Loads generated by jumping crowds: numerical modelling

B. R. Ellis
BSc, PhD, CEng, MIStructE
BRE Centre for Structural and Geotechnical Engineering

T. Ji
BSc, MSc, PhD, CEng, MIStructE
Manchester Centre for Civil and Construction Engineering, UMIST.

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Synopsis

This paper is concerned with modelling the loads generated by groups of people jumping rhythmically. The principal objective is to replicate the results that were obtained in an earlier experimental study in which measurements were made with groups of up to 64 people. The experiments showed how the Fourier components of the loads attenuate with increasing group size and this defines a load model which can be used to calculate structural response. The measurements also showed the variations that can occur for similar sized groups.

A model for the loads produced by an individual jumping is used as the basis of this study, with variations to three main parameters being examined. The first parameter being the jump height which the individual selects subconsciously; the second is the jumping frequency which may not align perfectly with the requested frequency; and finally the phase differences between individuals in a crowd. It is assumed that the variations in jump height and frequency will follow normal distributions and that the standard deviations of the distributions can be determined from the available measurements. A load-time history can then be generated for an individual jumping using the basic load model but including the chosen variables selected at random from the distributions. Groups of people are represented by the combination of the appropriate number of individual load-time histories and here the phase difference between individuals can be introduced. The variation in phase difference can be determined from the experimental data.

The modelling is based upon the measurements and attempts to reproduce the experimental data. Although this provides a method for determining a load model, it is not suggested that this should be used for calculating structural response because the model derived directly from the experiments is far easier to use. However, this serves to explain some of the characteristic variations that were observed in the experiments and provides a better understanding of this important load case. It also enables the loads produced by larger groups to be calculated.

Introduction

It is recognised that the dynamic loads generated by crowds need to be considered in the design of some structures, but the available information on this topic is limited. This has been highlighted by SCOSS and identified as an area of concern for engineers[1]. The largest dynamic loads are generated by rhythmic jumping, which may be encountered with some types of dancing, and this can be quite a severe load case especially as it may generate a resonant response in some structures. However, it is important to recognise the significant difference between the situation where everyone is jumping and the situation which is often encountered at real events, like pop concerts, where only some of the crowd are jumping. These two situations will lead to large differences in structural response. This paper considers the extreme condition when everyone in a crowd is jumping.

A reasonable load model for an individual jumping is available[2] but the determination of the loads generated by groups jumping is not a simple extrapolation based upon this model. A previous study[3] obtained experimental measurements of the response of two floors for groups of up to 64 people jumping and derived a load model based on this experimental data. Although this model provides a reasonably simple method for calculating the average structural response for various sized groups it fails to replicate the variations observed in the experiments. These variations are seen as varying peak amplitudes of response for any one event (i.e. not steady-state) and different averaged responses for similar sized groups, this being accentuated for the smaller groups. This paper considers the numerical modelling of the loads generated by crowds and relates this to the experimental measurements. It attempts to model the observed experimental variations in order to understand the key variables involved in the process. Although this provides an improved understanding of the topic, for calculations for small groups (say up to 64 people) it may be sensible to use a simpler load model, like that derived directly from the experiments.

As the objective of the work is to replicate the experimental measurements, some of the experimental data are analysed to provide information on a number of variables; e.g. the variation in an individual's jumping frequency during one event and the variation in jumping height. The model for an individual jumping is used as the basis of the study with variables introduced into this model in order to calculate a load-time history. Groups of people are represented by the combination of the appropriate number of individual load-time histories, but no variation in the weight of the different jumpers is considered. The modelling assumes that the sum of the response for a number of individuals jumping separately is the same as the response for them all jumping together, and this pragmatic approach provides a significant simplification of the situation.

In the following sections the basic load model is considered first as this introduces the terms used throughout the paper. The experimental work is then described. The numerical modelling is explained and calculations compared with the experimental measurements. The combined load model is used to calculate the floor's displacement-time history for 32 people jumping and compared with the experimental data. The load model is then used to determine the loads produced by larger groups and the significant points arising from the work are discussed.

