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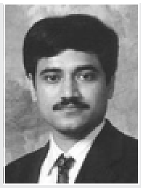

Parthasarathi Mandal
Lecturer, Manchester Centre for Civil & Construction Engineering, UMIST, Manchester

Tianjian Ji
Senior Lecturer, Manchester Centre for Civil & Construction Engineering, UMIST, Manchester

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Modelling dynamic behaviour of a cantilever grandstand

P. Mandal and T. Ji

This paper investigates the dynamic behaviour of a cantilever grandstand using two-dimensional (2D) frame, three-dimensional (3D) open-ended and 3D enclosed finite-element models. The effects of bracing systems, stiffness of seating decks and eccentricity of connections are examined. This comparative study also considers the relative advantages of the typical 2D and 3D models used in the study of dynamic behaviour and the response of grandstands to human loads. It is concluded that a 2D frame model is accurate enough for examining the dynamic behaviour of the grandstand in the vertical and front-to-back (also termed transverse) directions, and appropriate bracing systems can effectively increase structural stiffness of grandstands in the side-to-side (or longitudinal) and front-to-back directions. (The terms *side-to-side* and *front-to-back* have been used from the spectator's point of view.) While designing a grandstand for dynamic loads, it is customary to include the first few modes in a mode superposition based analysis. However, from the present study it has been observed that some vibration modes of interest could be at a much higher order than normally anticipated.

1. INTRODUCTION

Nowadays grandstands are used not only for sports events but also for pop concerts. During a pop concert, people may jump or bounce following the musical beat, which produces periodical crowd loads on grandstands and may cause excessive vibration. Some relatively modern grandstands in the United Kingdom are known to be vulnerable to such rhythmic crowd movements. The recently published 13th SCOSS report[1] on structural safety 2000–01 emphasised the significance of dynamic response of structures, in particular cantilever grandstands subject to rhythmic crowd loading and footbridges under pedestrian loading.

Grandstands, used for either sports events or pop concerts, need to be designed either to withstand human dynamic loading or to avoid resonance.[2,3] Either way requires an estimation of the relevant frequencies of grandstands by calculation or experiment. For a new design, a calculation of the frequencies has to be conducted. Thus it is important to model the structure correctly so that the natural frequencies can be correctly predicted and the dynamic behaviour of grandstands can be understood. Such a model becomes more significant when the response of grandstands to crowd dynamic loading needs to be assessed.

A 2D frame model may be used for such an analysis, which is normally believed to oversimplify the actual structure, and may lead to inaccurate prediction. In contrast, a 3D complete model may be used, which is thought to be adequate and accurate but complicated. However, there is little information available for a comparative study of these models used for dynamic analysis, such as the reliability of a 2D model, the efforts required for modelling a complete stand, and the relationship between 2D and 3D models of a grandstand. Thus this paper investigates the effects of different 2D and 3D finite-element (FE) models on the dynamic behaviour of a grandstand, considers the related modelling issues, and assesses the relative advantages of several FE models. The grandstand investigated is a combination of three existing grandstands, and thus reflects the general behaviour of cantilever grandstands but does not represent any particular grandstand. Three types of model are considered to represent the grandstand in this paper.

(*a*) *A 2D frame model*. This is a simple and typical representation of any one of the frames in the stand.
(*b*) *A 3D open-ended model*. Many grandstands in reality are constructed as open-ended 3D structures. Two adjacent stands may be linked by separate units. Though the stands may look completely enclosed, from the structural viewpoint they can be treated separately. This model may be considered as an independent grandstand or as part of the enclosed grandstand to be modelled.
(*c*) *A 3D enclosed model*. This provides a complete representation of the grandstand.

This being a numerical investigation, a commercial, general-purpose FE program, ABAQUS,[4] has been used. To ensure the validity of the modelling, output from ABAQUS was compared with the output from LUSAS[5] (another commercial FE program) for the study of the 2D frame model. For more computationally extensive cases, such as 3D modelling, only ABAQUS was used as it was run on a faster machine (SUN Enterprise HPC 4500 with eight 400 MHz UltraSPARC-II processors of 1·6 Gflops peak performance). The study includes calculations of the natural frequencies and vibration modes of a plane frame of the stand (about 100 nodes), an open-ended model of the stand with and without end supports (several thousand nodes), and an enclosed model of the stand (tens of thousands of nodes).

2. THE STRUCTURE AND FINITE-ELEMENT MODELS

Figure 1 shows an isometric view of the cantilever grandstand to be analysed. It consists of 90 frames with 21 frames at 6 m intervals on the longer sides (Fig. 2), 14 frames at 6 m intervals on the shorter sides, and five frames in each of the four corners. Each frame has a width of 40 m and a maximum height of 31 m (see Fig. 3). The dimensions and the member sizes of the stand are determined based on three different existing stands. There are three tiers to accommodate spectators. Two of them are cantilever tiers that are potentially sensitive to human-induced dynamic loads. The top tier uses cantilever steel beams and has a span of 6·7 m. To stiffen the tier, props are provided to connect the beams and frames. The middle tier is supported by 10·5 m long cantilever trusses. A pin fixing is placed at the end of the lower tier. The frames are connected by horizontal members at each connection and bracing members at suitable locations. Each level of the grandstand is covered by 200 mm thick concrete slabs with 75 mm topping with the exception of the upper tier, where the total thickness of the slab is 300 mm. The following loads, which are equivalent to the dead load of the slabs and the toppings, are used in the analysis when only the supporting frames are analysed without considering the slabs

- 0·48 kN/m^2 on the roof
- 7·00 kN/m^2 on the upper tier
- 6·50 kN/m^2 on the middle and lower tier, and floor areas.

The grandstand may be numerically modelled based on three possible types of model, as follows

(*a*) a single 2D frame model as shown in Fig. 3
(*b*) a 3D open-ended model as shown in Fig. 2
(*c*) a 3D closed model as shown in Fig. 1.

Possible variations in each type of model are also taken into account, which include the following.

(*a*) *The addition of slabs.* The slabs at each level are considered as shell elements in the model.
(*b*) *The consideration of eccentricity of beams and slabs.* The inclusion of the slab and beam eccentricities in the modelling ensures the appropriate composite action of beam–slab members.
(*c*) *The inclusion of bracing members.* Bracing members are added in the plane of the frame and also at the back of the structure, where applicable. For 3D models, the bracing members are provided in the plane of the frame for every sixth frame.

The modelling scheme and the relationships between all the models considered are summarised in Fig. 4. The arrows indicate subsequent modifications. For example, the box named 'Bracing' under the column '3D closed' means that the model is the basic one with added slab and bracing elements. The dashed horizontal lines indicate that the results from the different models are comparable. For example, the effect of slab and beam eccentricities in the three types of model can be compared from the results provided later in Table 2.

