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Horizontal movements of frame structures induced by vertical loads

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This paper considers the significance of vertical loads that can produce horizontal movements of frame structures. It is shown that, with a small number of exceptions, vertical loads can induce horizontal movements of symmetric, antisymmetric and asymmetric frames, which represent a wide range of engineering structures. The magnitudes of the horizontal movements depend on both the structural form and the location of the vertical loading. When vertical loads are applied dynamically, the movements of a structure can be significantly enlarged if one of the natural frequencies of the structure in the horizontal direction is close to one of the vertical load frequencies. These findings are illustrated by site measurements showing the horizontal movements of a framed building induced by vertical loading. Some implications of the findings are discussed briefly for several types of structure, including cantilever grandstands, temporary grandstands, cable-suspended bridges and rail bridges, in which horizontal response induced by vertical loading may need to be considered in design.

NOTATION

$A_1(t)$	magnitude of vibration contributed by the first mode
C_L	load factor that relates to the type and distribution of vertical loads
C_S	structural factor that is a function of structural form, α, and the distribution of member rigidities, β
C_{LS}	equivalent horizontal load factor
EI_b	flexural rigidity for beam
EI_c	flexural rigidity for column
F	horizontal concentrated force
M_A, M_B	bending moments at the two ends of a fixed beam induced by vertical loads
P	vertical concentrated static load
$P(t)$	vertical concentrated dynamic load
P_{TV}	total vertical load
f_p	frequency of a harmonic load
h	height of frame
h_1, h_2	heights of the left and right columns of an asymmetric frame
$\overline{m}$	mass density along element length
u, $\ddot{u}$	displacement and acceleration in the horizontal direction
α	ratio of the column height to the span of a frame
β	rigidity ratio of beam to column
γ	length ratio of the left column to the right column of a frame
θ_A, $\ddot{\theta}_A$	rotation and rotational acceleration at node A
θ_B, $\ddot{\theta}_B$	rotation and rotational acceleration at node B
ϕ_{21}, ϕ_{31}	the second and third components of the first normal mode vector

1. INTRODUCTION

When a structure moves horizontally, it is usually considered that this is in response to horizontal loads. However, vertical loads can also induce horizontal movements. This is because structures are three-dimensional and movements in the orthogonal directions are often coupled. For some structures such horizontal movements can be a significant design consideration, especially when dynamic response is an important factor.

Horizontal movements may result from the following:

- Horizontal loading (e.g. wind loading which will generate translational movement of tall buildings).
- Loading that, although primarily vertical, has a horizontal component, for example walking. The Millennium Bridge in London is a structure where significant horizontal movements were induced by people walking.[1]
- Vertical loading acting on asymmetric structures. Due to the structural geometry, vertical loads can induce both vertical and horizontal movements (i.e. vertical motion is coupled with a horizontal response). A simple example is a uniformly distributed vertical load on an inclined cantilever.
- Vertical loading acting asymmetrically on structures. Due to their location, vertical loads can induce both vertical and horizontal movements. An example is that of a train crossing a bridge and producing horizontal movements orthogonal to the rails; this will be discussed later.

This paper considers the last two situations where vertical loading can generate horizontal movements in frame structures. Initially a theoretical evaluation of the horizontal movement of a symmetric structure subject to asymmetric loading is considered and a number of load cases and structural combinations are examined. This is then extended to antisymmetric and asymmetric structures and the analytical results are verified by finite element (FE) calculations. An equivalent horizontal load factor is determined to represent the effect of the horizontal movements of frame structures due to vertical loads. For symmetric and antisymmetric frames the equivalent horizontal load factor can be expressed as a product of a load factor and a structural factor. Thus the effects of the distribution of vertical loads and structural geometry can be examined independently.

The examples presented consider static loading but the results are equally applicable to dynamic situations although here the possibility of resonance needs to be considered. An example is provided to illustrate that vertical dynamic loading can lead to a resonance that is primarily in the horizontal direction if the frequency of the dynamic load coincides with a horizontal natural frequency of the structure. To support the theoretical findings, measurements of the response of a floor subject to a crowd of people jumping rhythmically are presented which show that the vertical and horizontal responses of the floor occur at the same frequency as that of the vertical load.

The implications of these findings are discussed for grandstands and bridges, where horizontal responses induced by predominantly vertical loading may be key design considerations.

2. STATIC RESPONSE

2.1. A symmetric system

Consider a simple symmetric frame with no horizontal forces but subjected to any form of vertical load, such as a concentrated vertical load on a beam as shown in Fig. 1. The beam has a length of L and rigidity of EI_b, and the two columns have the same length of h and rigidity of EI_c.

If the axial deformations of the columns and the beam of the frame are negligible, the structure has three degrees of freedom, the horizontal displacement, u, and the rotations, θ_A and θ_B, at the connections of the beam and columns. Thus the equations of static equilibrium of the frame are given by:

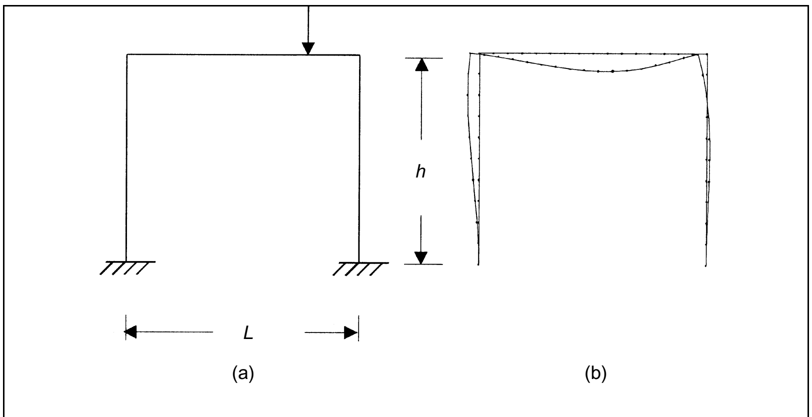


Fig. 1. A symmetric frame subject to an asymmetric vertical load: (a) frame and loading; (b) deformations

$$\frac{EI_c}{h^3}\begin{bmatrix} 24 & 6h & 6h \\ 6h & 4h^2(\alpha\beta+1) & 2h^2\alpha\beta \\ 6h & 2h^2\alpha\beta & 4h^2(\alpha\beta+1) \end{bmatrix}\begin{Bmatrix} u \\ \theta_A \\ \theta_B \end{Bmatrix} = \begin{Bmatrix} 0 \\ M_A \\ M_B \end{Bmatrix} \tag{1}$$

where

$$\alpha = h/L \qquad \beta = EI_b/EI_c \tag{2}$$

M_A and M_B are the fixed end moments of the beam arising from the vertical loading. The positive sign occurs when the end moment induces clockwise rotation. Equation (1) indicates that the vertical loads, which cause the moments M_A and M_B, induce rotations and horizontal displacements of the frame connections. As the stiffness matrix in equation (1) is fully populated, the horizontal displacement is coupled with the rotations.

