Floor vibration induced by dance-type loads: verification

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Synopsis

This paper describes both experimental tests and numerical calculations which were performed to verify an analytical method of calculating floor vibrations induced by dance-type loads[1]. The basic experimental set-up is described, and experiments which were undertaken to investigate the interaction between people and structures are discussed. To demonstrate the accuracy of the analytical load model, measured and calculated load time histories are compared. The response of a structure to dance-type loads is then checked by comparing finite element and analytical solutions and by comparing experimental and analytical solutions. To emphasise the importance of choosing an appropriate load model when calculating response, examples of resonant response caused by the sixth multiple of the dance frequency are provided. Finally, comparisons are made with the results from similar work conducted at NRC, Canada.

Notation

a	is the acceleration (m/s²)
A	is the generalised co-ordinate (m)
B_{11}	is the structural factor of the fundamental mode
C	is the damping matrix
f_p	is the load frequency (Hz)
f_{ij}	is the ij th natural frequency (Hz)
$F(t)$	is the load (N)
$\mathbf{F}(t)$	is the load vector (N)
G	is the weight load of dancers per unit area (N/m²)
K_p	is the dynamic impact factor
$\mathbf{K}$	is the stiffness matrix
L_x L_y	are the lengths of a floor along its x and y directions (m)
m	is the mass of a bare floor per unit area (kg/m²)
m_l	is the mass of people per unit area (kg/m²)
M	is the mass matrix
t	is the time (s)
t_p	is the contact duration (s)
T_p	is the period of dance type of load (s)
U	is the displacement vector
$\dot{U}$	is the velocity vector
$\ddot{U}$	is the acceleration vector
α	is the contact ratio
ζ	is the critical damping ratio

Introduction

For this paper, dance-type loads are defined as loads produced by rhythmic movement of an individual, or a group, usually in response to music. The maximum resulting structural response occurs when jumping is involved. These loads may cause either serviceability or safety problems on some structures – e.g. floors when dancing or aerobics is involved or temporary grandstands when jumping and stamping are encountered. This is of particular importance within the UK, as the dynamic loads from crowds – and hence the calculation of structural response to the crowd loads – are not covered by current standards, even though they are recognised as being important. The characterisation of these loads and the calculation of the structural response to these loads is considered in a separate paper[1].

In this paper, a range of experimental tests, conducted to identify the effect of human involvement on structural vibration, is described and used to confirm some of the basic assumptions adopted in the analysis in ref. 1. The data are obtained primarily from tests on a simply supported prestressed concrete beam. This experimental setup enables the effects of the interaction between the dancer and the beam to be evaluated and also load and response time histories to be obtained. Although this provides some experimental data, it is a simpler system than a real dance floor; therefore, to provide further validation, numerical examples are given, using the finite element method, which provide more detailed representations of floors. Physical measurements made by other groups of research workers on a variety of structures provide further confirmation of the validity of the analytical method.

When dealing with dynamic problems, the possibility of resonant excitation should be considered, and it is accepted that resonance may occur on lightweight floors subject to dance-type loads. This is axiomatic in the National Building Code of Canada[2] and in an international guide[3]. A simple design criterion which is used is that the fundamental frequency of a dance floor should be 2-3 times the highest expected frequency of the load. If this criterion is satisfied, it follows that no dynamic analysis is required. Also, for more detailed analysis, it has been suggested that the first three Fourier components of the load need be considered when calculating structural response. In this paper, these recommendations are examined and experimental and numerical evidence is provided to show that resonance can occur on a relatively stiff floor ($f_{11} > 10$ Hz), indicating that more Fourier components of the load may need to be considered, dependent on the dynamic characteristics of the floor.

Experimental verification of the effect of human involvement

Description of the experiment

The principal reason for undertaking the experimental work was to provide data of the effect on the fundamental characteristics of a structure, of its interaction with people – i.e. how the mass of a person affects the fundamental frequency of a structure. The experimental setup also served to check the accuracy of the load model and the calculated structural response to a typical load, as well as confirming one particular theoretical prediction[1].