While analysing the frame as a skeletal structure (for example, the basic 2D frame model without the slab elements), it is necessary to transfer the loads from concrete floors to the frames. The product of the interval distance and the loads applied on unit area gives the line loads applied on each frame. However, to calculate the natural frequencies by linear-eigenvalue analysis, it is assumed that these loads act as equivalent masses. Thus the mass density of the members directly supporting the loads is increased. For example, if a member directly supporting the roof (of dead load 0·48 kN/m^2) has an area A and mass density ρ_s, then the equivalent mass density of the member can be written as

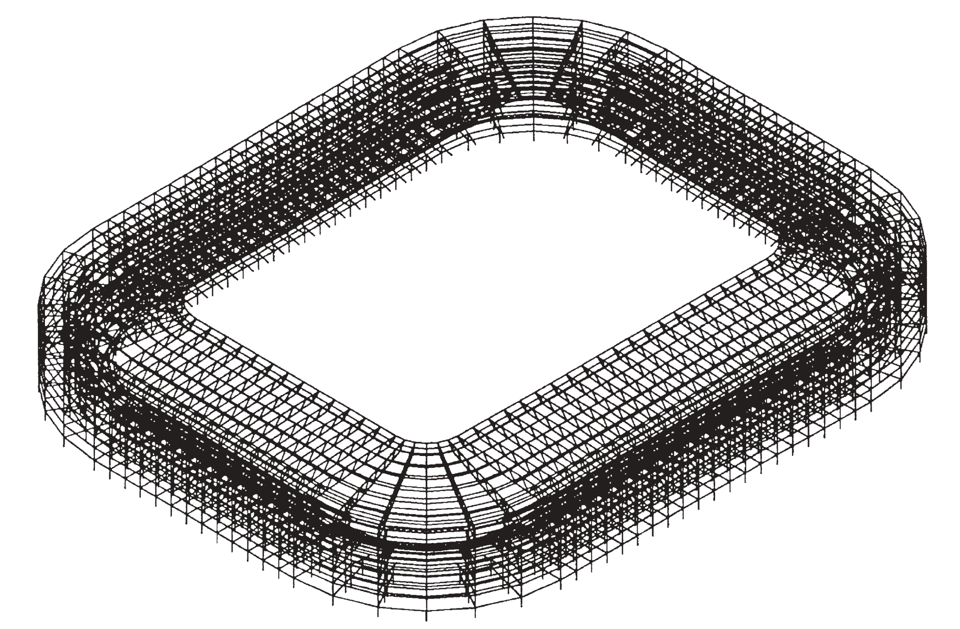

Fig. 1. Isometric view of the grandstands

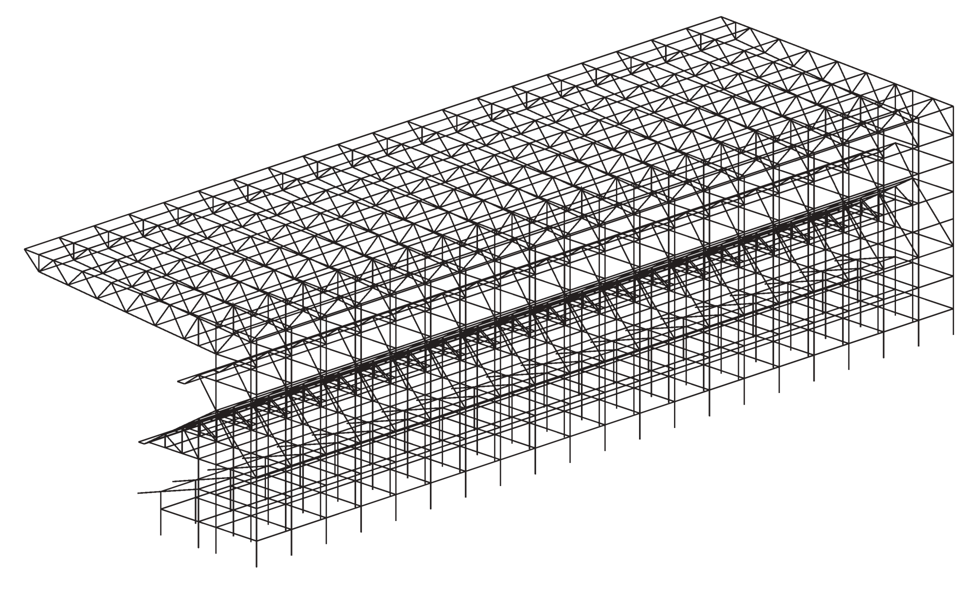

Fig. 2. The open-ended 3D model

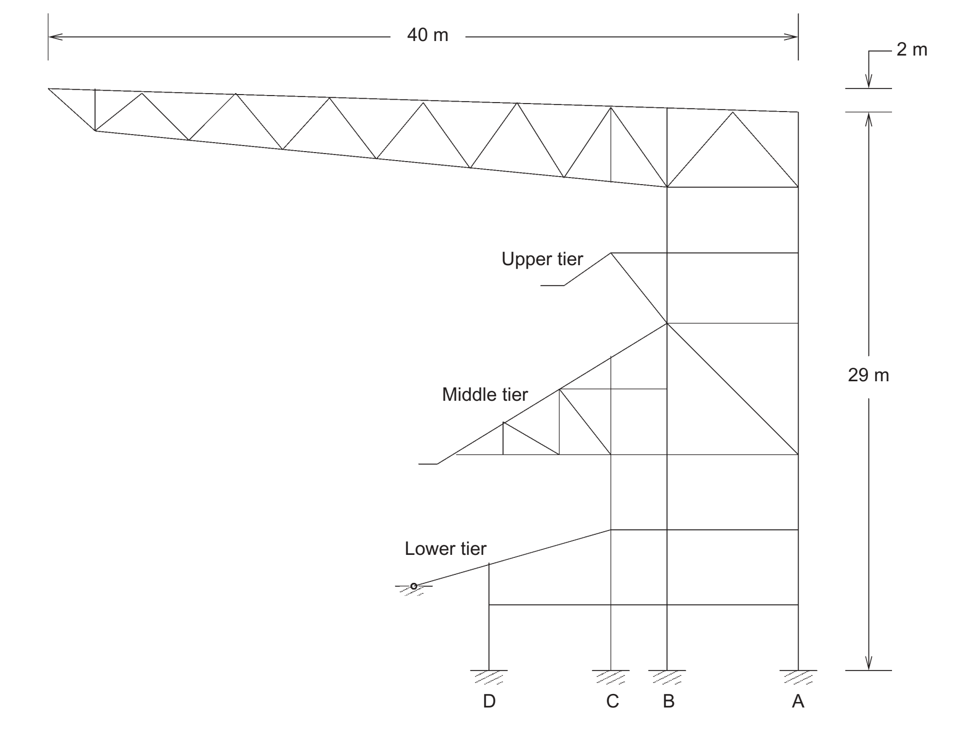

Fig. 3. Dimensions of a single frame (2D)

$$\rho_{eq} = \frac{6 \times 0{\cdot}48 \times 1000}{\boldsymbol{g} \times A} + \rho_s \qquad (1)$$

where $\boldsymbol{g}$ is the acceleration due to gravity, and the distance between two frames is 6 m. All the quantities are expressed in SI units.