Expanding the first row of equation (1) gives

$$u = -\frac{h(\theta_A + \theta_B)}{4} \tag{3}$$

from which it can be seen that u is zero only when $\theta_A = -\theta_B$. This occurs when symmetric loads are applied to the beam. Solving equation (1) gives the horizontal movement of the frame due to the vertical loads as

$$u = \frac{-(M_A + M_B)}{4(6\alpha\beta + 1)\alpha L}\frac{h^3}{EI_c} \tag{4}$$

The negative sign indicates that the movement of the frame is towards its left. If a horizontal force to the left, $-F$, is applied at one of the beam–column connections instead of the vertical load, solution of equation (1) gives the horizontal displacement as

$$u = \frac{-(3\alpha\beta + 2)F}{12(6\alpha\beta + 1)}\frac{h^3}{EI_c} \tag{5}$$

For the same horizontal displacement at the beam–column connections of the frame, equating equations (4) and (5) gives

$$\begin{aligned} F &= \frac{(M_A + M_B)12(6\alpha\beta + 1)}{4(6\alpha\beta + 1)(3\alpha\beta + 2)\alpha L} \\ &= \frac{(M_A + M_B)}{LP_{TV}}\frac{3}{(3\alpha\beta + 2)\alpha}P_{TV} \\ &= C_L C_S P_{TV} = C_{LS} P_{TV} \end{aligned} \tag{6}$$

in which

7	$$C_L = \frac{M_A + M_B}{LP_{TV}}$$

8	$$C_S = \frac{3}{(3\alpha\beta + 2)\alpha}$$

9	$$C_{LS} = C_L C_S$$

where P_{TV} is the total vertical load; C_L is a load factor that relates to the type and distribution of vertical loads; C_S is a structural factor that is a function of structural form, α, and the distribution of member rigidities, β; and C_{LS} is the equivalent horizontal load factor.

It can be seen that the smaller the values of α and β the larger the structural factor. It should also be noted that the load factor and the structural factor are independent for this case. Equation (6) indicates that the equivalent horizontal load can be expressed as a product of the load factor, the structural factor and the total vertical load. Table 1 provides values of the load factor for several vertical load distributions on the beam with two fixed ends. Table 2 shows the structural factor for a range of geometry and rigidity ratios.

Consider a particular case where $\alpha = 1$, $\beta = 1$ and a concentrated load, P, acts at a quarter of the length of the beam as shown in Fig. 1. The moments in equation (1) are

10	$$M_A = -\frac{3Ph}{64} \qquad M_B = \frac{9Ph}{64}$$

Substituting these into equation (6) gives

11	$$F = \frac{63P}{1120} = 0{\cdot}05625\,P$$

In this case the effect of the vertical load, P, on the horizontal movement of the frame is equal to that of a horizontal load of 5·625% P.

2.1.1. Example 1: Consider the frame shown in Fig. 1(a) with $h = 6{\cdot}0$ m, $L = 6{\cdot}0$ m, $E = 30 \times 10^9$ N/m^2, $I_b = I_c = 0{\cdot}25^4/12 = 3{\cdot}255 \times 10^{-4}$ m^4 and $P = 100$ kN (acting on a quarter of the length of the beam from the right end).

	$\beta = 0{\cdot}5$	$\beta = 1{\cdot}0$	$\beta = 2{\cdot}0$
$\alpha = 0{\cdot}5$	2·1818	1·7143	1·2000
$\alpha = 1{\cdot}0$	0·8570	0·6000	0·3750
$\alpha = 2{\cdot}0$	0·3000	0·1875	0·1071

Table 2. The structure factor, C_S, for different ratios of length and rigidity for a symmetric system

Figure 1(b) shows the deformed shape of the frame subject to the concentrated vertical load. The horizontal displacements calculated using equation (5) and the finite element method[2] are $-7{\cdot}406$ mm and $-7{\cdot}405$ mm, respectively.

The combined effect of the load and structural factors is shown in Table 3, which provides the equivalent horizontal load factor for three different load distributions on frames with $\alpha = 0{\cdot}5$, 1·0 and 2·0, and $\beta = 0{\cdot}5$, 1·0 and 2·0.

From Tables 1, 2 and 3 it can be concluded that:

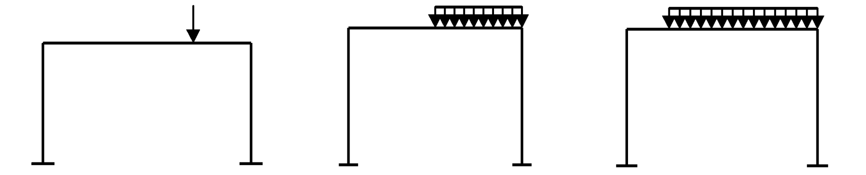
- The magnitude of the horizontal displacement induced by vertical loads (equation (4)) or the equivalent horizontal load (equation (6)) depends on the load distribution and the structural form.
- The structural factors are significantly larger than the load factors.
- The smaller the values of α and β, the larger the equivalent horizontal load (i.e. if the frame is relatively low and has a relatively large span, and/or the rigidity of the beam is smaller than that of the column, the frame will be subjected to a relatively large equivalent horizontal). Hence the load equivalent horizontal load for a taller frame is smaller than that for a similar lower frame if both are subjected to the same vertical loading.
- The height/length ratio, α, is more significant than the rigidity ratio, β, in determining the magnitude of the horizontal movement.
- The horizontal movement of a frame due to vertical loads is zero only when $M_A = -M_B$ (i.e. when concentrated loads act on the beam–column joints or when a symmetric load is applied to the beam.