A precast reinforced concrete beam was used in the experimental investigation. The beam had dimensions of 3.0 × 0.4 × 0.083 m and was simply supported 0.1 m from each end. The response of the beam was monitored using three transducers mounted under the centre of the beam, to measure acceleration, velocity and displacement. The output from these transducers was filtered before digitising and recorded on a computer. For most of the records a sampling frequency of 1000 Hz was used, and 8192 (8 k) datapoints were recorded per channel. The time histories presented in this paper are simply those recorded from the appropriate transducer, with the calibration factor included.

When the beam was ready for testing, it was subjected to a forced vibration test to obtain accurate values of the characteristics of its fundamental mode, i.e. frequency, damping, and stiffness. Further measurements of frequency were made throughout the investigation using impact tests. The procedure for both forced vibration and impact tests is described in ref. 12. Information on the frequency content of any particular signal was obtained by calculating an autospectral density function (autospectrum) from a Fourier transform of the recorded data. The actual autospectrum is composed of discrete points, with a step of 1000/8192 Hz (approx. 1/8 Hz) between each point. This gives the basic frequency resolution of the procedure.

Effect of human involvement

One of the difficulties in modelling floors with crowds of people is how to account for the mass of people, since it affects the structural characteristics and consequently the response to any given load. One frequent assumption has been that the crowd acts as a rigid mass, which implies that the added mass of the crowd will effectively reduce the fundamental frequency of the floor. Consideration of the basic physics of the problem suggests that people should really behave more like spring-mass-damper systems. To provide a better understanding of the problem a series of experiments were arranged. The following ideas are based on these relatively simple tests:

(1) *Human involvement as an additional mass-spring-damper system on the beam.* The fundamental frequency of the bare beam was measured at 18.68 Hz. When a person stood or sat on the beam, the frequency increased and the damping value increased significantly whereas, when a mass equivalent to the mass of the person was placed on the beam, the frequency decreased and the damping value remained unchanged. This suggested that the person acts as an additional mass-spring-damper system. It was also noted that the human spring stiffness varies with posture. Further complementary information has been recorded by the authors for a large crowd on a cantilevered part of a grandstand.

(2) *Human involvement as load on the beam.* For the case where people were jumping, stamping and running on the spot, their mass was not vibrating with the beam and it was found that the human involvement was as a load

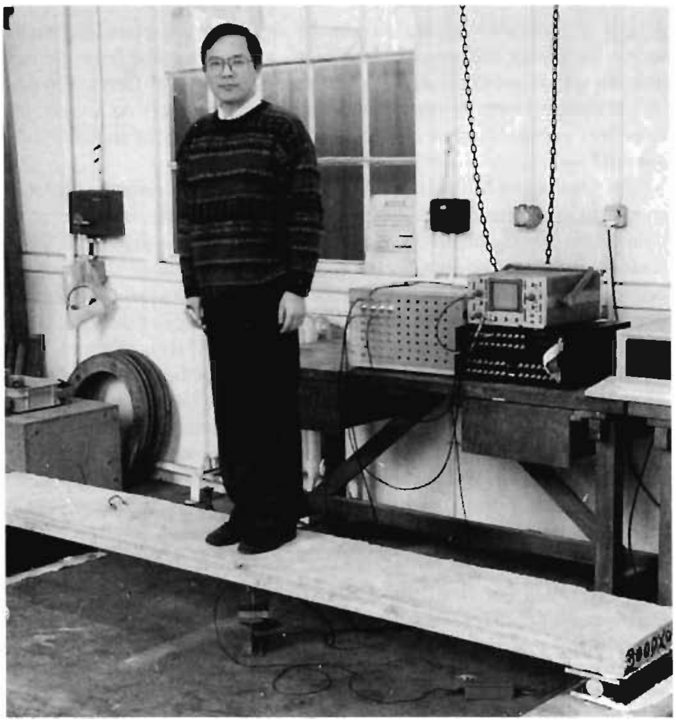

Fig 1. View of the beam used for the experiments

only, with the characteristics of the beam measured during the test being the same as the bare beam.