The load model for an individual jumping

The basic model for the vertical loads produced by an individual moving cyclically is:

$$F_s(t) = G_s\left(1.0 + \sum_{n=1}^{\infty} r_n \sin\left(\frac{2n\pi}{T_p}t + \phi_n\right)\right) \qquad \text{...(1)}$$

where
$F_s(t)$ = the time varying load
G_s = the weight of the individual
n = number of Fourier terms
r_n = Fourier coefficient (or dynamic load factor)
T_p = the period of the cyclic load or the inverse of the cyclic frequency
ϕ_n = phase lag of the n^{th} term

For jumping, the motion is defined by the ratio of the period that the person is on the ground to the period of the jumping cycle, which is termed the contact ratio 'α', with the load during the contact period being represented by a half-sine wave. This can be used to determine the Fourier components of equation (1)[2,4]

$$r_n = \begin{cases} \dfrac{\pi}{2} & \text{for } 2n\alpha = 1 \\ \left|\dfrac{2\cos(n\pi\alpha)}{1-(2n\alpha)^2}\right| & \text{for } 2n\alpha \neq 1 \end{cases} \qquad \text{...(2)}$$

and

$\phi_n = 0 \qquad if\ 2n\alpha = 1$

or

$$\phi_n = \begin{cases} \tan^{-1}\left(\dfrac{1+\cos(2n\pi\alpha)}{\sin(2n\pi\alpha)}\right) - \pi & if\ \dfrac{\sin(2n\pi\alpha)}{1-(2n\alpha)^2} < 0 \\ -\dfrac{\pi}{2} & if\ \sin(2n\pi\alpha) = 0 \\ \tan^{-1}\left(\dfrac{1+\cos(2n\pi\alpha)}{\sin(2n\pi\alpha)}\right) & if\ \dfrac{\sin(2n\pi\alpha)}{1-(2n\alpha)^2} > 0 \end{cases} \quad if\ 2n\alpha \neq 1 \qquad ..(3)$$

Hence if α is defined, the load model can be evaluated. From experimental observations $\alpha = 1/3$ was suggested as representing normal jumping[5]. However, this model is for perfect repetitive movement, but perfection is impossible to achieve and there will be variations about this ideal model.

Assume that an individual is jumping in response to an audible prompt at a given frequency and consider the possible variables.

- First, the contact ratio α and hence the height of jumping could vary, thus providing a variation in r_n and ϕ_n.
- Second, although the jumper will try to jump at the prompt frequency, the jumping may not be perfectly aligned with the prompt giving a variation in the jumping frequency.
- Finally there will be a phase or time difference between the prompt and the individual jumping and this difference will vary between individuals.

These variations may be too small to have a significant effect on the loading produced by an individual; however, their influence may be significant when a group of jumpers is considered. Within this paper these possible variations will be examined to see their influence and identify any characteristics that are likely to be observed in experiments.

Experimental data

Experimental procedure

In 1997 and 2001 controlled experiments were undertaken at BRE's Cardington laboratory to determine the loads generated by crowds jumping, focusing on the variation in load with crowd size. Tests were undertaken on two floors with different sized groups jumping at selected frequencies[3]. The group sizes were selected to be powers of 2 with 64 being the largest group that could be accommodated safely. The jumping was co-ordinated using a musical beat at selected frequencies and both the vertical acceleration and displacement of the centre of the floor were recorded.

For the experiments only the lowest three Fourier coefficients (FCs) were determined directly. The frequency of the 4th FC was near to the principal frequency of the floor and here the significant resonance effects would affect the accuracy of the evaluation method.

Variation in Fourier coefficients with group size

To determine the loads, the displacement measurements were used and subjected to band-pass filtering around the load frequency and two and three times that value. This provides the response to the different Fourier components of the load. The dynamic displacements for each Fourier component were then obtained by evaluating the rms value of the filtered displacement and multiplying this value by √2, to obtain an 'average' peak value rather than determining the absolute peak value. The individual FCs were determined by dividing the peak dynamic displacement for the selected Fourier term by the static displacement of the group, with a correction for any dynamic amplification. The FCs were then plotted against the number of people in the group on a log/log plot, together with regression lines for each FC based on a power relationship. The data are shown in fig 1.

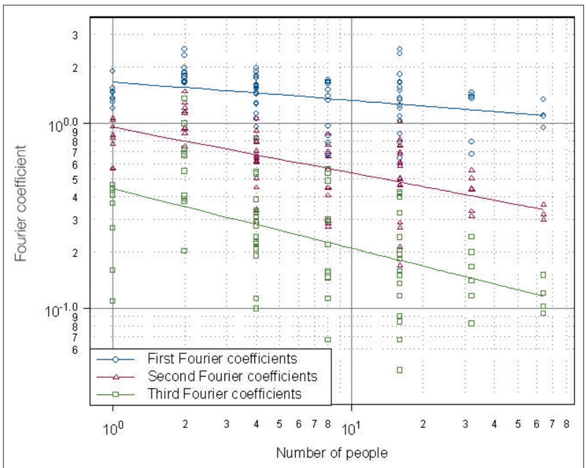


Fig 1. Fourier coefficients determined from the tests

The values which define the regression lines provide the numerical values of the FCs including their variation with crowd size and these were used in the load model to evaluate both acceleration and displacement for comparison with experiment. It was apparent that although this model gave similar amplitudes of acceleration and displacement to the average values that were measured it did not replicate the variations seen in the experimental data.

