

Two hours

THE UNIVERSITY OF MANCHESTER

MATH19832 MATHS 3 0C2

Date: XX May 2012

Time: XX.00 – XX.00

Answer any **FOUR** of Questions 1–6.

Formula Tables to be provided by the exams office. University-approved calculators may be used.

1.

(a) Calculate the area under the graph

$$y = x^{1/n}$$

and above the x -axis, between $x = 0$ and $x = 1$. (Here n is a given natural number.)

[6 marks]

(b) Find the angles of the right-angled triangle with sides 1, 1 and $\sqrt{2}$.

[6 marks]

(c) Use the Binomial Theorem to evaluate exactly

$$(1 + \sqrt{3})^4 + (1 - \sqrt{3})^4$$

without using a calculator.

[8 marks]

2.

(a) Find the unique primitive $F(x)$ of the function

$$f(x) = e^{7x}$$

satisfying the condition $F(0) = 0$.

[6 marks]

(b) Calculate the differential:

$$d \arctan \ln(5x) = ?$$

(Assume that $x > 0$ for the logarithm to be well-defined.)

[6 marks]

(c) Split the proper rational function

$$\frac{2x + 1}{(5x - 3)^2}$$

into partial fractions.

[8 marks]

3.

(a) Evaluate the definite integral:

$$\int_0^{\pi/6} \sin^2 x \, dx.$$

(You should give exact numbers in the answer, using radicals, π , etc., and not an approximate value.)

[6 marks]

(b) For an arithmetic progression $\{a_k\}$ it is known that the sum of the first 10 terms is 75 and the seventh term is zero. Find the sixth term.

[6 marks]

(c) Use the Taylor expansions to evaluate the limit

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+3x} - \sqrt[4]{1+4x}}{x^2}.$$

[8 marks]

4.

(a) Find the indefinite integral by applying integration by parts:

$$\int x \ln x \, dx .$$

[6 marks]

(b) Use polynomial long division, or otherwise, to write the improper rational function

$$\frac{x^4 + 7x^3 + 9x^2 - 5x + 17}{x^2 + 5x}$$

as the sum of a polynomial and a proper rational function.

[6 marks]

(c) Find all solutions of the equation

$$\sin^2 3x = \frac{3}{4} .$$

(You should give exact numbers in the answer.)

[8 marks]

5.

(a) Use table integrals or a suitable substitution to evaluate

$$I = \int_{-1}^1 \frac{dx}{1+x^2}.$$

(You should give an exact number as the answer, not an approximate value.)

[6 marks]

(b) Find the sum of the first 10 terms of the following geometric progression:

$$1, 3, 9, 27, 81, \dots$$

[6 marks]

(c) Find the undetermined constants in the identity

$$\frac{x^3 + x^2 - x + 5}{x^2(x^2 + 2x + 5)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 2x + 5}.$$

[8 marks]

6.

(a) Find the indefinite integral

$$\int \frac{dx}{x(x^2 + 1)}$$

by applying expansion into partial fractions.

[6 marks]

(b) Use the addition formulas and other identities for sine and cosine to express

$$\cos 3x$$

as a polynomial in $\cos x$.

[6 marks]

(c) Calculate the derivative dy/dx of the function

$$y = x^x,$$

where $x > 0$.

[8 marks]