(3) *Human involvement as a load and an additional mass-spring-damper system on the beam.* This was observed when people were sitting and stamping on the beam.

For the theoretical study of floor response to dance loads where jumping is involved[1], the second case where the human involvement can be represented as a load provides the appropriate model.

Experimental verification of jumping load model

The analytical model for dance-type loads, involving jumping, is described by eqns. (6) and (7) in ref. 1. To demonstrate the accuracy of this load model, it is useful to compare it with some measurements obtained from the response of the beam to a person jumping. To help the 'dancer' jump at a particular frequency, a computer program was written which, for a given input frequency, provides both audio and visual timing signals. The force time history for the experiment was generated from the measured response using the equation

$$M_1\ddot{A}(t)+C_1\dot{A}(t)+K_1A(t)=F(t) \qquad ....(1)$$

where $\ddot{A}(t)$, $\dot{A}(t)$ and $A(t)$ are the recorded time histories for acceleration, velocity and displacement; the structural parameters (M_1, C_1 and K_1) were obtained from the force vibration test where the subscript indicates that those parameters correspond to the fundamental mode. Eqn. (1) gives the modal force which, for one person jumping at the centre of the beam, equals the force generated by the jumper. Since the antisymmetric modes are not involved in the vibration, the response from higher modes is negligible. Fig 1 shows one of the authors jumping on the beam. In the test, the jumper was trying to jump at a frequency of 2.0 Hz. The recorded results showed that the actual jumping frequency was 2.05 Hz and the contact ratio was 0.49. Fig 2 displays the measured load filtered to remove the response over 12 Hz and normalised so that 1.0 corresponds to the static weight of the dancer. Fig 2 also shows the calculated load using eqns. (6) and (7) in ref. 1 with the recorded frequency and contact ratio and adopting the first five terms of the Fourier series. It can be seen that the analytical model compares reasonably well with the measurements.

Verification of response induced by dance-type loads

The analytical method[1] provided a means of calculating the response of a floor to dance loads. To verify this method, it is useful to compare, for one particular case study, the results obtained using the analytical method with results obtained from a finite element analysis and from experimental measurements.

Correlation between the analytical and numerical solutions

The basic dynamic equilibrium equation for a finite element system is

$$M\ddot{U}+C\dot{U}+KU=F(t) \qquad ....(2)$$

where

M, C and K are the mass, damping and stiffness matrices

$F(t)$ is the external load vector in which the variation with time is defined by eqn. (1) in ref. 1

U, $\dot{U}$ and $\ddot{U}$ are the displacement, velocity and acceleration vectors of the finite element assemblage

To solve the above equation, the direct integration method[4] was adopted, using the finite element program, LUSAS[5].