The regression lines are defined by the following equations which characterise the variation in FCs with the number of people 'p' in the group and these can be used within the load model:

First Fourier coefficient	$r_{1,p} = 1.61 \times p^{-0.082}$
Second Fourier coefficient	$r_{2,p} = 0.94 \times p^{-0.24}$
Third Fourier coefficient	$r_{3,p} = 0.44 \times p^{-0.31}$

Variations in frequency and contact ratio for individuals

From the measurement or observation of a group of people jumping in response to a rhythmic beat it would be very difficult to determine the variation in jumping frequency of the individuals within that group. However, determining variations in frequency is possible for an individual jumping alone. For this work it has been assumed that the load generated by a group of people jumping is the same as the sum of the loads generated by those same people jumping individually. This hypothesis removes the difficulty of examining the frequency variations between individuals in a group.

To represent a group the jumping frequencies generated by several individuals should be considered. Therefore, the frequency variation of one person jumping for a number of cycles is examined first. Then the same procedure is used for several individuals to generate a set of data which describes the frequency variation of jumping loads.

The method for investigating the variation of jumping frequency can also be applied to study the variation of the contact ratio.

There were eight records for individuals, four jumping to a beat of 1.90Hz and four to a beat of 2.15Hz. For analysis the recorded vertical displacement time histories were band–pass filtered to leave the response to the first Fourier component of the load. For, example for the first test, the person was jumping at 1.90Hz, so the data were band-pass filtered between 0.95Hz and 2.85Hz. The filtered data were then divided by the static displacement for the individual and the appropriate amplification factor for excitation of the principal mode by a load at 1.90Hz, to yield a normalised load in which the peaks are equivalent to the first FC of the load. The resulting time history is shown in fig 2.

From the figure it can be appreciated that the amplitude of the peaks varies throughout the test. It should be recognised that the digital filtering does affect the amplitude of the first and last cycles. A computer program was used to determine the amplitude of each peak and the time interval between the peaks. For this example the peaks vary about a mean value of 1.61 which equates to a contact ratio of 0.47.

The examination of the individual time histories suggests that the jumper selects a particular jump height (or contact ratio) and retains this throughout the short jumping event,

