0C2 Class test 1, 19.2.2018
ANSWER ALL FIVE QUESTIONS

You may use electronic calculators and one A4 sheet of revision notes.

You must hand your revision sheet in with your test paper.

Duration: 40 minutes.

Name & ID Number:

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1. Find each of the following indefinite integrals, **and check your answer in each case**, by differentiating the answer!

(i) $\int 5x^4\,dx$ [2 marks]

$$\int 5x^4 dx = \underline{\underline{x^5 + C}}$$

<u>Check:</u> $D(x^5+C) = 5x^4$

(ii) $\int x^2 - e^{2x}\,dx$ [4 marks]

$$\int x^2 - e^{2x} dx = \int x^2 dx - \int e^{2x} dx$$

$$= \underline{\underline{\tfrac{1}{3}x^3 - \tfrac{1}{2}e^{2x} + C}} \qquad [\text{chain rule}]$$

<u>Check:</u> $D(\tfrac{1}{3}x^3 - \tfrac{1}{2}e^{2x} + C) = x^2 - \tfrac{1}{2}e^{2x}\cdot 2 = x^2 - e^{2x}$

(iii) $\int \frac{2x}{x^2-1}\,dx.$ [4 marks]

$$\int \frac{2x}{x^2-1} dx = \underline{\underline{\ln|x^2-1| + C}}$$

<u>Check:</u> $D(\ln|x^2-1| + C) = \frac{1}{x^2-1}\cdot D(x^2-1)$

chain rule $= \frac{2x}{x^2-1}$

2. Find the antiderivative $F(x)$ of the function $f(x) = \cos(2x)$ satisfying $F(\frac{\pi}{2}) = 1$. [5 marks]

All antiderivatives of $f(x) = \cos(2x)$:

$$F(x) = \frac{1}{2}\sin(2x) + C$$

Check: $F'(x) = \frac{1}{2}\cos(2x)\cdot 2 = \cos(2x) = f(x)$

Want: $F(\frac{\pi}{2}) = 1 \iff \frac{1}{2}\underbrace{\sin(2\cdot\frac{\pi}{2})}_{=\sin(\pi) = 0} + C = 1$

$$\iff 0 + C = 1$$

$$\iff C = 1$$

Hence desired antiderivative is

$$\underline{\underline{F(x) = \frac{1}{2}\sin(2x) + 1}}.$$

3. Compute the definite integral $\int_0^1 x^3 + 2x\,dx$. [5 marks]

$$\int_0^1 x^3 + 2x\,dx = \int_0^1 x^3\,dx + \int_0^1 2x\,dx$$

$$= \left[\tfrac{1}{4}x^4\right]_0^1 + \left[x^2\right]_0^1$$

Check:

$$\begin{cases} D\frac{1}{4}x^4 = x^3 \\ \& \\ D x^2 = 2x \end{cases}$$

$$= \left(\tfrac{1}{4}\cdot 1^4 - \tfrac{1}{4}\cdot 0^4\right) + \left(1^2 - 0^2\right)$$

$$= \frac{1}{4} + 1 = \underline{\underline{\frac{5}{4}}}$$

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4. Use the substitution $u = x^2 + 1$ to find $\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx$. [5 marks]

First find $\displaystyle\int \frac{x}{x^2+1}\,dx$

Substitution $u = x^2+1$

$$\Rightarrow \frac{du}{dx} = 2x \Rightarrow du = 2x\,dx \qquad D\ln|u| = \frac{1}{u}$$

Hence

$$\int \frac{\overbrace{x\,dx}^{\frac{1}{2}du}}{\underbrace{x^2+1}_{=u}} = \int \frac{1}{u}\cdot\frac{1}{2}\,du = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|x^2+1| + C$$

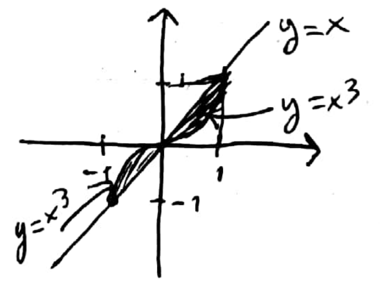
$u = x^2+1$

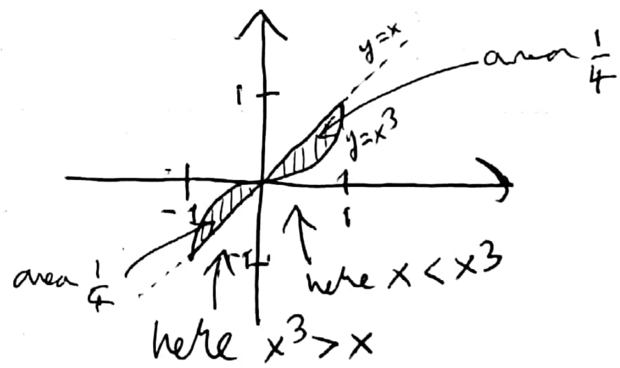
Thus

$$\int_0^1 \frac{x}{x^2+1}\,dx = \left[\frac{1}{2}\ln|x^2+1|\right]_0^1 = \frac{1}{2}\underbrace{\ln|1^2+1|}_{=\ln 2} - \frac{1}{2}\underbrace{\ln|0^2+1|}_{=\ln 1 = 0}$$

$$= \frac{1}{2}\ln 2$$

5. Calculate the area bounded by $y = x$ and $y = x^3$. Make a sketch of the two functions indicating the area. [5 marks]





2 parts: 1° area from 0 to 1:

(as $x<x^3$ here) $\displaystyle\int_0^1 x\,dx - \int_0^1 x^3\,dx = \left[\frac{1}{2}x^2\right]_0^1 - \left[\frac{1}{4}x^4\right]_0^1$

$$= \left(\frac{1}{2}\cdot1^2 - \frac{1}{2}\cdot0^2\right) - \left(\frac{1}{4}\cdot1^4 - \frac{1}{4}\cdot0^4\right)$$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

total area $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

2° area from -1 to 0: (Same as above but negative sign:)

(as $x^3 > x$ here) $\displaystyle\int_{-1}^0 x\,dx - \int_{-1}^0 x^3\,dx = \left[\frac{1}{2}x^2\right]_{-1}^0 - \left[\frac{1}{4}x^4\right]_{-1}^0$

$$= \left(\frac{1}{2}\cdot0^2 - \frac{1}{2}\cdot(-1)^2\right) - \left(\frac{1}{4}0^4 - \frac{1}{4}\cdot(-1)^4\right)$$

$$= -\frac{1}{2} - \left(-\frac{1}{4}\right) = -\frac{1}{2} + \frac{1}{4} = -\frac{1}{4}$$