In the numerical analysis, the damping matrix is of the form

$$C=\alpha_c M+\beta_c K \qquad ....(3)$$

where α_c and β_c are constants to be determined from two given damping

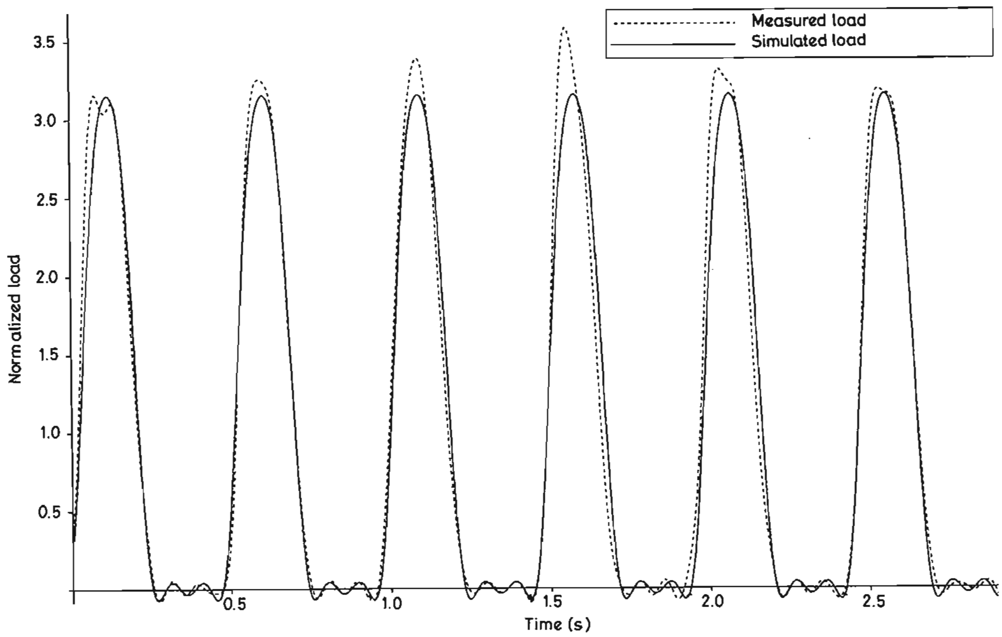

Fig 2. Measured and simulated loads for one person jumping

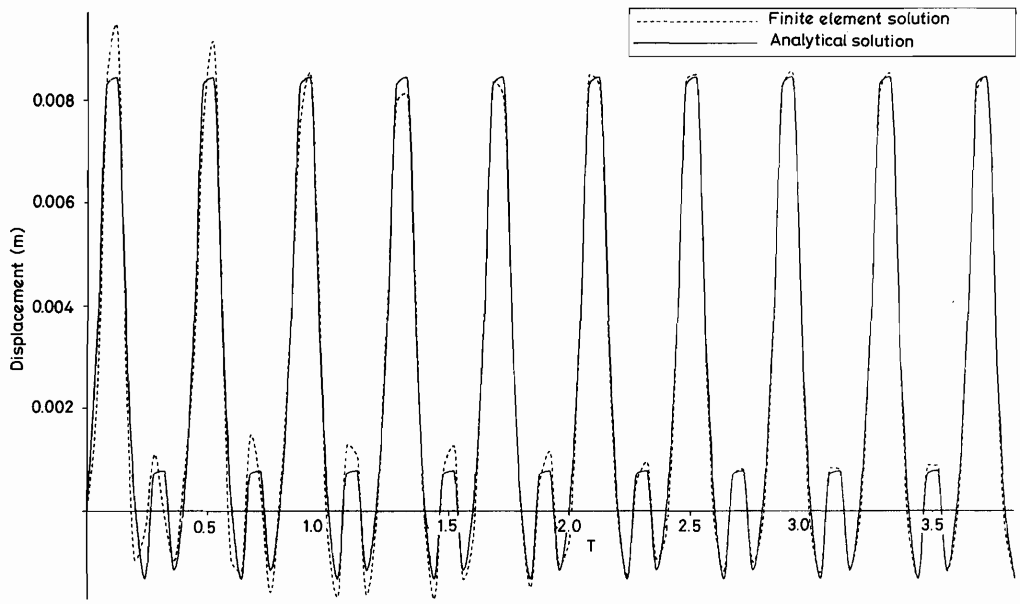

Fig 3. Comparison of numerical and analytical solutions for displacement at the centre of the floor

ratios (ζ_1 and ζ_2) that correspond to two unequal frequencies of vibration. By utilising the orthogonality of eigenvectors, these constants can be expressed as follows:

$$\left.\begin{aligned}\alpha_c &= \frac{2\omega_1\omega_2(\zeta_1\omega_2 - \zeta_2\omega_1)}{\omega_2^2 - \omega_1^2}\\ \beta_c &= \frac{2(\zeta_2\omega_2 - \zeta_1\omega_1)}{\omega_2^2 - \omega_1^2}\end{aligned}\right\} \quad ....(4)$$

To verify the analytical solution, an example was calculated using both FEM and eqn. (31) in ref. 1. The simply supported floor can be described by the following data:

dimension of the floor:	$L_x = L_y = 8.0$ m
damping ratios of first two modes:	$\zeta_1 = \zeta_2 = 0.02$
floor material:	reinforced concrete (2400 kg/m³)
floor thickness:	$h = 0.15$ m
period of excitation load:	$T_p = 0.4$ s
dancing load:	$G = 1177.2$ N/m²
contact ratio:	$\alpha = 0.5$