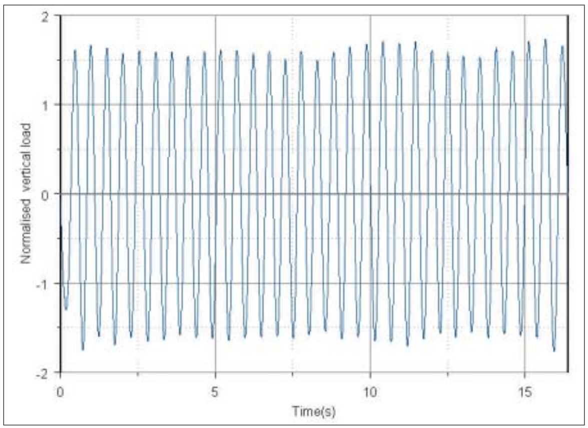


Fig 2. Normalised load for first Fourier component, test 1

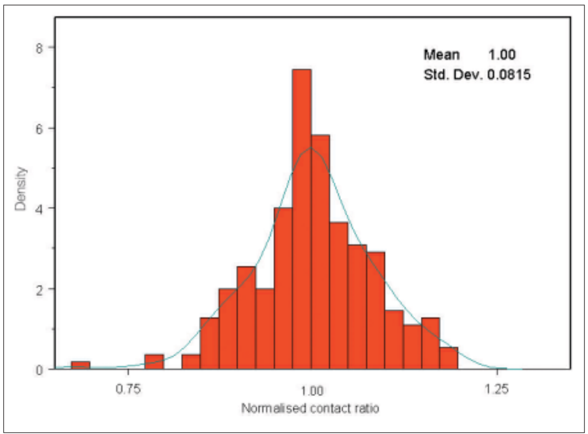


Fig 3. Normalised contact ratios for all single person events

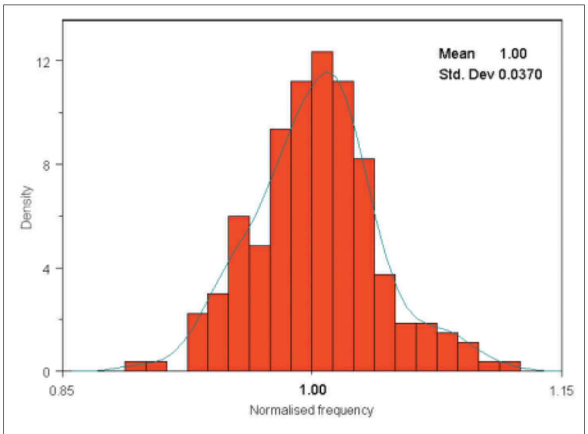


Fig 4. Normalised frequencies for all single person events

albeit with slight variations about the selected value. The distribution of contact ratios when normalised by the average contact ratio for the event is shown in fig 3. It can be appreciated that this can reasonably be represented by a normal distribution with a standard deviation of 0.082.

Although the jumping is co-ordinated by a musical beat, it may not be perfectly in time with the prompt giving a variation in the jumping frequency about the prompt frequency. The jumper probably tries to align the jumping with the prompt frequency and hence tries to compensate for any error in one jump by a slight adjustment to the next jump, thus leading to a process of successive corrections. This may be more severe at jumping frequencies which the jumper finds difficult to achieve, but for this study frequencies around 2Hz were selected which are reasonable comfortable jumping frequencies.

By examining the time of each peak response in fig 2 the variation in jumping frequency throughout the displacement-time history is revealed. The average frequency throughout this record is 1.91Hz (the requested frequency being 1.90Hz). The distribution for the jumping frequencies, when normalised by the requested frequency is shown in fig 4. It can be appreciated that this can be reasonably represented by a normal distribution with a SD of 0.037.

Numerical modelling

The variables to consider and determination of Fourier coefficients

In the introduction three possible variations in jumping were mentioned, namely contact ratio, jumping frequency and relative phase. In this section these variables will be examined. This is still a simplification of the real situation because these variables may themselves change whilst an individual is jumping and be affected by factors like the individual's ability and physical state, and possibly his interaction with others within a crowd. However, it is reasonable to assume that the variables can be specified for a particular event and hence their influence on the generated loads can be assessed.

The basic analysis procedure is to determine a load-time history for an individual jumping based on the load model given in equation 1, but taking account of variability in the parameter being considered. To determine a load-time history for a crowd, an appropriate number of individual load-time histories are combined, with the final load-time history being divided by the number of individual records. The combined load-time history is then analysed using a similar procedure to that used for the experimental data, i.e. the data are band-pass filtered and the average peak load for each Fourier component determined.

Generation of data obeying the normal distribution

The calculations consider variations to several parameters based upon a normal distribution with a specified standard deviation. To generate a normal distribution a method similar to that developed for ref 6 is used. It makes use of a random number generator that generates numbers between 0 and 1. The process generates 8192 numbers between 0 and 1 in 32 bands of equal width. The number of elements in each band is calculated, to the nearest integer, based on the normal distribution, and covers ±3 standard deviations. The random numbers are generated and stored within each band. When a band has the required number of elements, further numbers that would fall into the band are discarded. The process continues until all bands are full. All the numbers are then combined in a single array and the elements re-arranged in a random order to remove the effects of the selection process. The data are then normalised to leave the array with an average value of 0.0 and a Standard Deviation of 1. The choice of ±3 standard deviations includes 99.7% of values within the true distribution.

To determine a random number for a given normal distribution with average value X and standard deviation Y, a value from the array is selected, multiplied by Y and then added to X.

Modelling variations in contact ratio and frequency for an individual

To replicate the load for an individual jumping two variables are considered. First the contact ratio is selected and the target frequency is specified. Then random variables (based on the selected normal distributions) are introduced