3. DYNAMIC BEHAVIOUR

The eigenvalue analysis has been conducted to determine the natural frequencies and mode shapes of the stand. These provide important information for assessing the dynamic response of the structure. At the outset, the frequencies and mode shapes of an isolated 2D frame are calculated using both ABAQUS and LUSAS. The use of two different packages may eliminate possible systematic errors in the modelling. Then an

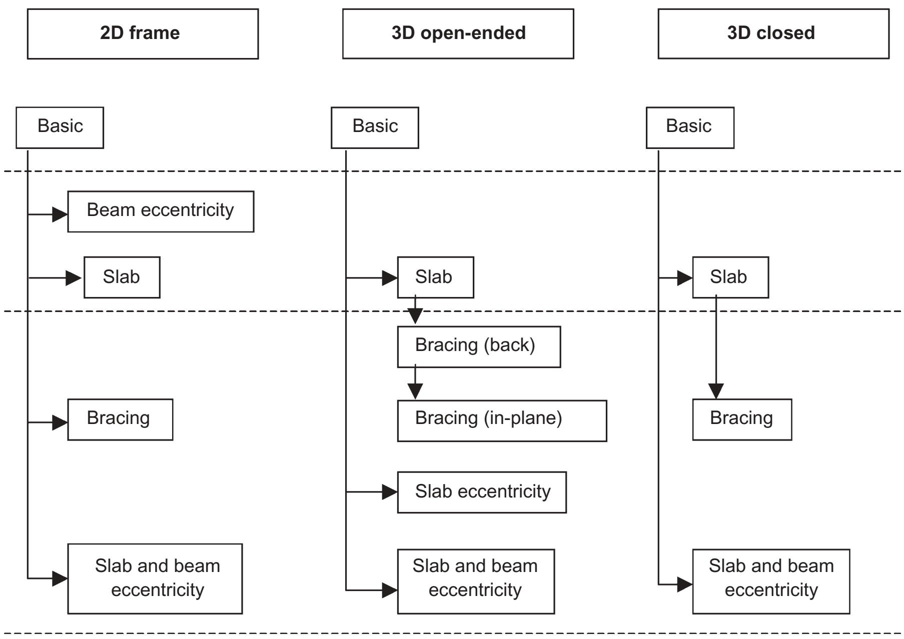

Fig. 4. Summary of the numerical modelling scheme (the arrows indicate subsequent modifications)

open-ended and an enclosed 3D model are analysed using ABAQUS. In all the analyses it is assumed that

(*a*) the structure behaves in the linear-elastic range
(*b*) there are no imperfections in the beams and columns
(*c*) all elements are rigidly connected.

3.1. The 2D frame model

The model consists of a single 2D frame, which means that any movement in the sway direction (side to side) is constrained. The first six frequencies computed using both the packages are given in Table 1. The mode shapes from both the packages also agree with each other. The basic dynamic behaviour of the frame can be described as follows.

The first mode shows the *front-to-back* movement of the whole frame at a frequency of 0·88 Hz (Fig. 5). The columns marked A and B (shown in Fig. 3) are predominantly vibrating like cantilevers fixed at the level of the lower tier. The upper tier and middle tier move as if rigid arms are attached to the columns, whereas the roof shows some vertical movement. As a result the movements of the frame in the front-to-back direction are approximately proportional to the vertical height from the level of the lower tier. This mode also indicates that stiffness between level 3 and level 4 is relatively low, and an increase of stiffness in this part will effectively increase the natural frequency. In the second mode the vibration of columns A and B is also predominant—like the second mode of a cantilever column. The vibrations of the roof, upper and middle tiers are similar to those shown in mode 1. There is no appreciable vibration in the lower tier as a pin support is arranged at its end. The third mode shows dominant vertical movements of the roof and the upper tier. The middle tier moves horizontally with relatively small amplitude. The fourth mode indicates mainly vertical movement of the middle tier at a frequency of 2·88 Hz (Fig. 6). The upper tier and the roof also move in the vertical direction, whereas the lower tier and the columns A and B have little movement. In the fifth mode the vibration is confined to the upper part of the frame. The upper tier moves vertically, coupled with a horizontal movement of the roof. The sixth mode is essentially the front-to-back movement of the columns below the lower tier.

	ABAQUS	LUSAS	% difference
Mode 1	0·885	0·882	−0·34
Mode 2	1·575	1·571	−0·25
Mode 3	1·890	1·886	−0·21
Mode 4	2·880	2·863	−0·59
Mode 5	3·896	3·883	−0·33
Mode 6	4·385	4·387	+0·04

Table 1. Comparison of frequencies for an isolated 2D frame model

In the previous eigenvalue analysis it was assumed that the grandstand is an assembly of one-dimensional structural members such as beams and columns. Although the dead load of seating decks was included in the calculation of effective mass density, the stiffness of the deck was not considered. The seating decks may be modelled by *plate* or *shell* elements, but that is not possible for the plane model. To consider the stiffness contributed from the deck slabs, an increased second moment of area of the beams has been used. The effective width of the slab for this calculation was considered as recommended by the design codes.[6] The results are shown in row 1(b) of Table 2. The fundamental natural frequency in the front-to-back direction increases from 0·88 Hz to 0·94 Hz, while the natural frequency in the vertical direction moves up from 2·88 Hz to 2·99 Hz. However, this calculation does not include the eccentricity of the neutral axis for bending of the beam–slab *composite* cross-section. The neutral axis of this composite section lies somewhere near the beam–slab interface. Instead of calculating the exact position of the

Model type		Horizontal		Vertical	Torsional	CPU: s
		Front-to-back	Side-to-side			
1. 2D frame	(a) Basic	0·88 (1)		2·88 (4)		1
	(b) Slab	0·94 (1)		2·99 (4)		1
	(c) Beam eccentricity	0·94 (1)		3·00 (4)		1
	(d) Slab and beam eccentricity 1	0·99 (1)		3·32 (4)		1
	(e) Slab and beam eccentricity 2	1·04 (1)		4·19 (4)		1
	(f) Bracing + (a)	1·53 (1)		4·35 (4)		1
2. 3D open	(a) Basic	0·84 (4)	0·25 (1)	2·80 (60)	0·70 (3)	116
	(b) Slab	0·98 (2)	0·61 (1)	3·55 (19)	1·00 (3)	126
	(c) Slab eccentricity	0·99 (2)	0·61 (1)	3·57 (19)	1·00 (3)	116
	(d) Slab and beam eccentricity	1·00 (2)	0·61 (1)	3·66 (19)	1·01 (3)	104
	(e) Bracing (back) + (b)	0·99 (2)	1·53 (3)	3·56 (17)	0·96 (1)	411
	(f) Bracing (in-plane) + (e)	1·34 (2)	1·55 (3)	4·16 (19)	1·30 (1)	420
3. 3D closed	(a) Basic	0·41 (2)		2·74 (370)	0·31 (1)	12 600
	(b) Slab	0·75 (1)		3·42 (62)	0·91 (3)	2 300
	(c) Slab and beam eccentricity	0·75 (1)		3·47 (63)	0·91 (3)	2 328
	(d) Bracing (partly in back and in-plane) + (b)	1·31 (1)		4·03 (61)	1·39 (2)	2 345

Table 2. Summary of frequencies calculated using different models. The frequencies are in Hz. The number in parentheses indicates the order of mode of vibration. The difference between models 1(d) and 1(e) is in the width of the slabs considered in the analysis. All these frequency values have been obtained by using ABAQUS