The analytical solution was undertaken first, and the dynamic response, together with the first two relevant frequencies (7.67 Hz and 38.34 Hz, corresponding to symmetric modes), were obtained. The constants α_c and β_c, which were determined using eqn. (4) and the above calculated frequencies, are 1.606 and 1.383×10^{-4}, respectively. For numerical analysis the load was defined by eqn. (1), and the dynamic impact factor K_p was calculated using eqn. (5) in ref. 1 and is equal to π. Thus all the required data are available for performing the numerical analysis.

Fig 3 shows the displacements at the centre of the floor calculated using numerical and analytical solutions, respectively. A slight difference is observed over the first half second, because the free vibration component is included in the numerical analysis but is excluded in the analytical solution. However, the free vibration disappears quickly in a damped system and then the two solutions match well. In the analytical solution, only the response from the fundamental mode was involved whereas, in the numerical analysis, the full response was evaluated. The comparison also indicates that the response from higher modes is negligible.

Comparison between the analytical and experimental results

The test setup described earlier was used to measure the response of the beam

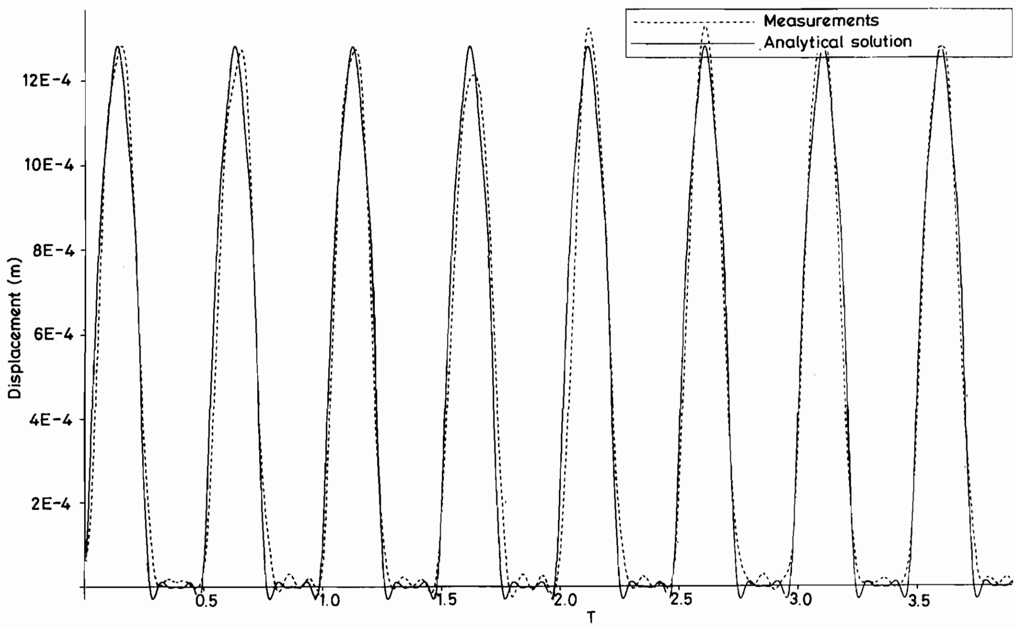

Fig 4. Comparison of experimental and analytical solutions for displacement at the centre of the beam

