0C2 Exercise Sheet 1
Integration

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1. Complete the following sentence:

An antiderivative of a function $f(x)$ is a function $F(x)$ whose ______________ is equal to ______________.

2. For each of the following pairs of functions use the chain rule to show that $F(x)$ is an antiderivative of $f(x)$:

(i) $F(x) = (7x-1)^5$, $f(x) = 35(7x-1)^4$

(ii) $F(x) = (3-2x)^3$, $f(x) = -6(3-2x)^2$

(iii) $F(x) = -3e^{(1-2x)}$, $f(x) = 6e^{(1-2x)}$

(iv) $F(x) = \ln(12x-1)$, $f(x) = \dfrac{12}{12x-1}$

(v) $F(x) = \tan\left(\frac{3x+3}{2}\right)$, $f(x) = \dfrac{3}{2}\sec^2\left(\dfrac{3x+3}{2}\right)$

(vi) $F(x) = e^{x^2}$, $f(x) = 2xe^{x^2}$

3. Find all antiderivatives of $f(x)$.

(i) $f(x) = \cos(11x)$

(ii) $f(x) = \dfrac{3}{x}$

(iii) $f(x) = e^{7x}$

4. For each of the corresponding functions in question 3, find the antiderivative $F(x)$ satisfying the extra condition:

(i) $F(0) = \frac{1}{11}$

(ii) $F(1) = 5$

(iii) $F(\frac{1}{7}) = 2$

5. Find the following indefinite integrals:

(i) $\displaystyle\int x^5 \, dx$

(ii) $\displaystyle\int \sec^2(x) \, dx$

(iii) $\displaystyle\int e^{2x} \, dx$

(iv) $\displaystyle\int \frac{5}{x} \, dx$

(v) $\displaystyle\int (x+1)^4 \, dx$

(vi) $\displaystyle\int \sin(x) + \cos(x) \, dx$

6. Evaluate the following definite integrals:

(i) $\displaystyle\int_{-3}^{7} (x^2 - x + 7) \, dx$

(ii) $\displaystyle\int_0^{\pi} \cos\left(\frac{x}{4}\right) dx$

(iii) $\displaystyle\int_1^2 \frac{1}{x} \, dx$

(iv) $\displaystyle\int_0^1 e^{5x} \, dx$

7. Complete the following sentences using the phrases: **positive, negative, number, the area, function, below, above, minus one times the area, antiderivatives**

The indefinite integral of a function $f(x)$ is a ________________ defined up to the addition of a constant, which represents all ________________ of the function $f(x)$.

For given constants a and b, the definite integral $\int_a^b f(x)\,dx$ is a ________________.
If $f(x)$ lies entirely ________________ the x-axis between $x = a$ and $x = b$, then $\int_a^b f(x)\,dx$ is ________________ and represents ________________ between the curve $y = f(x)$ and the x-axis.
If $f(x)$ lies entirely ________________ the x-axis between $x = a$ and $x = b$ then $\int_a^b f(x)\,dx$ is ________________ and represents ________________ between the curve $y = f(x)$ and the x-axis.

8. Calculate the following definite integrals:

(i) $\int_0^{\pi} \sin(x)\ dx$ (ii) $\int_{\pi}^{2\pi} \sin(x)\ dx$ (iii) $\int_0^{2\pi} \sin(x)\ dx\,.$

Make a sketch of $y = \sin(x)$ and interpret your answers in terms of the areas between the curve and the x-axis.

9. Find the areas of the regions bounded by the given curves (make a sketch in each case):

(i) $y = \sqrt{x}$ and $y = x^2$ (ii) $y = x^2$ and $y = 4$

10. Make a suitable substitution to solve each of the following integrals:

(i) $\displaystyle\int \frac{dx}{2x+1}$ (ii) $\displaystyle\int \frac{3x^2+1}{x^3+x}\,dx$ (iii) $\displaystyle\int \sin(2x-5)\,dx$ (iv) $\displaystyle\int x(x^2+3)^7$

11. Use the substitution $u = \ln(x)$ to find

$$\int \frac{dx}{x\ln(x)}\,.$$

12. Use the substitution $u = \frac{1}{x}$ to find

$$\int \frac{1}{x^2}\cos\left(\frac{1}{x}\right)\,dx\,.$$