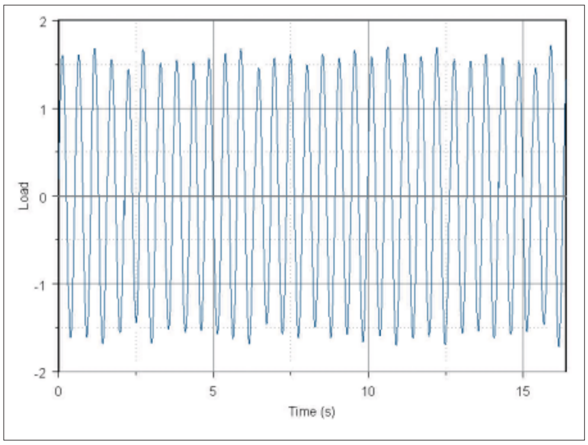


Fig 5. Data generated with characteristics of test 1

to modify the contact ratio and the frequency. One load cycle is then determined. Values for frequency and contact ratio are then selected for the next load cycle and the load-time history calculated and appended to that of the previous cycle. This process is repeated for the duration of the jumping. The variation in the contact ratio is selected from the experimentally determined normal distribution but that for the frequency is based on a series of successive corrections upon which the random variation is applied. The successive correction procedure determines the time for one cycle based on the selected frequency which will be slightly different to the time defined by the musical prompt. The next frequency is calculated based on the time required to re-align with the musical prompt, but with another variation imposed, etc. This method of determining frequency is due to the fact that the person is jumping in response to a prompt and hence the jumper tries to compensate for any error in one jump by a slight correction to the next jump, thus leading to a process of successive corrections. This may be considered to be a feedback process. This contrasts with the variations for the contact ratio where there is no prompt or target. An example of the generated output for the first Fourier component for the first test is shown in fig 5, with the initial selection of alpha being 0.47 and the frequency being 1.90Hz. It can be seen that the artificially generated data exhibits similar characteristics to the measurements (fig 2).

Modelling variations in contact ratio and frequency for groups

From the measurements it appears that the individual selects a jumping style and tries to maintain this throughout any one event. Hence in the previous section it was possible to select values of contact ratio and frequency for the event that was considered. For the group tests, the target frequency is known, but the selection of the contact ratio (α), or jump height, is the choice of the individual jumpers. Based on the data shown in fig 1 and focusing on the single jumpers, an average value of α of 0.4 is selected to model the observed variations in the three FCs. From analysis of the first FCs a standard deviation of 0.08 was derived. Whether a normal distribution is appropriate here is questionable, but as the objective is to illustrate how the observed experimental variations can occur, an exact distribution of the loading is not required and, in fact, this cannot be derived from the available measurements. Thus for each individual a value of α is selected, the target frequency is known and the variations based upon the experimental results for the individuals is known, hence the load-time history can be calculated.

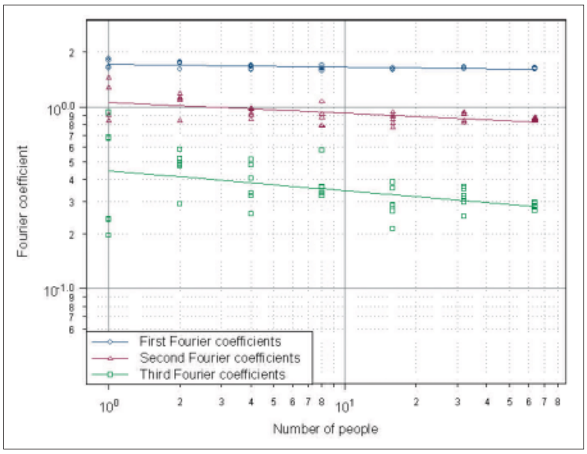
If various sized groups with these characteristics are considered, and analysed in a similar manner to the experimental data, then the resulting variation in FCs with group size can be obtained. The load was evaluated six times for groups of 1, 2, 4 …64 and the results shown in fig 6. The figure indicates that small variations in the contact ratio and frequency do not produce a significant reduction in FCs for increasing group sizes in comparison with the measurements shown in fig 1. Increases in contact ratio and corresponding decreases in FC will be balanced by the equally likely decreases in contact ratio and corresponding increases in FC.

In contrast to the results shown in fig 6, if the frequency feedback loop is removed, to look at a random selection of frequencies (within the distribution), a severe reduction in FCs with number of people is seen (in comparison with the measurements) especially for the first FC. This would represent the situation without a musical prompt and must therefore be rejected.

Variation in phase within a group

Phase is the parameter that those involved with modelling have previous considered[6, 7]. Equation 4 provides the model for an individual jumping but includes a further phase difference ψ.

Fig 6.
FCs against group size – no phase variation



$$F_s(t) = G_s\left(1.0 + \sum_{n=1}^{\infty} r_n \sin\left(\frac{2n\pi}{T_p}t + \phi_n + n\psi\right)\right) \quad \text{...(4)}$$

In this model it is assumed that the individual jumps at the correct frequency and at a constant height (or α) but there is a phase difference ψ between the prompt and the response which is different for individuals. By considering the individual Fourier terms, it is relatively simple to determine the FCs for groups of people when the variation in ψ is defined. This has been undertaken for $\alpha = {}^1/_3$ and evaluated for various sized groups. Following a few trials a SD of 0.22π was found to provide a reasonable correlation with the measurements based on the attenuation in FCs with group size. The results are given in fig 7.