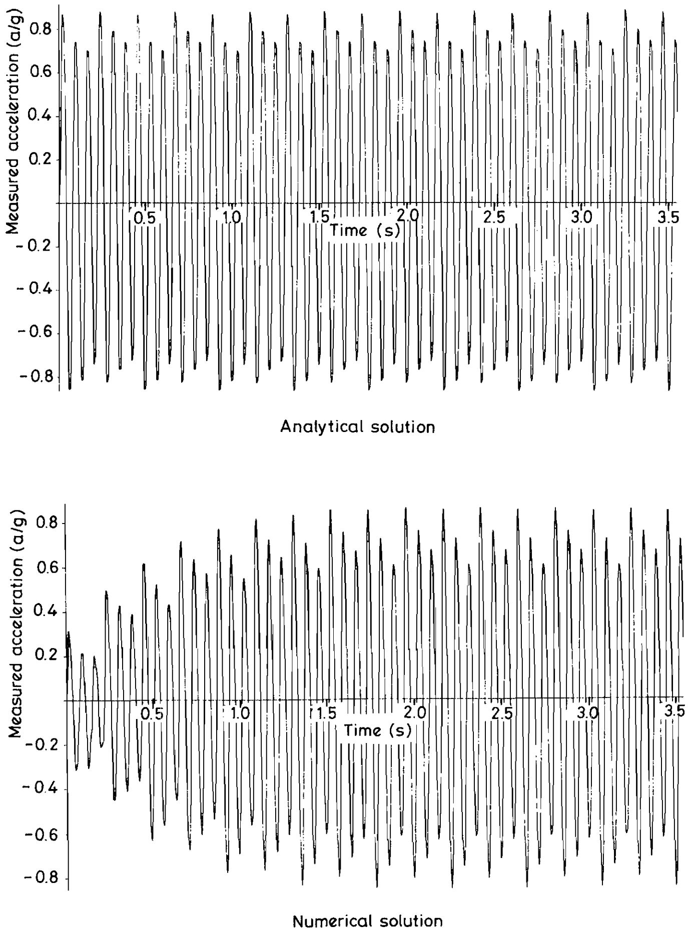

Fig 5. Comparison of resonant accelerations at the centre of the floor

to a jumping load. One of the authors jumped at the centre of the beam at approximately 2 Hz. The information obtained from the test is summarised as follows

measured damping	$\zeta_1 = 0.0035$
measured frequency of jumping load:	$f_p = 2.08$ Hz
measured modal stiffness:	1.61×10^6 N/m
dancer's mass:	$G_s = 65$ kg
measured frequency of the bare beam:	18.68 Hz
measured frequency of the loaded beam:	18.68 Hz
contact ratio:	$\alpha = 0.565$

Fig 4 gives the comparison of the displacement time history at the centre of the beam from the analytical solution and experimental measurements. The measured displacement is filtered to remove the response above 12 Hz. The measured modal stiffness, fundamental frequency, and damping, were used for calculating the displacement using eqn. (36) in ref. 1, and five Fourier terms were used in the load model.

An advantage of using the measured modal stiffness and frequency in the calculation is that structural modelling errors, which may be caused by imperfectly defined boundary conditions, or inaccurate material properties or dimensions, are avoided. This may explain why the calculated results compare so favourably with the measurements, but it does serve to demonstrate the validity of the analytical model.

Verification of a resonance caused by higher Fourier components

Vibration problems relating to either safety or serviceability of floors are usually due to resonance. Therefore, any possible resonance should be identified and avoided. A possible resonance is predicted in ref. 1, though it has not been reported in practice. It is suggested that resonance may occur on a relatively stiff dance floor ($f_{11} > 10$ Hz). The possibility is investigated by recalculating the example presented earlier and also simulating the situation experimentally.

Numerical verification

To create a stiffer floor, a clamped boundary condition was adopted instead of the simply supported one reported earlier. Thus, the fundamental frequency was 13.99 Hz in the analytical solution and 13.96 Hz in the numerical solution, instead of 7.67 Hz for the simply supported floor. Using eqn. (4) the constants α_c and β_c are 2.794 and 4.55×10^{-4}. In order to see the response produced by the sixth component of the load, the time step adopted for the integration method was one-twentieth of the fundamental period of the floor, i.e. 0.003577 s. Nearly 1000 time steps were used to calculate a 3.5 s acceleration time history. Fig 5 shows the corresponding analytical solution (top) and the numerical solution (bottom), for the acceleration at the centre of the floor. It can be seen that

(1) the pattern of the acceleration histories indicates a resonant situation (the steady state patterns are similar in both analytical and numerical solutions);
(2) in the numerical calculation the acceleration quickly builds up and reaches a steady state in 1.5 s;
(3) the peak acceleration calculated using the numerical method (0.86 g) is almost the same as that calculated using the analytical method (0.89 g).

