0C2 Class test 2, 23.4.2018
ANSWER ALL FOUR QUESTIONS

You may use electronic calculators and one A4 sheet of revision notes.

You must hand your revision sheet in with your test paper.

Duration: 40 minutes.

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1. (i) Find the **differential** dy of the following function:

$$y = x\sin(\ln(x)), \quad x > 0$$

$$dy = \left(\sin(\ln(x)) + x\cos(\ln(x)) \cdot \frac{1}{x}\right)dx$$ [5 marks]

product rule & chain rule

$$= \left(\sin(\ln(x)) + \cos(\ln(x))\right)dx$$

(ii) Suppose real numbers $x, y > 0$ satisfy the equation

$$x^2 + y^2 + x = 1.$$

Use **implicit differentiation** to find $\frac{dy}{dx}$ as a function of x. [5 marks]

$$x^2+y^2+x = 1$$ || Differentiate both sides

chain rule $\Rightarrow 2x+2yy'+1=0$

$$\Rightarrow y' = -\frac{1+2x}{2y}$$

Solve y from $x^2+y^2+x=1$:

$$\Rightarrow y^2 = 1-x-x^2$$

$$\Rightarrow y = \pm\sqrt{1-x-x^2}$$

As $y>0 \Rightarrow y = \sqrt{1-x-x^2}$

$$\therefore y' = -\frac{1+2x}{2y} = -\frac{1+2x}{2\sqrt{1-x-x^2}}$$

$\left(\text{“}\frac{dy}{dx}\text{”}\right)$

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2. (i) In the following geometric progression

$$1, 2, 4, 8, 16, 32, 64, \ldots$$

after how many terms is the **sum** greater than 10000? [5 marks]

First term $a = 1$, ratio $r = 2$

Sum of n terms: $S_n = \sum_{k=1}^{n} a r^{k-1} = a \frac{1-r^n}{1-r} = 1 \cdot \frac{1-2^n}{1-2}$

$= 2^n - 1$

Sum $S_n > 10000 \iff 2^n - 1 > 10000$

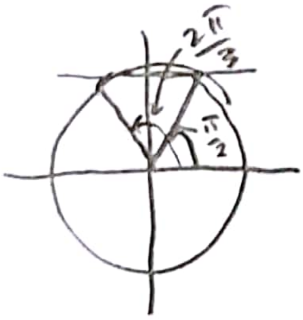
$\iff 2^n > 10001$

$\iff n > \log_2(10001) \approx 13.29$ in 2 dec. pl.

Hence for $\underline{\underline{n = 14}}$ the sum $S_n > 10000$

(ii) Find **all** the solutions to the equation $\sin(2x) = \frac{\sqrt{3}}{2}$ expressing your answer in terms of exact numbers, not approximate values. Recall that the principal value $\sin^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$. [5 marks]

Solve first $\underbrace{\sin(y) = \frac{\sqrt{3}}{2}}$ for $y = 2x$



- One solution $y = \sin^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$
- other solution $y = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\Rightarrow$ General solution:

$y = \frac{\pi}{3} + 2\pi k, \quad k \in \mathbb{Z}$

or $y = \frac{2\pi}{3} + 2\pi k, \quad k \in \mathbb{Z}$

Substitute $y = 2x$:

$2x = \frac{\pi}{3} + 2\pi k, \quad k \in \mathbb{Z}$

or $2x = \frac{2\pi}{3} + 2\pi k, \quad k \in \mathbb{Z}$

$\iff$ $\boxed{x = \frac{\pi}{6} + \pi k, \; k \in \mathbb{Z} \quad \text{or } x = \frac{\pi}{3} + \pi k, \; k \in \mathbb{Z}}$

3. Use the **binomial theorem** to expand the brackets in the expression $(x+2)^5$. [5 marks]

$$(a+b)^5 = \sum_{k=0}^{5} \binom{5}{k} a^{5-k} b^k$$

Pascal's triangle (Row 5)

$$= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Set $a = x$, $b = 2 \Rightarrow$

$$= x^5 + 5x^4 \cdot 2 + 10x^3 \cdot 2^2 + 10x^2 \cdot 2^3 + 5x \cdot 2^4 + 2^5$$

$$= \underline{x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32}$$

4. Apply **integration by parts** to find the indefinite integral $I = \int x^2 e^x \, dx$. [5 marks]

Set $u = x^2$ & $dv = e^x dx$

$\Rightarrow du = 2x\,dx$ & $v = e^x$

By integration by parts,

$$\int x^2 e^x dx = \int u\,dv = uv - \int v\,du$$

$$= x^2e^x - \int e^x 2x\,dx$$

$$= x^2e^x - 2\int xe^x dx$$

Again, let now

$f = x$ & $dg = e^x dx$

$\Rightarrow df = 1\,dx$ & $g = e^x$

By integration by parts,

$$\int xe^x dx = \int f\,dg = fg - \int g\,df = xe^x - \int e^x dx$$

$$= xe^x - e^x + C$$

$$\therefore \int x^2e^x dx = \underline{x^2e^x - 2xe^x + 2e^x + C}$$