It can be seen that although this is exhibiting attenuation of FCs with increasing group size similar to the experimental values, there is no variation for FCs for the individuals as variations of jumping frequency and contact ratio are not considered here, which is clearly at variance with the measurements. Also the group-time histories would provide a steady-state loading, again in contrast to the measurements.

Fig 7.
FCs against group size considering only phase variation

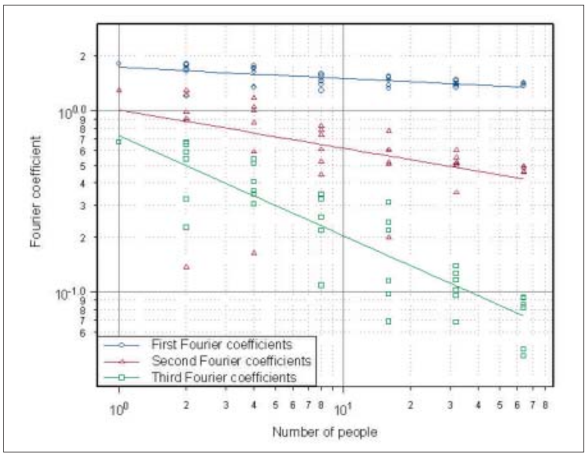
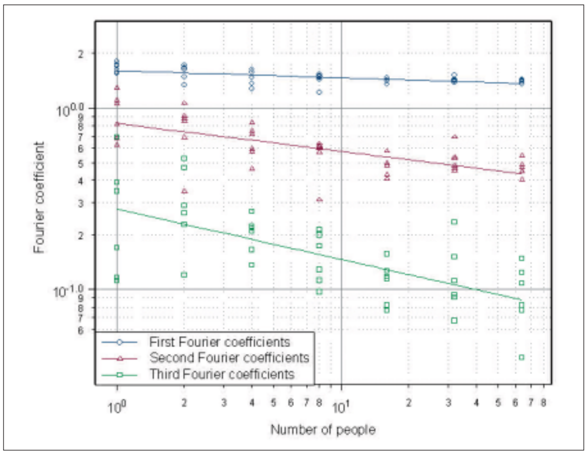


Fig 8.
FCs against group size – combined model



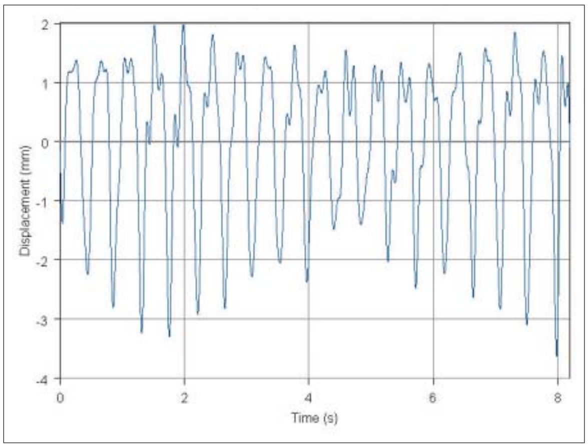


Fig 9. (left) Measured displacement-time history for 32 people

Table 1: Evaluation of FCs for different values of phase variation for groups of 8192

SD Phase	Fourier coefficients of the resulting loads FC1	FC2	FC3	FC4	FC5	FC6
0.18 π	1.40	0.469	0.072	0.0070	0.0024	0.0018
0.12 π	1.52	0.655	0.157	0.0281	0.0073	0.0019
0.00 π	1.62	0.852	0.283	0.0831	0.0434	0.0245