Load distribution	M_A	M_B	C_L
Uniformly distributed load over full length	$-qL^2/12$	$qL^2/12$	0
Concentrated load acting at a quarter of the span from the right	$-3PL/64$	$9PL/64$	3/32
Uniformly distributed load over a half of the span from right	$-5qL^2/192$	$11qL^2/192$	1/16
Uniformly distributed load over three quarters of the span from right	$-63qL^2/1024$	$81qL^2/1024$	3/128

Table 1. The load factor, C_L, for different load distributions for a symmetric system

2.2. An antisymmetric system

If the left column of the frame shown in Fig. 1(a) is rotated through 180° around its connection to the beam, it becomes antisymmetric as shown in Fig. 2(a). The equivalent horizontal load can be found, as in section 2.1, by solving its equilibrium equations as



	$\beta = 0{\cdot}5$	$\beta = 1{\cdot}0$	$\beta = 2{\cdot}0$	$\beta = 0{\cdot}5$	$\beta = 1{\cdot}0$	$\beta = 2{\cdot}0$	$\beta = 0{\cdot}5$	$\beta = 1{\cdot}0$	$\beta = 2{\cdot}0$
$\alpha = 0{\cdot}5$	0·2045	0·1607	0·1125	0·1364	0·1071	0·0750	0·0511	0·0402	0·0281
$\alpha = 1{\cdot}0$	0·0804	0·0563	0·0352	0·0536	0·0375	0·0234	0·0201	0.0141	0.0088
$\alpha = 2{\cdot}0$	0·0281	0·0176	0·0100	0·0188	0·0117	0·0067	0·0070	0·0044	0·0025

Table 3. The equivalent horizontal load factor, C_{LS}, for a symmetric system

In addition to the conclusions drawn from section 2.1, which are also valid for the antisymmetric system, it can be deduced that:

- The load factor and structural factor for the antisymmetric systems are significantly larger than those for the symmetric system. Hence, the magnitude of the horizontal movement due to vertical loads depends on the structural form.
- Equation (12) indicates that the antisymmetric frame has no horizontal movement when $M_A = M_B$, which requires a particular distribution of antisymmetric vertical loading. For any other vertical loading situation, there will be a resulting horizontal movement.

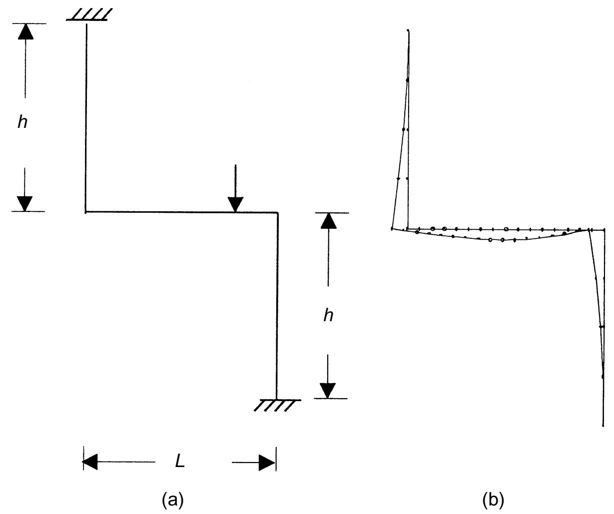


Fig. 2. An antisymmetric frame subject to an asymmetric vertical load: (a) frame and loading; (b) deformations

$$F = \frac{(M_B - M_A)}{4(2\alpha\beta + 1)\alpha L} \frac{12(2\alpha\beta + 1)}{(\alpha\beta + 2)} = \frac{(M_B - M_A)}{LP_{TV}} \frac{3}{(\alpha\beta + 2)\alpha} P_{TV} = C_L C_S P_{TV} = C_{LS} P_{TV} \quad (12)$$

where

$$C_L = \frac{M_B - M_A}{LP_{TV}} \quad (13)$$

$$C_S = \frac{3}{(\alpha\beta + 2)\alpha} \quad (14)$$

and C_{LS} is defined by equation (9). Equations (12), (13) and (14) have the same form as equations (6), (7) and (8). For comparison similar tables for the load factor, structural factor and equivalent horizontal load factor of the antisymmetric frame are given in Tables 4, 5 and 6.

Example 2: Consider the frame shown in Fig. 2(a) with similar data to that used for Example 1: $h = 6{\cdot}0$ m, $L = 6{\cdot}0$ m, $E = 30 \times 10^9$ N/m^2, $I_b = I_c = 0{\cdot}25^4/12 = 3{\cdot}255 \times 10^{-4}$ m^4 and $P = 100$ kN (acting on a quarter of the length of the beam from the right end).

The equivalent horizontal load can be evaluated using equation (12) as 18·75 kN. The finite element method is used to calculate the horizontal displacements induced by the vertical load of 100 kN and the horizontal load of 18·75 kN, respectively. These displacements are identical and have a value of −34·56 mm. Fig. 2(b) shows the deformed shape of the frame subject to the concentrated vertical load.

2.3. An asymmetric system

If the lengths of the columns of the frame shown in Fig. 1(a) are different, the frame becomes asymmetric as shown in Fig. 3(a). The ratios given in equation (9) are redefined as follows:

$$\alpha = h_1/L \qquad \beta = EI_b/EI_c \qquad \gamma = h_1/h_2 \quad (15)$$

and the equivalent horizontal load becomes

$$F = \frac{3[(\alpha\beta(2 - \gamma^2) + 2\gamma]M_A + 3[\alpha\beta(2\gamma^2 - 1) + 2\gamma^2]M_B}{\alpha L[4(\alpha\beta + 1)\gamma + \alpha\beta(3\alpha\beta + 4)]P_{TV}} \times P_{TV} = C_{LS} P_{TV} \quad (16)$$

where

$$C_{LS} = \frac{3[(\alpha\beta(2 - \gamma^2) + 2\gamma]M_A + 3[\alpha\beta(2\gamma^2 - 1) + 2\gamma^2]M_B}{\alpha L[4(\alpha\beta + 1)\gamma + \alpha\beta(3\alpha\beta + 4)]P_{TV}} \quad (17)$$

C_{LS} is the equivalent horizontal load factor, which is a function of load distribution, location and structural form. In contrast to the symmetric and antisymmetric frames considered

