. Complete the following sentences using the phrases:
ratio, $\mathbf{a_k = ar^{k-1}}$, $\mathbf{r = \frac{a_{k+1}}{a_k}}$, **sum,** $\mathbf{\sum_{k=1}^{n} a_k}$, $\mathbf{a\frac{1-r^k}{1-r}}$, **first term,** $\mathbf{\frac{a}{1-r}}$

We say that the sequence $a_1, a_2, a_3, \ldots$ is a geometric progression if each pair of consecutive terms have a common _____ r. That is _____. If the geometric progression has first term a and common _____ r, then the kth term can be expressed as _____. The _____ of the first n terms of a geometric progression is given by _____. The sum of the first n terms of a geometric progression with _____ a and common _____ $r \neq 1$ is given by _____. If $-1 < r < 1$, then the (infinite) sum of all terms in the geometric progression is _____.

5. Find the sum of the first 10 terms of the following geometric progression:

$$1,\ \tfrac{2}{3},\ \tfrac{4}{9},\ \tfrac{8}{27},\ \tfrac{16}{81},\ \ldots$$

What is the sum of the infinite progression?

6. Find the sum of the first 8 terms of the following geometric series:

$$1 + 3 + 9 + 27 + 81 + \ldots$$

After how many terms is the sum greater than a million?

7. Use the Binomial Theorem to expand the following:

(i) $(a+b)^7$, (ii) $(\sqrt{3}-\sqrt{2})^4$.

(Note: for part (ii), give the exact answer using a radical; don't give approximate values.)

8. Expand $(1+x)^6$, and use this to approximate $(1.01)^6$ to four decimal places.