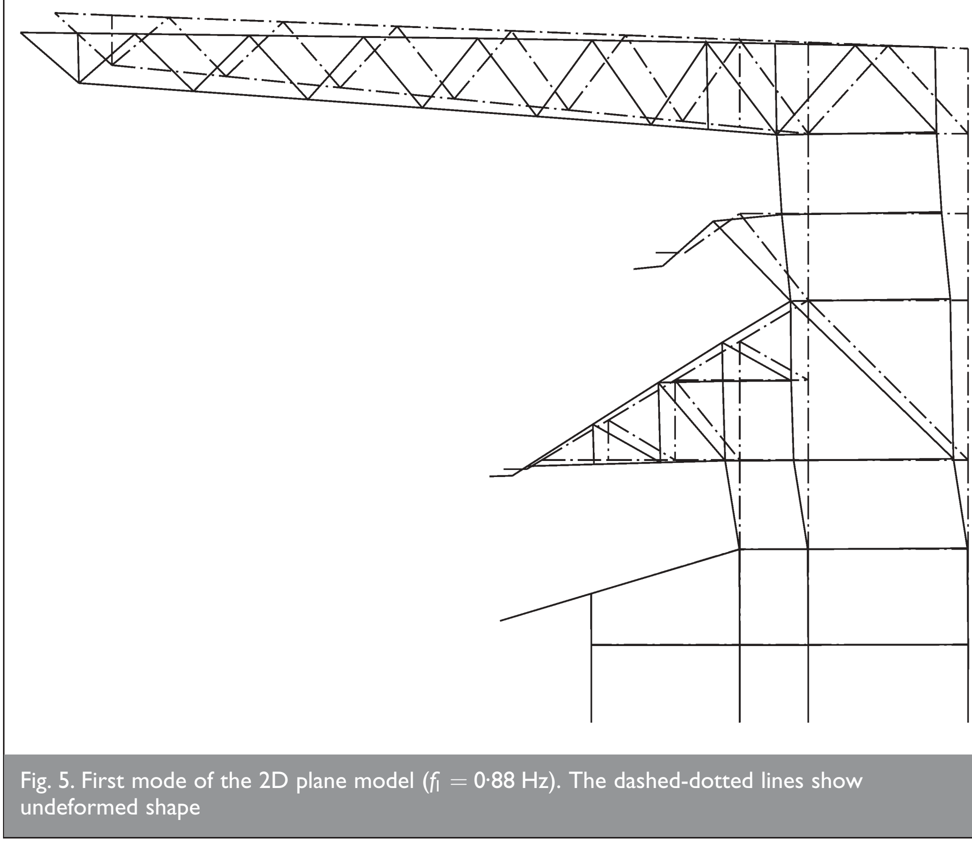

Fig. 5. First mode of the 2D plane model ($f_1 = 0{\cdot}88$ Hz). The dashed-dotted lines show undeformed shape

neutral axis, it is assumed that it is located in the beam–slab interface. Thus the increased second moment of area is calculated using the *parallel axis theorem*. The results are shown in row 1(d) in Table 2. Two concerned natural frequencies in the two perpendicular directions further increase to 0·99 Hz and 3·32 Hz respectively, which gives an increase in frequency with respect to the basic model of about 12% and 15% respectively, or an increase in stiffness of approximately 25% and 33% respectively. In this study, the effective width of the slab was considered as per the recommendations of British

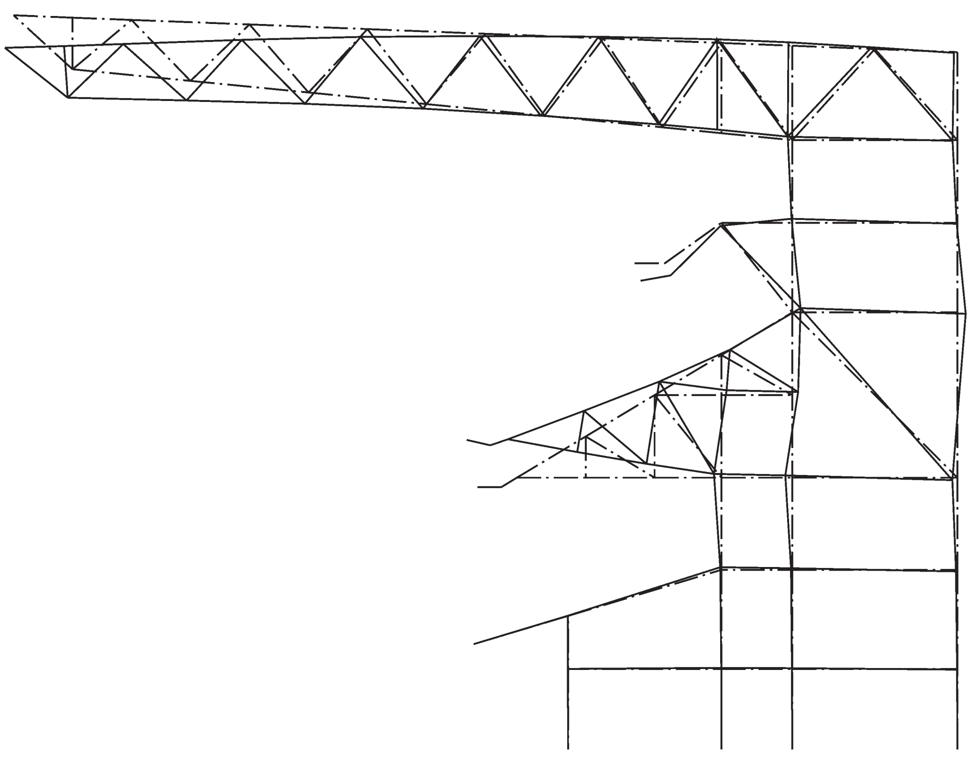

Fig. 6. Fourth mode of the 2D plane model ($f_4 = 2{\cdot}88$ Hz). The dashed-dotted lines show undeformed shape

Standards.[6] However, it seems that this was too conservative for the eigenvalue analysis. Another calculation was carried out by taking the effective width of the slab, 6 m, as the interval distance between two adjacent frames. The real situation may lie in between these two. This is because the second moment of area is overestimated in the latter case. The results are shown in row 1(e) of Table 2. Although the second moment of area increases considerably, the change of the frequency in the front-to-back direction is not substantial. However, the vertical frequency increases significantly. Row 1(c) indicates the frequency of the frame if only the beam eccentricity is considered. However, the mode shapes for these models have little difference.

After studying different mode shapes it was realised that the stiffness in the front-to-back direction can be significantly increased by adding extra bracings. A suitable in-plane bracing system is shown in Fig. 7. By adding nine bracing members, the natural frequencies in the two perpendicular directions become 1·53 Hz and 4·35 Hz (row 1(f) in Table 2), which are 1·7 and 1·5 times the corresponding frequencies of the basic model (row 1(a), Table 2).