Experimental verification

The simply supported concrete beam described earlier was again used to provide confirmation of the analytical and numerical observations given above. The intention was to measure the response of the beam when excited at resonance by the sixth Fourier coefficient of the load. As the fundamental frequency of the bare beam was too high for this experiment, it was reduced to 14.40 Hz by placing a few steel weights on the beam. The damping was evaluated from a measurement of the response of the beam to an impact. With this setup, jumping at exactly 2.40 Hz should provide the case where the sixth Fourier component of the load excites the fundamental mode of the beam. To help the person jump at a frequency of 2.4 Hz a timing signal was provided. The displacement, velocity and acceleration of the centre of the beam were monitored, with an analogue filter being used to remove the response above 50 Hz. The contact ratio was calculated from the measurements and found to be 0.60.

(1) *Frequency content of the response.* The displacement and acceleration at the centre of the beam to jumping were recorded for just over 8 s. To see the frequency content of the signals, autospectra of the time histories were derived and are shown in Fig 6. It can be seen that, for the displacement (the upper figure), the first two Fourier terms dominate the response but, for the acceleration, the sixth component is dominant. This confirms that it would not be sufficient to consider only three Fourier terms in order to

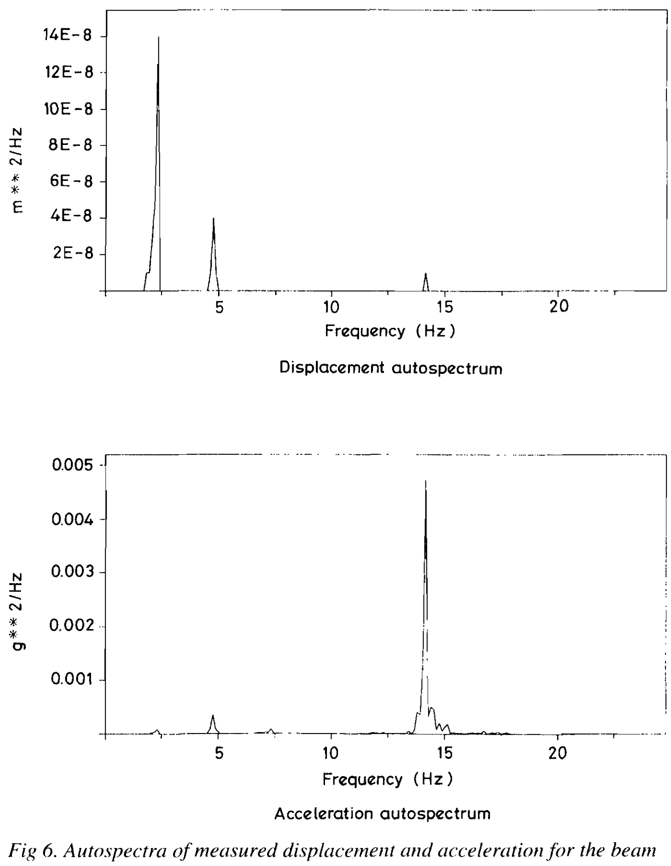

Fig 6. Autospectra of measured displacement and acceleration for the beam

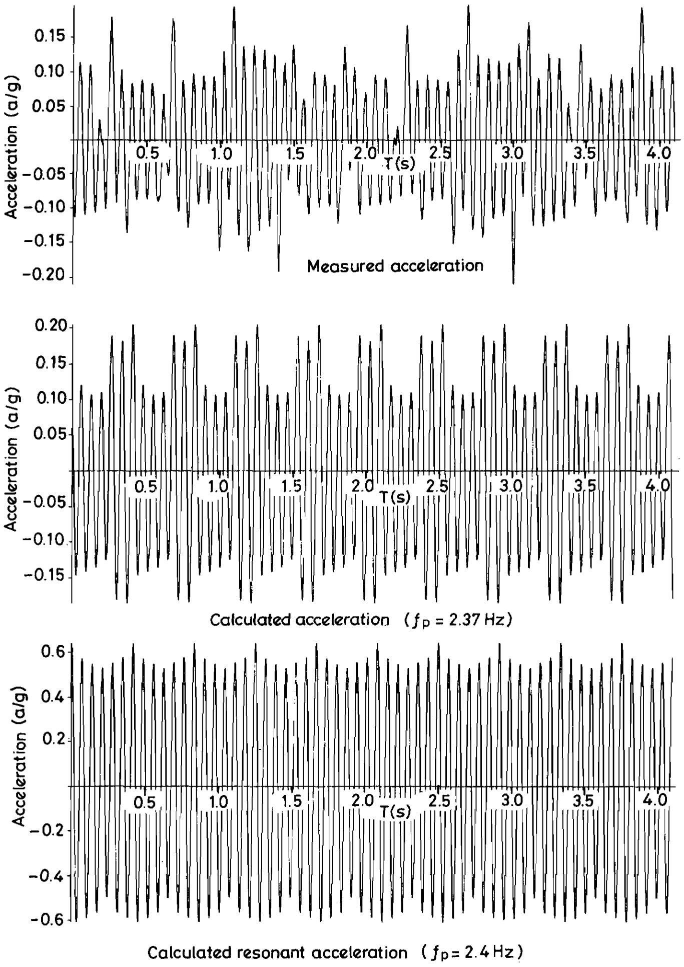

Fig 7. Comparison of measured and calculated accelerations for the beam

calculate the acceleration in this case.

