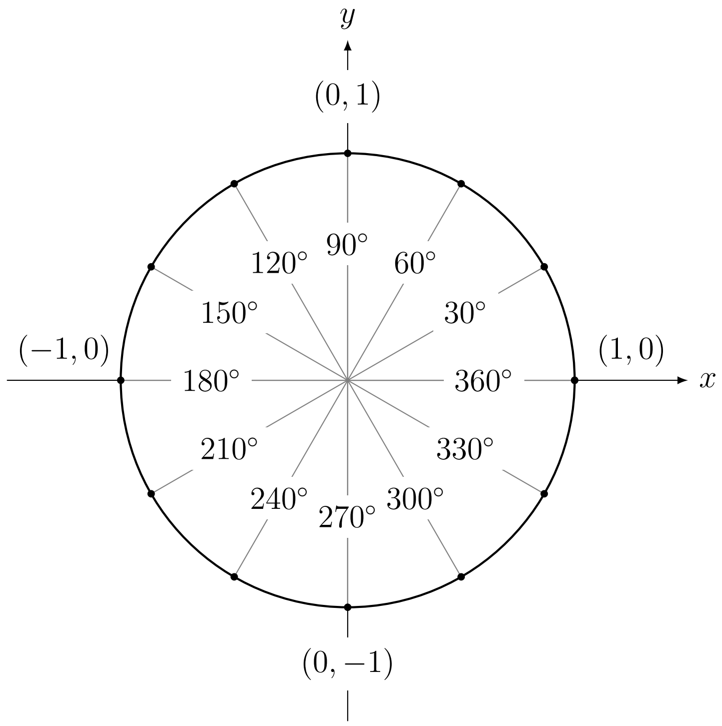
0C2 Exercise Sheet 2
Trigonometry

---

1. Complete the following sentences using: $\sin(\theta)$ and $\cos(\theta)$

The point on the unit circle corresponding to the angle θ has x-co-ordinate equal to ______________, and y-co-ordinate equal to ______________. For all angles θ, we also define:
$\tan(\theta) =$ ______________ $\div$ ______________,
$\text{cosec}(\theta) = 1 \div$ ______________,
$\sec(\theta) = 1 \div$ ______________, and
$\cot(\theta) = 1 \div$ ______________.

2. Convert each angle to radians, and find the (x, y) co-ordinates of the marked points.



3. Use the substitution $u = \sec(x) + \tan(x)$ to find $\displaystyle\int \frac{\sec(x)\tan(x) + \sec^2(x)}{\sec(x) + \tan(x)}\,dx$.

4. Use the substitution $u = \text{cosec}(x) - \cot(x)$ to find $\displaystyle\int \frac{\text{cosec}^2(x) - \text{cosec}(x)\cot(x)}{\text{cosec}(x) - \cot(x)}\,dx$.

5. Find:

(i) $\displaystyle\int \sec(x)\,dx$ (ii) $\displaystyle\int \text{cosec}(x)\,dx$ (iii) $\displaystyle\int \cot(x)\,dx$

(Hint: Simplifying the expressions in questions 3 and 4 may be helpful.)

6. Use trigonmetric identities to find:

(i) $\displaystyle\int \frac{\sin^2(x)+\cos^2(x)}{\cos^2(x)}\,dx$ (ii) $\displaystyle\int \tan^2(x)\,dx$

(iii) $\displaystyle\int \cos(x+\pi)\cos(x)+\sin(x+\pi)\sin(x)\,dx\,.$ (iv) $\displaystyle\int \sin^2(x)\,dx$

(v) $\displaystyle\int (\cos(x)-\sin(x))^2\,dx$ (vi) $\displaystyle\int (\cos(x)+\sec(x))^2\,dx\,.$

7. (i) Show that $\sin(3x) = 3\sin(x) - 4\sin^3(x)$.
(ii) Find $\displaystyle\int \sin^3(x)\,dx$.

8. (i) Express $\cos(4x)$ as a polynomial in $\cos(x)$.
(iii) Find $\displaystyle\int \cos^4(x)\,dx$

9. Complete the following sentences using: $\sin^{-1}(c)$, $\cos^{-1}(c)$, $\tan^{-1}(c)$, $\text{Arctan}(c)$, $\text{Arccos}(c)$, $\text{Arcsin}(c)$, π, 2π, x-co-ordinate, y-co-ordinate.

We write ______________ to denote the angles θ such that $\cos(\theta) = c$. These angles can be found by first identifying the points on the unit circle with ______________ equal to c. The corresponding angles are $\pm$ ______________. The remaining solutions can be found by adding multiples of ______________ to these two values.

We write ______________ to denote the angles θ such that $\sin(\theta) = c$. These angles can be found by first identifying the points on the unit circle with ______________ equal to c. The corresponding angles are ______________ and $\pi -$ ______________. The remaining solutions can be found by adding multiples of ______________ to these two values.

We write ______________ to denote the angles θ such that $\sin(\theta) = c$. One such value is ______________. The remaining solutions can be found by adding multiples of ______________ to this value.

10. Solve the following for all values of θ:

(i) $\sin(\theta) = \frac{1}{2}$ (ii) $\cos(2\theta) = 1$ (iii) $\sec(\theta) = 10$

(iv) $\tan(\theta) = 3$ (v) $2\cos^2(\theta) - 3\cos(\theta) + 1 = 0$ (vi) $\sin(8\theta) = 8$

11. Find all solutions in the range $0 \le \theta < 2\pi$:

(i) $\sin(\theta) = \cos(\theta)$ (ii) $\sin(5\theta) = 0$ (iii) $\cos^2(\theta) = \frac{1}{4}$