Load distribution	M_A	M_B	C_L
Uniformly distributed load over full length	$-qL^2/12$	$qL^2/12$	1/6
Concentrated load acting at a quarter of the span from the right	$-3PL/64$	$9PL/64$	3/16
Uniformly distributed load over a half of the span from right	$-5qL^2/192$	$11qL^2/192$	1/6
Uniformly distributed load over three quarters of the span from right	$-63qL^2/1024$	$81qL^2/1024$	3/16

Table 4. The load factor, C_L, for different load distributions for an antisymmetric system

	$\beta = 0{\cdot}5$	$\beta = 1{\cdot}0$	$\beta = 2{\cdot}0$
$\alpha = 0{\cdot}5$	2·6667	2·4000	2·0000
$\alpha = 1{\cdot}0$	1·2000	1·0000	0·7500
$\alpha = 2{\cdot}0$	0·5000	0·3750	0·2500

Table 5. The structure factor, C_S, for different ratios of length and rigidity for an antisymmetric system

in the previous two sections, the load factor and the structural factor are coupled for the asymmetric frame.

Consider $\gamma = 3/2$. The equivalent horizontal load factors for the same loading cases, length ratios and rigidity ratios, as for the symmetric and antisymmetric frames, are given in Table 7.

Example 3: Consider the frame shown in Fig. 3(a) with $h_1 = 6{\cdot}0$ m, $h_2 = 4{\cdot}0$ m, $L = 6{\cdot}0$ m, $E = 30 \times 10^9$ N/m^2, $I_b = I_c = 0{\cdot}25^4/12 = 3{\cdot}255 \times 10^{-4}$ m^4 and $P = 100$ kN (acting on a quarter of the length of the beam from the right end).

The equivalent horizontal load for this case is evaluated using equation (16) and is 15·73 kN. The horizontal displacements calculated using the FE method for the vertical and horizontal loads have the same value of $-10{\cdot}59$ mm. Fig. 3(b) shows the deformed shape of the frame.

Comparing the results in Tables 3 and 7, it can be seen that the equivalent horizontal load factors for the asymmetric frame are significantly larger than those for the symmetric frame. This again shows that structural form affects the magnitudes of horizontal movements of frame structures subject to vertical loads.

2.4. Further comparison

Table 8 summarises the ranges of the equivalent horizontal load factors for the three types of frame subject to three types of vertical loading limiting the variations of α and β between 0·5 and 2·0. From Table 8 it can be seen that:

- The equivalent horizontal load factors for the antisymmetric frame have the largest values, but this type of structure may not be common.
- The equivalent horizontal load factors of the asymmetric frame are at least double those of the symmetric frame for the same loading conditions.

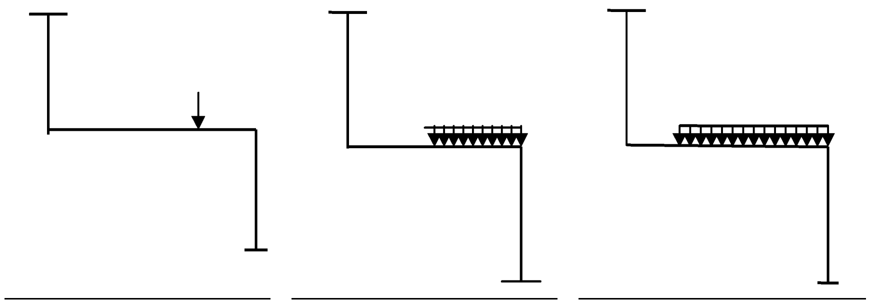
3. DYNAMIC RESPONSE

When a structure is subjected to cyclic dynamic loading, resonance may occur with a consequent, and potentially significant, increase in response. The possibility of vertical loading resulting in a resonant horizontal response therefore must be considered.

Consider the frame discussed in section 2.1 and shown in Fig. 1(a) subjected to a simple sinusoidal vertical load, $P(t)$, with maximum amplitude P_0:

$$P(t) = P_0 \sin 2\pi f_p t \tag{18}$$

where f_p is the frequency of the load and t is time. The mass densities for the columns and the beam are assumed to be m and $10m$ respectively, with the high density of the beam representing added loads that may arise from floors, etc. The equation of the undamped forced vibrations of the frame is, when $\alpha = \beta = 1{\cdot}0$,



	$\beta = 0{\cdot}5$	$\beta = 1{\cdot}0$	$\beta = 2{\cdot}0$	$\beta = 0{\cdot}5$	$\beta = 1{\cdot}0$	$\beta = 2{\cdot}0$	$\beta = 0{\cdot}5$	$\beta = 1{\cdot}0$	$\beta = 2{\cdot}0$
$\alpha = 0{\cdot}5$	0·5000	0·4500	0·3750	0·4444	0·4000	0·3330	0·5000	0·4500	0·3750
$\alpha = 1{\cdot}0$	0·2250	0·1875	0·1406	0·2000	0·1667	0·1250	0·2250	0·1875	0·1406
$\alpha = 2{\cdot}0$	0·0938	0·0703	0·0469	0·0833	0·0625	0·0417	0·0938	0·0703	0·0469

Table 6. The equivalent horizontal load factor, C_{LS}, for an antisymmetric system

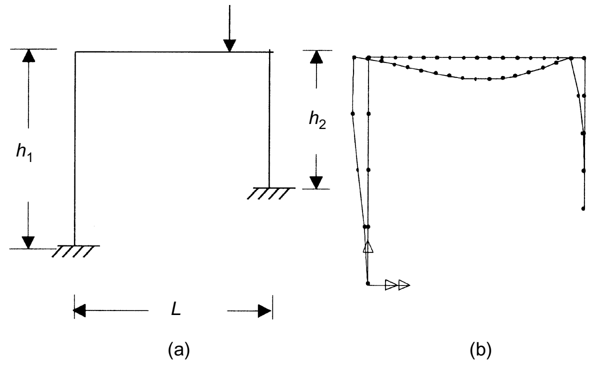


Fig. 3. An asymmetric frame subject to a symmetric vertical load: (a) frame and loading; (b) deformations

$$\frac{\bar{m}h}{420}\begin{bmatrix}4512 & 22h & 22h \\ 22h & 44h^2 & -30h^2 \\ 22h & -30h^2 & 44h^2\end{bmatrix}\begin{Bmatrix}\ddot{u} \\ \ddot{\theta}_A \\ \ddot{\theta}_B\end{Bmatrix} + \frac{EI_c}{h^3}\begin{bmatrix}24 & 6h & 6h \\ 6h & 8h^2 & 2h^2 \\ 6h & 2h^2 & 8h^2\end{bmatrix}\begin{Bmatrix}u \\ \theta_A \\ \theta_B\end{Bmatrix} = \begin{Bmatrix}0 \\ M_A \\ M_B\end{Bmatrix}\sin(2\pi f_p t) \tag{19}$$