3.2. The 3D open-ended model

The open-ended grandstand consists of 21 frames (Fig. 2) at an interval of 6 m. The basic model of the stand consists of the frames and the *horizontal beams* between the frames, which are modelled as 3D beam elements. The 2D frame model studied earlier can be considered as an open-ended 3D model in which all movements in the side-to-side direction are restrained. Now all the constraints are released, it will lead to more degrees of freedom and the fundamental frequency of the 3D open-ended mode will not be larger than the lowest frequency of a single 2D frame. The fundamental mode, with a frequency of 0·25 Hz, shows a side-to-side motion in Fig. 8. The second mode is similar to the first except that the roof and the middle tier move in opposite directions horizontally. The mode shape of a single frame viewed from the back resembles the second mode of a cantilever column. The third mode is a torsional mode of frequency 0·70 Hz. The first in-plane vibration (that is, in the front-to-back direction) occurs at the fourth mode at a frequency of 0·84 Hz (row 2(a), Table 2), which is approximately 5% less than the first frequency of an isolated 2D frame. This is because the mass of the horizontal beams between the frames was not considered in the 2D frame model. This explanation is confirmed by running another analysis that assumes the mass of these beams to be zero and yields the corresponding frequency of 0·88 Hz, the same as that of the isolated frame. The vertical motion of the middle tier occurs at the sixtieth mode with a frequency of 2·80 Hz. Compared with the 2D frame model, the frequency is 3% lower. This is due to the combined effects of considering the added mass of the horizontal beams and the reduced mass on the two end frames. The frames at the two ends take half of the mass that is on the other frames.

In the basic model the slabs are considered as contributing only to the dead loads of the stand. However, the slabs have large in-plane stiffness, which acts like a bracing system. To investigate the effect on the frequency of the whole structure, the slabs are considered as *shell elements* in the 3D model. The mass is now revised to the self-weight of the members without any indirect mass. The fundamental frequency then becomes 0·54 Hz in the side-to-side direction. The mode shows that the

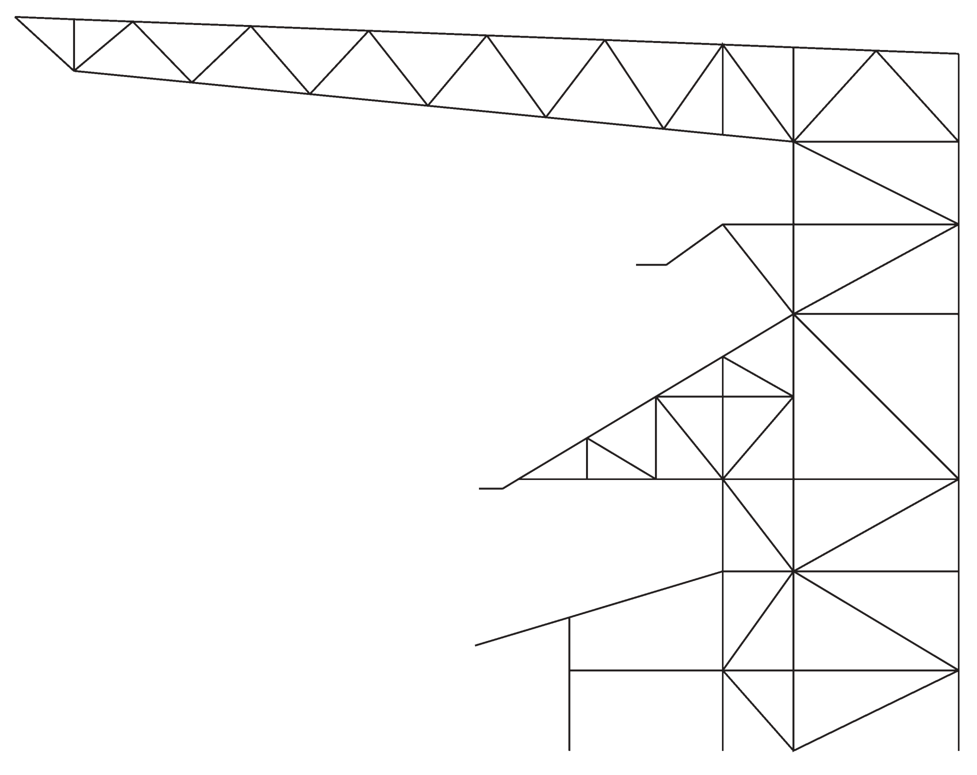

Fig. 7. In-plane bracing arrangements of the 2D plane model

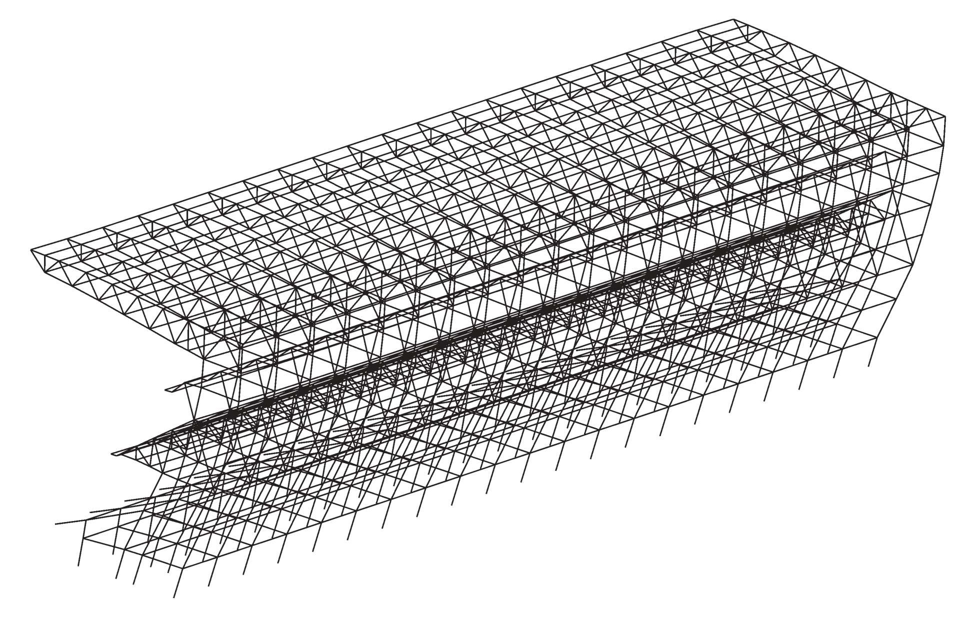

Fig. 8. First mode of a 3D open-ended model showing the dominant movement in the side-to-side direction ($f_1 = 0{\cdot}25$ Hz)

movement is restricted mainly to the roof. In reality, however, there will be roof sheeting, and this will increase the stiffness of the roof in the longitudinal direction. If a roof cover of flexural rigidity 33·2 kNm is considered in the calculation, the fundamental frequency in the side-to-side direction increases to 0·61 Hz. The second mode is the front-to-back vibration mode with a frequency of 0·98 Hz (row 2(b), Table 2). The previous calculations considered that the slabs and supporting beams are connected at the nodes that are at the centroids of the elements. If the eccentricity is considered, the natural frequencies will increase in the front-to-back and vertical directions, but not in the side-to-side direction, as shown in rows 2(c) and 2(d) of Table 2.