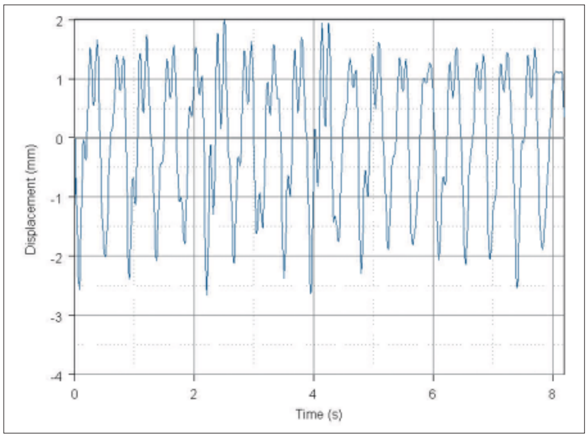


Fig 10. (right) Calculated displacement-time history for 32 people

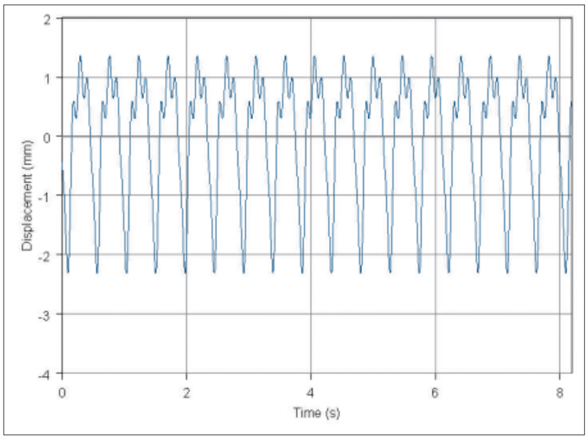


Fig 11. Displacement-time history for 32 people – experiment model

A combination of all of the variations

Although the variables have been examined in isolation, it is apparent that they actually occur together. Hence all the variables have been included in one program and a series of load-time histories calculated. It is assumed that the frequency is determined by a process for successive corrections upon which a variation is imposed, rather than as a purely random variable. The values of the variables that appear to give similar characteristics to those observed in the experiments are: a frequency variation with normalised SD of 0.037, a phase variation with a SD of 0.18π, and a value of α of 0.4 with a normalised SD of 0.08, with a variation throughout each individual time history with a SD of 0.082.

A plot of the variation in FCs with crowd size is given in fig 8, which compares reasonably well with the experimental data. However, with so many variables in the numerical generation of the data, some differences in the results are to be expected, in the same way that slight differences would inevitably result if the experiments were repeated. The SD of the phase variation is slightly smaller than that seen in the previous section, primarily because of the weighting due to the FCs for the tests with single jumpers, which are based on an average α of $^2/_5$ here, but on an α of $^1/_3$ in the previous section.

Although the results presented are for the first three FCs, the calculation procedure can evaluate more Fourier terms, and this will be considered later.

Calculated response

Having developed a method of generating a load-time history it is of interest to see how this can be used to evaluate response. The normalised load intensity for 32 people is determined for the first test floor using the method given in ref 3.

$$G_{32}=G_{ave}\gamma(32)=67.6\times 9.81\times 32^{0.79}=10249\text{N}$$

The calculated load-time history, which is based on unity load and includes four Fourier terms, is increased by the above load intensity to represent the actual load due to 32 people. This is used along with the floor's characteristics and the frequency measured for the particular tests to determine a displacement-time history using the Duhamel integral method. Measured and calculated displacement time histories are given in fig 9 and 10.

It can be appreciated that the calculations are broadly inline with measurements, although an exact match is not to be expected. For comparison the displacement-time history calculated using the load model derived directly from the experimental results is shown in fig 11.

Evaluating the loads for larger groups

The testing was restricted to 64 people because of safety concerns related to the test floor, but the development of the load model allows evaluation for larger groups. This has been undertaken for groups of up to 8192, using the values defined in the section entitled *a combination of all of the variations* and selecting the group sizes to be powers of 2. These are shown graphically in fig 12 for the first three FCs.

With the results for up to 64 people based upon six calculations for each group size, it was seen that the spread in calculated FCs became less for the larger groups. This was probably due to the statistical variations being smoothed by a larger sample size. This becomes even clearer when groups up to 8192 people are examined. It can be seen that the FCs are no longer reducing with increasing crowd size but attain a constant value, suggesting that the power relationships derived from the experimental data are only appropriate for the smaller groups.

Although it is possible to calculate loads for such large groups, it is important to realise that other factors, which are not included in the study, may become important. For

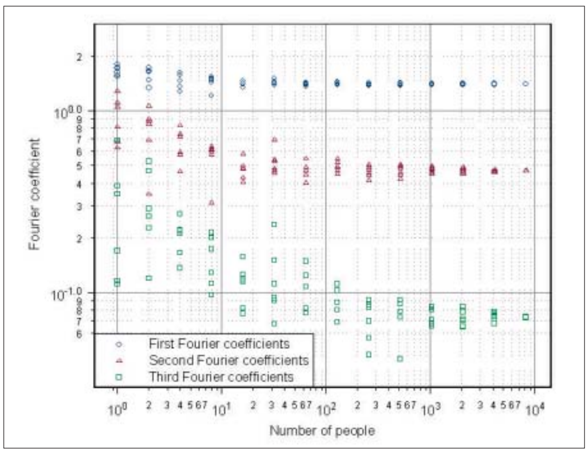


Fig 12. FCs against group size – combined model for up to 8192 people

example, for a large group there could be a time delay between the people nearest the loudspeakers hearing the musical prompt and those furthest away and this has been observed on a grandstand[8]. This can result in significant phase differences and therefore different loads.