The elements in the mass matrix are obtained in the same manner as those in the stiffness matrix. The mode shapes and frequencies of the structure can be found by solving the eigenvalue problem associated with equation (19). Taking the mass density, $\bar{m}$, equal to 150 kg/m and other data as used in example 1, the three natural frequencies of the frame are 1·39 Hz, 5·00 Hz and 14·5 Hz, and the corresponding mode shapes are shown in Fig. 4. The first mode shows horizontal movements of the frame while the two other modes give symmetric and antisymmetric rotations of the beam–column joints respectively. The response in the first mode of the frame is[3]

$$A_1(t) = \frac{\phi_{21}M_A + \phi_{31}M_B}{K_1}\frac{1}{1-(f_p/f_1)^2}\sin 2\pi f_p t \tag{20}$$

where $A_1(t)$ is the amplitude of the horizontal motion of the frame and $\phi_{21}M_A + \phi_{31}M_B$ is the modal load for the first mode, which acts in the horizontal direction. Equation (20) indicates that if the modal load is not equal to zero and the load frequency, f_p, is close to the fundamental natural frequency, f_1, the vertical load will induce resonant vibration of the frame in the horizontal direction. This conclusion can be verified numerically.

Example 4: Consider the frame defined in example 1 with $\bar{m} = 2400\ \text{kg/m}^3 \times (0{\cdot}25\ \text{m})^2 = 150\ \text{kg/m}$, $P = P_0 \sin 2\pi f_p t$ and $P_0 = 100$ kN.

Dynamic analysis was carried out using LUSAS[2] with the critical damping set to zero. Fig. 5 shows the time history of the horizontal motion of the frame, up to 10 s, due to the vertical load. A typical resonance situation is encountered.

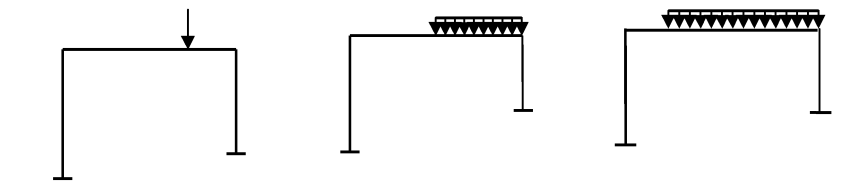
Although the example is simple, it illustrates the important phenomenon that *if the frequency of a vertical load is close to one of the horizontal natural frequencies of a structure, resonance in the horizontal direction can occur as a result of vertical excitation.* This situation should be recognised in the design of some structures.

The necessary condition for no horizontal movement occurs when the vertical loads are applied either symmetrically on the beam or at the beam–column joints. For any other distributions of vertical dynamic loads, resonance can occur in the horizontal direction.

4. SITE MEASUREMENTS

In the introduction the example of an inclined cantilever was used to provide an illustration where a vertical load can lead to both vertical and horizontal movements. With cantilever grandstands the potential problems induced by people jumping or bouncing in time to music is topical. This type of cyclic loading induces a characteristic type of response, which occurs at the load frequency (the jumping frequency) and whole number multiples thereof. So when the structural response is presented as a spectrum (i.e. response plotted against frequency), the response at specific frequencies is shown. Measurements on a cantilever grandstand are presented in reference 4 and this characteristic response could be seen in both the vertical and the horizontal directions.

For this paper, another, perhaps less obvious, example is considered. A



	$\beta = 0{\cdot}5$	$\beta = 1{\cdot}0$	$\beta = 2{\cdot}0$	$\beta = 0{\cdot}5$	$\beta = 1{\cdot}0$	$\beta = 2{\cdot}0$	$\beta = 0{\cdot}5$	$\beta = 1{\cdot}0$	$\beta = 2{\cdot}0$
$\alpha = 0{\cdot}5$	0·4269	0·3800	0·3146	0·3197	0·2892	0·2442	0·2251	0·2162	0·1952
$\alpha = 1{\cdot}0$	0·1900	0·1573	0·1184	0·1446	0·1221	0·0938	0·1081	0·0976	0·0796
$\alpha = 2{\cdot}0$	0·0786	0·0592	0·0400	0·0616	0·0469	0·0322	0·0488	0·0398	0·0285

Table 7. The equivalent horizontal load factor, C_{LS}, for an asymmetric system

	Symmetric frame	Antisymmetric frame	Asymmetric frame
Concentrated load acting at a quarter of the span from the right	0·2045 – 0·0100	0·5000 – 0·0469	0·4269 – 0·0400
Uniformly distributed load over a half of the span from right	0·1364 – 0·0067	0·4444 – 0·0417	0·3197 – 0·0322
Uniformly distributed load over three quarters of the span from right	0·0511 – 0·0025	0·5000 – 0·00469	0·2251 – 0·0285

Table 8. Comparison of the ranges of the equivalent horizontal load factor

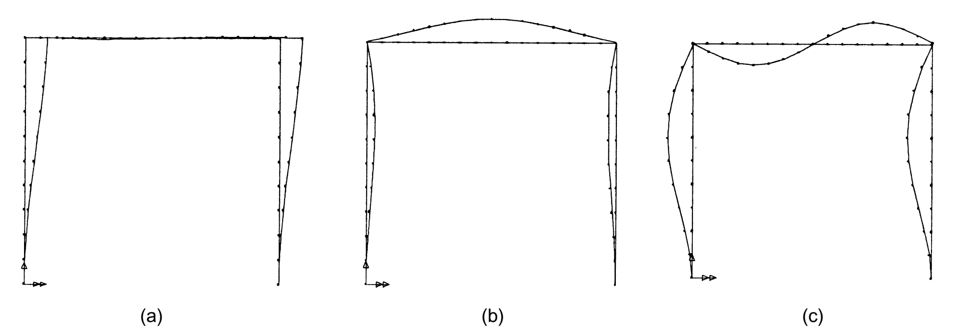


Fig. 4. The three normal modes of the symmetric frame: (a) horizontal movement; (b) symmetric rotation; (c) antisymmetric rotation

Although this is given as an example of vertical loading on an asymmetric structure, it is worthwhile considering whether people jumping actually generate horizontal forces. For example, it is recognised that people walking actually generate a horizontal force, normal to the direction of walking, of approximately 10% of the vertical dynamic force. This is a result of a horizontal push from each foot with each step; a process of continued correction to maintain a balanced progress. Equally an individual jumping will produce a small horizontal force simply to correct for any lateral movement and so maintain a selected jumping location. However, with a group jumping on a level floor, the overall sum of these apparent random horizontal forces must tend to zero as the number of people jumping increases.