The effects of bracing systems are then examined. An effective bracing system[7] is arranged based on the slab model (row 2(b) in Table 2), as shown in Fig. 9. The calculation shows that the torsional mode becomes the fundamental mode with a

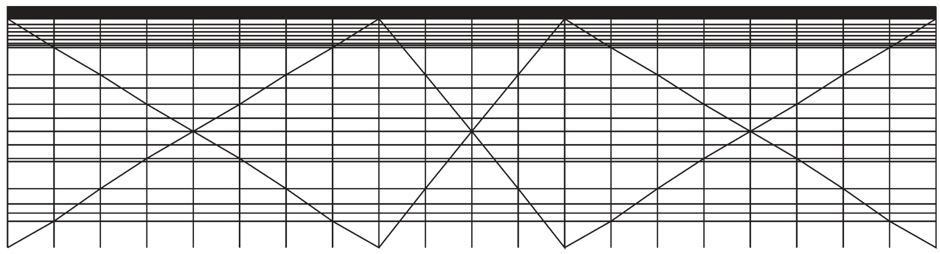

Fig. 9. 3D open model with bracing members at the back

frequency of 0·96 Hz. The frequencies in the front-to-back and vertical directions change by only 1·00% and 0·28%. This is understandable because the bracing members at the back of the stand do not impart any effective stiffness in the two perpendicular directions. However, it is observed that the bracing system effectively restrains the motion in the side-to-side direction. The first side-to-side mode is the third mode at a frequency of 1·53 Hz. Although the additional material of the bracing members takes only a very small percentage of the total weight of the grandstands, the stiffness of the grandstand in the side-to-side direction is increased significantly.

In the next step in-plane bracing members are added (as shown in Fig. 7) in every sixth frame (that is, frame No. 1, 6, 11, 16 and 21). This measure will effectively increase structural stiffness in the front-to-back, vertical and torsional directions. The first frequencies in these three directions increase from 0·99 Hz, 3·56 Hz and 0·96 Hz to 1·34 Hz, 4·16 Hz and 1·30 Hz respectively (rows 2(e) and 2(f), Table 2). The mode shapes in case 2(f) (see Fig. 10) remain similar to those in the previous case 2(e). However, the difference in the frequency in the side-to-side direction is not significant because the bracing in the plane of the frame hardly contributes to the stiffness in the side-to-side direction.

3.3. The 3D closed model

The complete grandstand consists of 90 frames: 21 frames on each of two longer sides at an interval of 6 m, 14 on each of the other two sides at an interval of 6 m, and 5 each at four corners at 15° intervals (see Fig. 1). The first vibration mode of

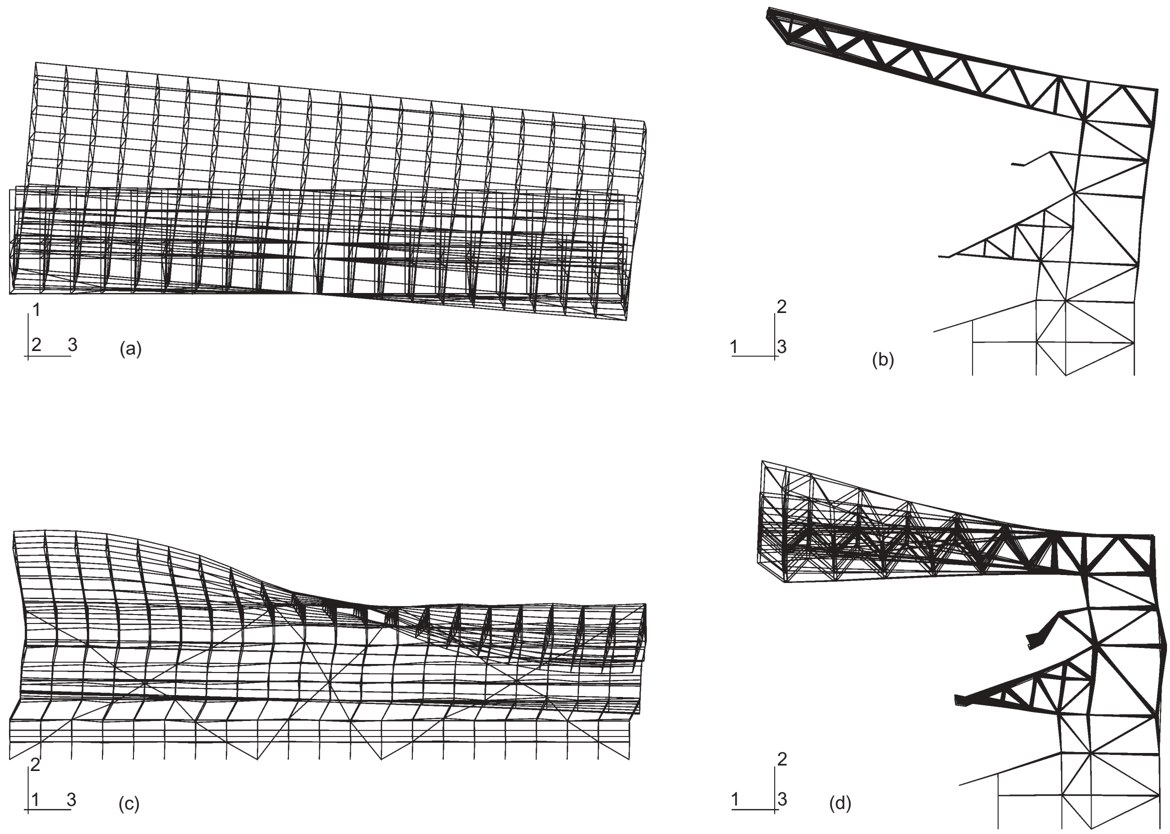

Fig. 10. Mode shapes of the 3D open model with bracing members at the back and in the plane of the frame. The frequencies are given in row 2(f), Table 2. The directions 1, 2 and 3 in the figures correspond to front-to-back, vertical, and side-to-side respectively. (a) First mode shows torsional motion ($f_1 = 1{\cdot}30$ Hz); (b) second mode is the motion in the front-to-back direction ($f_2 = 1{\cdot}34$ Hz); (c) third mode shows vibration in the side-to-side direction ($f_3 = 1{\cdot}55$ Hz); (d) vertical vibration of the middle tier appears at the nineteenth mode ($f_{19} = 4{\cdot}16$ Hz)

the basic model shows a torsional movement (Fig. 11) at a frequency of 0·31 Hz. This is because the individual plane frame has less stiffness in the side-to-side direction. Therefore all the frames move in the direction perpendicular to their own planes, which constitutes an overall torsional mode. This mode corresponds to the side-to-side mode of the 3D open-ended model, which has a frequency of 0·25 Hz. It can be appreciated that the corner frames provide some restraint to this movement, which leads to an increase in the frequency of the closed model.

The second mode (Fig. 12) shows a combined side-to-side and front-to-back movement. The longer sides move in the side-to-side direction whereas the shorter sides move in the front-to-back direction. The frequency of the mode is 0·41 Hz, which is lower than the frequency of a single frame vibrating front-to-back, 0·88 Hz, but higher than the frequency in the side-to-side direction of model 2(a), 0·25 Hz. When vibrating in the side-to-side direction the 42 frames in the longer sides have a frequency of 0·25 Hz, whereas the 28 frames in the shorter sides have frequencies about 0·84 Hz. Therefore the overall effect is in between the two values. Similarly, in the third mode, the longer sides move front-to-back whereas the shorter sides move in the side-to-side direction, resulting in a frequency of 0·43 Hz.

