

Two hours

To be provided by the examinations office: Mathematical formula tables.

THE UNIVERSITY OF MANCHESTER

MATH19832 (MATHEMATICS 0C2)

xx May 2017

Time: xx:00 – xx:00

Answer **ALL EIGHT** questions (80 marks in total).

University approved calculators may be used.

- 1.
- (a) It is known that the ninth term of an arithmetic progression is $a_9 = 1$, whilst the sum of the first nine terms is $S_9 = 45$. Find the 100th term of the arithmetic progression.

[6 marks]

- (b) Find the sum of the first eight terms of the geometric progression with first term 1,000,000 and common ratio $1/10$. What is the sum of the infinite series?

[4 marks]

- 2.
- (a) Use the binomial theorem to expand the brackets in the expression $(2x - 2)^5$.

[5 marks]

- (b) Hence write $\frac{(2x - 2)^5}{x^2 + 1}$ as the sum of a polynomial and a proper rational fraction.

[5 marks]

- 3.
- (a) Split the proper rational function $\frac{4x - 1}{x^3 + 3x^2 + 2x}$ into partial fractions.

[6 marks]

- (b) Hence find the indefinite integral $\int \frac{4x - 1}{x^3 + 3x^2 + 2x} dx$.

[4 marks]

- 4.
- (a) Use the identities for $\sin^2(A)$ and $2 \cos(A) \sin(B)$ to express $\sin^3(2x)$ in the form

$$\sin^3(2x) = D \sin(2x) + E \sin(6x),$$

where D and E are constants to be determined.

[6 marks]

- (b) Hence find $\int \sin^3(2x) dx$.

[5 marks]

5.

- (a) Find the angles of the right-angled triangle with sides of length 3, 4 and 5, giving your answers to four decimal places. Make a sketch of the triangle, labelling the angles you have found.

[3 marks]

- (b) Find all solutions of the equation $\sin^3(2x) = 2 \sin(2x)$, expressing your answer in terms of exact numbers, not approximate values.

[6 marks]

6.

- (a) Find the differential dy of $y = \sec(x) \cos(x^2)$.

[4 marks]

- (b) Calculate the derivative dy/dx of the function $y = x^{x^3-x^2}$, for $x > 0$.

[6 marks]

7.

- (a) Find a Taylor expansion to degree 3 of the function $f(x) = \sin(x)$ at $x = \frac{\pi}{2}$.

[8 marks]

- (b) Hence evaluate $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sin(x) - 1}{2x - \pi} \right)$.

[2 marks]

8.

- (a) Use integration by parts to evaluate $\int_0^1 e^x(5x - 2) dx$.

[5 marks]

- (b) Use a suitable substitution to evaluate $\int_0^\pi \frac{\cos(x)(1 - 2 \sin(x))}{\cos^2(x) + \sin(x)} dx$.

[5 marks]

END OF EXAMINATION PAPER