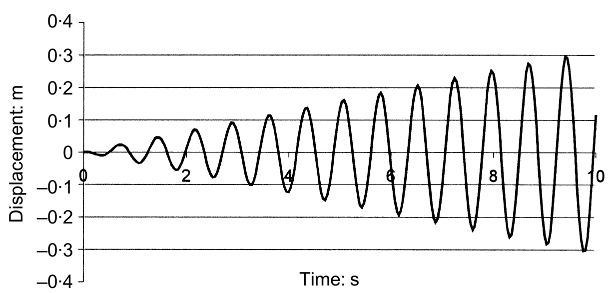


Fig. 5. Resonant response of the frame

Fig. 6. 64 students jumping on a floor in response to music

composite floor of area 9 m by 6 m, was tested and the structural response was measured for a group of people jumping. Sixty-four students, evenly distributed over the floor, were asked to jump in time to a musical beat (Fig. 6). At the centre of the floor, the vertical acceleration was recorded for just over 16 s, as was the horizontal acceleration in the direction orthogonal to the direction in which the students were facing. The peak vertical acceleration was 0·48g and the corresponding horizontal acceleration was 0·03g. The autospectra for these records are shown in Fig. 7, and the characteristic response can be seen in both directions. The test area was in fact part of the much larger flooring system shown in Fig. 8 and the loading was thus applied asymmetrically on the whole structure, which induced the horizontal motion.

5. IMPLICATIONS

It has been shown that vertical loading can produce both vertical and horizontal motion and, if the loading is dynamic, resonance can occur in either the horizontal or vertical direction. Actual frame structures will be more complicated than the simple frames studied, but they can be considered as an assembly of these basic units and will possess the features revealed in sections 2 and 3. It is therefore useful to consider the implications of these findings for some common types of structure.

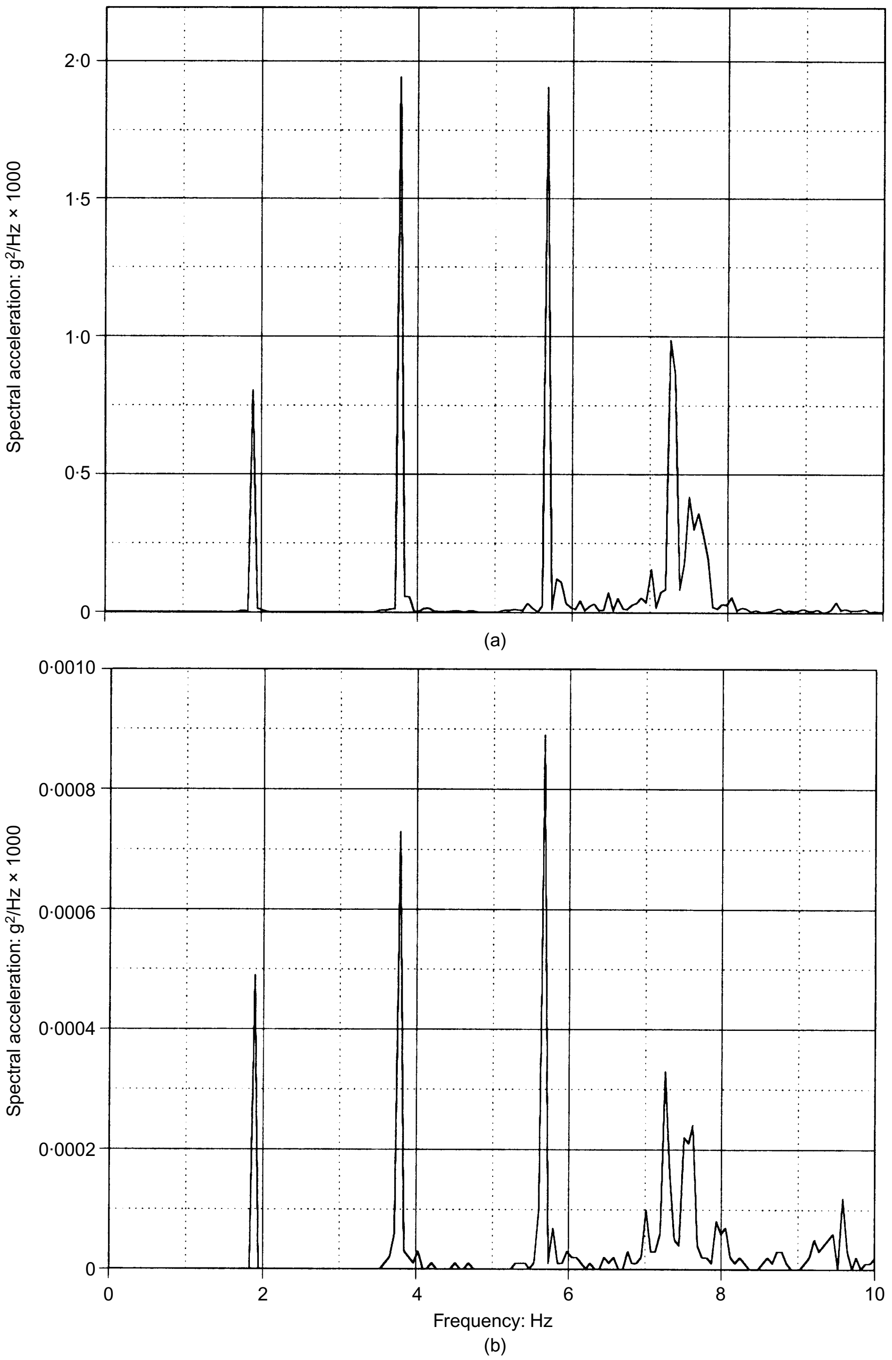


Fig. 7. The acceleration spectra for 64 people jumping on a floor: (a) vertical direction; (b) side-to-side direction

Although the mechanisms have been illustrated using static models, it is likely to be the dynamic situations that are of principal concerns, especially when resonance occurs. Hence, most of the following examples relate to cyclic dynamic loading.