The calculation of the vertical frequency of the middle tier requires a significant amount of computational effort. It occurs at the 370th mode. It was necessary to check all the intermediate mode shapes before identifying this mode. The computed frequency is 2·74 Hz. This is about 5% less than the vertical frequency calculated from a single 2D frame (model 1(a)).

In the next model (3b), the slabs are included. The fundamental mode shape changes from the torsional to the combined side-to-side and front-to-back motion mode, with a significantly increased frequency of 0·75 Hz. The third mode is torsional with a frequency of 0·91 Hz, which is about 2·9 times the corresponding frequency in model 3(a). If the bracing members are added both at the back and in the plane of the frames at suitable intervals as shown in Figs 7 and 9, the

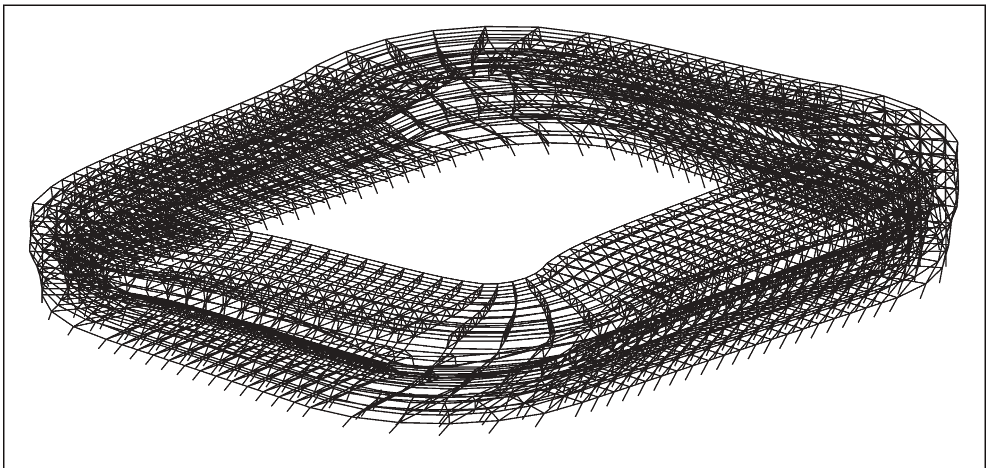

Fig. 11. First mode (torsional) of a 3D closed model of the grandstand ($f_1 = 0{\cdot}31$ Hz)

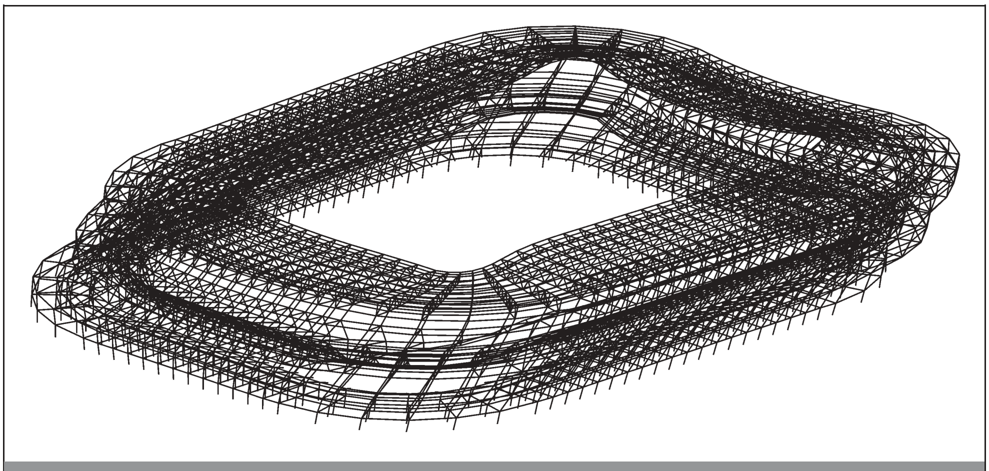

Fig. 12. Second mode (horizontal) of a 3D closed model ($f_2 = 0{\cdot}41$ Hz)

fundamental frequency increases from 0·75 Hz to 1·31 Hz but the mode shape remains unchanged. The second mode is torsional with a frequency of 1·39 Hz, compared to 0·91 Hz without bracing.

All the calculated natural frequencies of the above models are summarised in Table 2 for comparison in conjunction with the model brief and relations shown in Fig. 4.

4. DISCUSSION

4.1. 2D models versus 3D models

It is observed that the dynamic behaviour (at least in terms of fundamental mode shapes) differs if the modelling is done by a single 2D plane frame, a 3D open model (a series of plane frames), or a complete closed 3D model. A 2D frame model is simple and is able to identify the frequencies and vibration modes in the front-to-back direction of the frame and in the vertical direction of the tiers. The in-plane stiffness of the seating deck is normally not considered in the 2D model, which is quite significant for correctly predicting the frequencies in the front-to-back and vertical directions. The way for such a consideration in the plane model has been suggested by taking the equivalent second moment of area of the beam and deck sections. The quality of the 2D frame models can be appreciated by the comparison of frequencies in the directions concerned with the full 3D models in Table 3, which is abstracted from Table 2.

However, the 2D frame model ignores the vibrations in the side-to-side and torsional directions. A full 3D model needs to be used if the frequencies and modes in these two directions are of concern. Nowadays many prestigious structures are analysed using 3D models. The 3D models identify vibration modes in the side-to-side and torsional directions, which cannot be observed in a 2D model. Therefore such modelling provides complete information on the dynamic behaviour of the grandstand. The 3D models also allow examination of the effects of bracing systems that are arranged in the plane of selected frames and at the back of the stand. The 3D enclosed model also shows the effects of the corner frames, which link the frames in the two perpendicular directions and provide elastic constraints to these frames. It is observed that the fundamental frequency increases from 0·25 Hz for the open-ended model to 0·31 Hz for the closed model.

The problem size increases significantly from 2D to 3D modelling. Besides the complications in modelling, the output size also becomes much larger. It is a tedious task and sometimes difficult to find information for a particular mode. For example, in the present study, if one is interested in the vertical frequency of the middle tier, the 2D analysis provides the answer within just four modes. However, in a 3D open model the same mode occurs at the order of sixtieth mode. One needs to carefully check all the 59 modes preceding it. The situation becomes almost unmanageable for the 3D closed structure. The same vibration shape occurs at the 370th mode.

The computation (CPU) times for calculating all the models are also provided in Table 2. This refers to the CPU time taken by ABAQUS calculations run on a SUN Enterprise HPC 4500 machine with eight 400 MHz UltraSPARC-II processors of 1·6 Gflops peak performance. The CPU time increases from 1 s for a 2D model to 116 s for a 3D open-ended model and 12 600 s for the 3D enclosed basic model. The analysis also indicates that less CPU time is required when the structure possesses enough stiffness. The high CPU time for model 3(a) occurs because 400 modes are required in the calculation in order to identify the vertical vibration mode of the middle tier. The values of the related frequencies listed in Table 3 are within 5% differences between the 2D and 3D basic models. Therefore it is probably unnecessary to stick to 3D modelling if the same information within some tolerance could be obtained from an equivalent, simpler 2D analysis.

The comparative study indicates that it is suitable and accurate enough to use a 2D frame model to predict the frequencies and mode shapes in the front-to-back direction of the stand and in the vertical direction of the tiers. If any effects outside the plane of a 2D model need to be assessed, a 3D model should be considered.