With the finding that the FCs settle to final values for large crowds it is possible to quantify the effects of changing the Standard Deviations for the phase variation. For illustration purposes the calculations will be repeated for groups of 8192 people, but examining three values for phase variation, i.e. $0.18\,\pi$, $0.12\,\pi$ and 0.00. The first of these values is the best-fit value based on the experimental results; the second is an hypothetical value to reflect a much better co-coordinated group of jumpers, and the third value assumes no phase variation (as considered earlier). The results of the calculations are given in table 1 and six FCs are presented.

It might reasonably be assumed that a group of professional dancers would have these improved all round characteristics, hence a lower phase variation ($0.12\,\pi$) than that seen in the experiments ($0.18\,\pi$), but it is of interest to see that this has only a relatively small effect on the first FC, but a larger effect for the some of the higher FCs. However, no information is available for the co-ordination of different groups of people, so these cannot yet be quantified.

Discussion

The work presented in this paper has used variations on the load model for an individual jumping and combinations of load-time histories for individuals in an attempt to replicate some experimental results that were obtained for groups of people jumping on two floors. It is not suggested that this is the best method for determining a load model, because the model derived directly from the experiments is far easier to use for calculating structural response for small groups and the values given in table 1 are easier to use with large groups. However the modelling serves to explain some of the characteristic variations that were observed in the experiments. It is clear that there are many variables to consider with a group of people jumping, and each load-time history will be different even for the same group of people jumping at different times. The modelling suggests that the variations will be smaller for larger groups. However, this model is based upon one set of measurements and some variations are to be expected with different groups, hence there will never be one definitive solution.

The range of variables used in the modelling is based on limited experimental results. It is apparent that different groups of people will exhibit different characteristics. For example a group of professional dancers would be far better co-ordinated than a group of construction workers and therefore generate much higher loads. For the experiments the groups were students with an interest in music, and this may be reasonably representative of people at discotheques. Whether the same group of students would achieve better co-ordination if the same exercise was repeated several times remains unanswered; but it does seem probable that co-ordination would improve with experience.

The data on the variations in frequency and contact ratio have been determined from a relatively small number of jumping records for individuals, although it would be relatively simple to obtain a large amount of data on these items from a few more experiments. More data could be used to provide information on the differences between individuals and a better understanding of the statistical distributions; but this was not undertaken here as the intention was to limit the study to replicating the data measured at Cardington. Nevertheless, the data presented do help to explain the significant variations that are seen in the experiments. The information on phase variation has been determined from the experimental data with the different sized groups and it would be difficult to get a reasonable estimate of this parameter by studying individuals or small groups. This is perhaps the most important variable for determining attenuation in loads with increasing group size and it may be difficult to obtain more data here unless experiments like that undertaken at Cardington are repeated.

One item that the modelling assumes is that everyone is jumping. In reality this is an extreme case which generates very large dynamic loads, but in many situations some members of a group may not be jumping. Stationary people would serve to reduce the structural response significantly[9] as they not only fail to contribute to the load but also provide a large increase in the system damping. However, for some structures, like dance floors, it would seem wise to consider the situation with everyone jumping because this extreme load case may be encountered. For other structures, like inclined seating decks on grandstands, this load case may not be appropriate even for pop concerts.

Conclusions

The paper investigates the numerical modelling of the loads produced by crowd jumping and considers variations of contact ratio, frequency and phase lag in the models based on measurements obtained on two floors with groups of up to 64 jumpers. The main findings from this work are:

- A numerical method has been developed for determining the loads generated by groups of people jumping. The calculated loads correspond reasonably well with those measured in an earlier experimental study.
- The standard deviations for contact ratios and jumping frequencies have been determined from records of individuals jumping. The normalised standard deviation for contact ratio is 0.082 and for frequency is 0.037.
- The variations in frequency and contact ratio do not significantly affect the FCs for different group sizes although they explain some of the variations observed in the experiment.
- The standard deviation for phase lags determined from the group tests is 0.18π.
- The phase lags have a significant effect on the variation of FCs with crowd size. For small groups (say up to 64 people) the reduction in FCs is more marked for higher FCs, but for large crowds the FCs reach a constant value.

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