5.1. Cantilever grandstands

The inclined cantilever was used in the introduction to illustrate a simple example of vertical loading inducing horizontal movement. Cantilever grandstands are frequently subjected to dynamic crowd loads, which at certain events, like concerts, are cyclic in nature. Fig. 9 shows the coupled vertical and front-to-back vibration of the cross-section of a grandstand in one typical mode of vibration. It can be seen that the front-to-back movement is larger than the vertical movements of the two tiers for this particular mode. Based on this observation and the finding in section 3, the frame is likely to experience resonance in the front-to-back direction if one of the frequencies of vertical loading on a tier is close to the natural frequency associated with the mode, even though the vertical movement will be small. The fact that the whole grandstand moves in this mode means that the modal mass will be large and hence the resonance may not always lead to excessive movement.

Although the coupling between vertical movements and horizontal (front-to-back) movements is easily understood, measurements have indicated that sway (side-to-side) movements can also be induced by vertical loading on some grandstands. The significance of the sway movement due to vertical loads depends on the structural form. To date there have been few measurements of sway movement of permanent grandstands, but this, somewhat less obvious coupling, should not be overlooked.

As a simple alternative to evaluating structural response, frequency limits are sometimes given suggesting that structures with fundamental frequencies above the limit will not encounter problems from the specific form of loading. For example, guidance was issued for permanent grandstands indicating that structures with a vertical frequency above 6 Hz should be suitable for concerts.[5] The rationale was that this avoids resonance from the first or second Fourier components of the cyclic loading, as the higher Fourier components have not been observed to cause problems on this type of structure. However, if this logic is applied to horizontal movement, a difficulty arises and this is discussed in the next section.

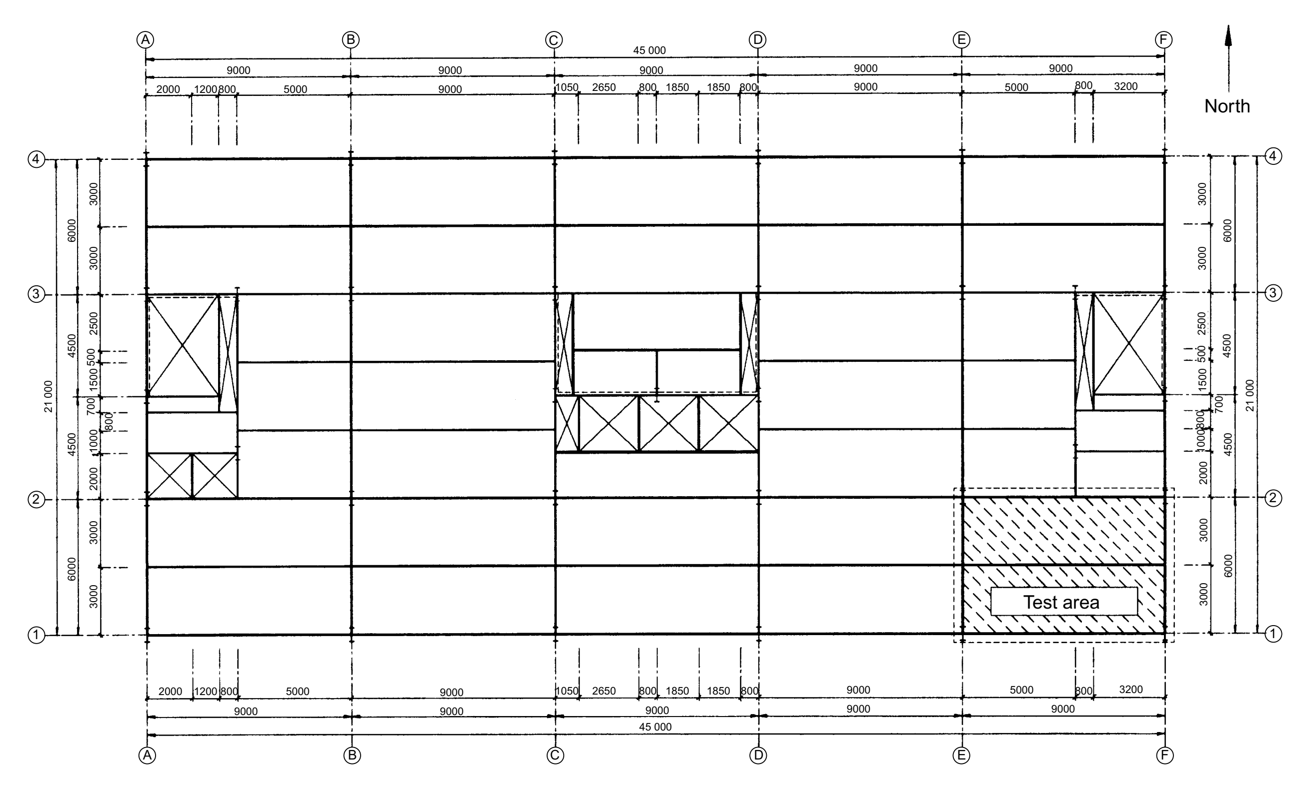


Fig. 8. Floor plan of the test building showing the test area

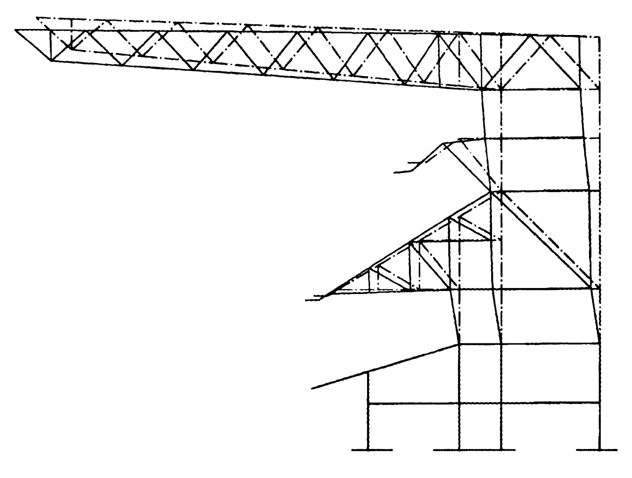


Fig. 9. Typical mode of vibration of a frame model of a cantilever grandstand showing coupled vertical and front-to-back movements