In most cases grandstands are built in stages, and the owner sometimes cannot afford to build a complete stand at a time. Two isolated stands on two adjacent sides are not uncommon situations. Some 3D enclosed stands are also not fully integrated. The corners and the side stands are all separate

		Model		
		2D frame	3D open	3D closed
Basic	Front-to-back	0·88 (1)	0·84 (4)	
	Vertical	2·88 (4)	2·80 (60)	2·74 (370)
Slab–beam eccentricity	Front-to-back	0·99 (1) 1·04 (1)	1·00 (2)	
	Vertical	3·32 (4) 4·19 (4)	3·66 (19)	3·47 (63)

Table 3. Comparison of the front-to-back and vertical frequencies for different models. The numbers in parentheses indicate the corresponding mode numbers. This is a subset of Table 2. The two values for 2D plane frames for slab–beam eccentricity indicate two different models employed in calculating the effective width of slabs

units, which may have been built at different times (e.g. Old Trafford, Manchester), resulting in 3D open-ended stands. An analysis of the whole stand is always preferable; however, it will require huge computational effort. The results from a 2D analysis also provide much useful information, which could also be obtained from a 3D analysis at the expense of careful and painstaking observations.

4.2. The stiffness of seating decks

Consideration of the beam and slab eccentricities in the modelling increases the natural frequencies. In the 2D model, the inclusion of beam eccentricity increases the front-to-back and vertical frequencies by 7% and 4% respectively. When the slab and slab eccentricities are considered, these increments become 12% and 15% with respect to the basic model. However, the consideration of eccentricities is not significant in the 3D models because the stiffness of the slab has not been considered by any other means in the 2D model.

The inclusion of slab as shell elements in the model significantly improves the dynamic behaviour of the stand, which is only possible in 3D models. For example, the slab model increases the frequencies of the 3D open-ended model by 17%, 140%, 27% and 43% in the front-to-back, side-to-side, vertical and torsional directions respectively. The increments are also significant in the 3D closed models, which are 83%, 25% and 190% in the horizontal, vertical and torsional directions respectively.

4.3. The effects of bracing systems

The in-plane bracing increases both front-to-back and vertical frequencies of the 2D model. The frequencies increase by 74% and 51% in the front-to-back and vertical directions for the chosen bracing arrangement. These values will be different for other arrangements. In the case of 3D models the in-plane bracing is provided at every sixth frame so as not to restrict accessibility and usability of the stand. Comparing the frequencies in models 2(e) and 2(f), these bracing members, using a negligible amount of material, increase the frequencies from 0·99 Hz, 3·56 Hz and 0·96 Hz to 1·34 Hz, 4·16 Hz and 1·30 Hz in the front-to-back, vertical and torsional directions respectively. Comparing the frequencies between models 1(a), 1(f) and 2(f) it is observed that the bracing in plane arranged in every sixth frame effectively increases the frequencies in both the vertical and front-to-back directions.

It has been observed in practice that the side-to-side vibration is not critical for cantilever grandstands owing to human-induced rhythmic load. Normally the spectators' loading is concentrated on the tier that tends to initiate front-to-back and/or vertical vibration. However, the side-to-side mode is important for temporary grandstands, where the spectator loading is spread over the whole grandstand. The movement can be reduced by suitable bracing arrangements. It is interesting that the bracing system arranged at the back brings the frequency from 0·61 Hz to 1·53 Hz in the side-to-side direction (compare models 2(b) and 2(e)). This is probably because the structure itself is relatively less stiff in this direction before being braced and the bracing arrangement provided is very effective, which creates a direct force path leading to a stiffer structure.[7] Although the use and arrangement of bracing members at the back of a grandstand may be restricted owing to aesthetic requirements, it provides a way to make grandstands stiffer.

The extension from a 3D open-ended model to a 3D enclosed model with the same type of bracing does not change the frequencies significantly (models 2(e) and 3(d)), although the fundamental mode changes from a torsional to horizontal one. For the 3D closed model the front-to-back and the side-to-side motions are coupled. Therefore the two columns for horizontal frequencies are merged in Table 2.

4.4. Coupled vertical and front-to-back vibration

The geometry of the frame of the grandstand is not symmetric; the fundamental mode of the frame model shows a coupled front-to-back and vertical vibration where the movement in the front-to-back direction is much larger than that of the tiers in the vertical direction. This observation implies that, if the vertical load applied on the tiers has a frequency close to the fundamental frequency of the frame, excessive vibration in the front-to-back direction of the frame may occur.

4.5. Number of modes required in a dynamic analysis

In a mode-superposition-based dynamic analysis it is normally sufficient to consider the first few modes. However, in the present study it has been observed that important modes such as vertical vibrations of the 3D open and closed models appear at a much higher order. For example, the vertical vibration of the 3D open-ended basic model is observed at mode 60, which means that it is necessary to study all the 59 modes preceding it. Moreover, for a dynamic analysis at least 60 modes need to be considered. When considering the 3D closed basic model, the vertical vibration of the middle tier appears at mode 370.

5. CONCLUSIONS

This paper investigates the comparative dynamic behaviour of a cantilever grandstand using 2D frame models, 3D open-ended models and 3D enclosed models. The conclusions of the study are as follows.

5.1. Selection of numerical models

(*a*) A 2D frame model can be used to examine the behaviour of the stand in the vertical and front-to-back directions.
(*b*) A 3D model should be used when the motion in the side-to-side and/or torsional directions is to be assessed.
(*c*) The eccentricities of the seating decks and supporting beams to their neutral axis should be considered in the analysis: they not only correctly reflect the actual behaviour of the stand, but also effectively increase the stiffnesses of the cantilever tiers in the vertical direction.

5.2. Coupled vertical and front-to-back vibration

Owing to the geometry of the cantilever grandstand, the movement of the stand in the front-to-back direction is coupled with the vertical vibration of the upper and middle tiers, as shown in Fig. 5. This observation indicates that excessive vibration in the front-to-back direction may occur when one of the load frequencies in the vertical direction matches the natural frequency of the stand in the front-to-back direction.

5.3. Bracing arrangement in the front-to-back direction

Bracing members arranged in this direction (Fig. 7) can effectively increase stiffnesses (frequencies) in both vertical and front-to-back directions. It may be possible to arrange such a braced frame an interval of every five or six frames without affecting serviceability requirements.

5.4. Bracing arrangement in the side-to-side direction

Effective bracing systems arranged at the back of the stand can significantly increase the frequency of the stand in the side-to-side direction.

5.5. Number of modes in the analysis

The modes of interest to the designer of a grandstand could be at a higher order than in the usual cases. This is because of the occurrence of many insignificant modes. If a complete dynamic analysis (mode-superposition based) is to be undertaken, one needs to be certain about the inclusion of all the modes of primary interest—that is, vibration in the vertical, torsional and the two horizontal directions.

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Please email, fax or post your discussion contributions to the secretary by 1 December 2004: email: daniela.wong@ice.org.uk; fax: +44 (0)20 7799 1325; or post to Daniela Wong, Journals Department, Institution of Civil Engineers, 1–7 Great George Street, London SW1P 3AA.