5.2. Temporary grandstands

Temporary grandstands may also encounter dynamic crowd loads but here horizontal motion in the sway direction is usually critical. To avoid dynamic problems the fundamental natural frequencies of the structures should be greater than the related load frequencies. For these structures it has been suggested that sway motion of the crowd will generate horizontal loads with a frequency up to 0·9 Hz.[5] However, if vertical jumping or bouncing is encountered this would generate horizontal motions in a higher frequency range, typically between 1·8 and 2·3 Hz.[7]. Thus setting a frequency limit to avoid resonance needs to consider both vertical and horizontal forms of loading.

When horizontal movements of a grandstand subject to human loads are observed, they are likely to be induced by vertical components of human loads rather than their horizontal components. The study in section 2 shows that the equivalent horizontal load factor for an asymmetric frame (Table 8) is larger than the ratio of the horizontal to vertical components of the loads. From measurements on many temporary grandstands, Littler[6] has shown that the largest horizontal accelerations exhibited the same frequencies as the vertical loading similar to the measurements of the floor response to jumping.

5.3. Cable-suspended bridges

Consider a cross-section of a cable-suspended bridge where the two cables are perpendicular to the bridge deck and a vertical load is applied asymmetrically on the deck as shown in Fig. 10(a). It can be shown that there is no horizontal movement under the vertical load. This is because the vertical and horizontal movements are not coupled and the vertical load produces vertical movements, which are due to elastic elongation of the cables.

If the two cables are inclined as shown in Fig. 10(b), it can be shown that the deck will experience both horizontal and rotational movements when it is subjected to an asymmetrically applied vertical load. Due to the inclination of the cables the vertical and horizontal movements of the deck are coupled. Thus an asymmetrically applied vertical load will induce both vertical and horizontal movements, which are

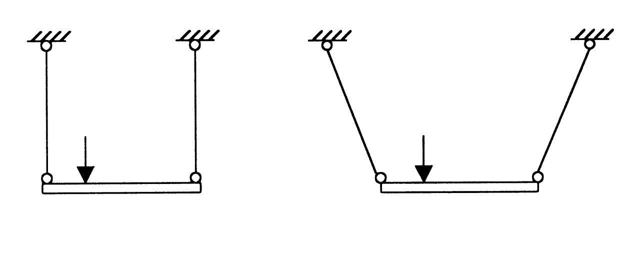


Fig. 10. Cross-sections of cable-suspended bridges

mainly due to the geometry of the system. This can be demonstrated using a simple model. Fig. 11 shows a metal plate held by two inclined strings. When a concentrated load acts at a quarter point of the plate, it produces horizontal and rotational movements.

The behaviour of cable-suspended bridges is different to that of the frame structures studied in sections 2 and 3. However, cable-suspended bridges are likely to be more sensitive to horizontal movements induced by vertical loads than frame structures. Thus it is pertinent to examine the horizontal movements induced by both vertical and horizontal loads.

5.4. Railway bridges

Horizontal movements of some railway bridges in China have been observed due to the increasing speed of trains and a number of bridges are now being reassessed for safety. As there are often two or more rail tracks on a bridge, the loading from any one train is effectively asymmetrical on the structure and hence horizontal motions as described in section 2.1 are generated. There will also be some horizontal forces generated by lateral movement of the railway vehicles, even along straight tracks. With the increasing speed of trains, the loading frequency will increase and this may be a problem if resonance occurs. Therefore it is appropriate to check horizontal as well as vertical natural frequencies of bridges to ensure that both are above the likely loading frequencies associated with trains running at higher speeds.

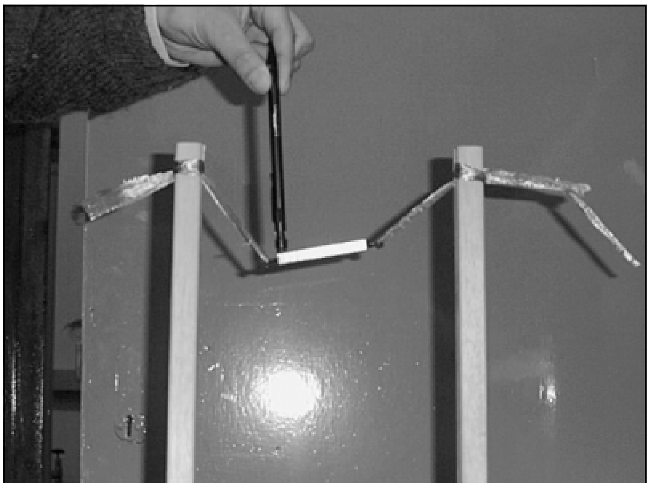


Fig. 11. Demonstration of horizontal movement induced by a vertical load

6. CONCLUSIONS

The paper has examined the horizontal movements of frame systems induced by vertical loads. The significance of such horizontal movements is represented by the equivalent horizontal load factor. It is concluded that:

(a) With few exceptions, vertical loads acting on frame structures induce horizontal movements of the structures. Exceptions are symmetric structures subject to symmetric vertical loads and (rarely) antisymmetric structures subject to antisymmetric loads.
(b) The magnitudes of the horizontal movements of frame structures due to vertical loads depend on the load distribution and the structural geometry.
(c) Structural form is more significant than load distribution to the magnitude of the horizontal movements.
(d) The taller the frame, the smaller the equivalent horizontal loading.
(e) When the frequency of a vertical dynamic load is close to one of the natural frequencies of a structure in its horizontal direction, resonance in the horizontal direction can occur.
(f) It is likely to be dynamic load situations that are of principal concern, especially when resonance may occur.

These findings may be useful when examining the horizontal response of structures, such as grandstands and bridges, which are subjected to predominantly vertical loads. The implications of the findings in practical structures have been discussed. An awareness of these findings may help to identify and avoid some potential problems.

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