(2) *Measured and calculated acceleration.* Fig 7 shows part of the measured acceleration time history (the upper figure) and the corresponding calculated accelerations. The measured maximum acceleration is about 20 % g, whereas the calculated one at resonance is about 60 % g. One of the difficulties with this experiment was in providing the precise dancing frequency, which is especially important when dealing with a system with such a low damping value (0.35 % critical). The experiment was actually repeated three times and the event producing the largest accelerations used for the analysis. If the calculation is repeated using a dancing frequency of 2.37 Hz (instead of 2.40 Hz), the calculated maximum acceleration is 20.5 % g.

Although this experiment has concentrated on resonance produced by the sixth Fourier component of the load, it is possible for various Fourier components ($n > 3$) to excite resonance. Referring to the experiment discussed earlier, it can be seen that the load frequency ($f_p = 2.08$) is one-ninth of the fundamental frequency of the beam ($f_{11} = 18.68$ Hz). The measured acceleration was filtered to remove the response over 20 Hz and is shown in Fig 8. This shows an acceleration resonance induced by the ninth Fourier component of the load.

TABLE 1 – Comparison of dynamic load factors or Fourier coefficients

Activities	Dynamic load factors 1	2	3	4
Dancing (no jumping)[7,8]	0.5			
$\alpha = 1$	0.667			
Pedestrian movements[9]	1.10	0.20	0.10	0.05
Low-impact aerobics[7]	1.20	0.22	0.06	
$\alpha = 2/3$	1.286	0.164	0.133	0.036
Rhythmic exercises[9]	1.60	0.60	0.20	0.10
High-impact aerobics[7]	1.50	0.60	0.10	
$\alpha = 1/2$	1.571	0.667	0.000	0.133
Normal jumping (2 Hz)[3]	1.80	1.30	0.7	
Normal jumping (3 Hz)[3]	1.70	1.10	0.05	
$\alpha = 1/3$	1.80	1.286	0.667	0.164
High jumping (2 Hz)[3]	1.90	1.60	1.10	
High jumping (3 Hz)[3]	1.80	1.30	0.80	
$\alpha = 1/4$	1.886	1.571	1.131	0.667

Comparison with findings of NRC in Canada

The main body of information on the response of floors to dance-type loads is attributable to a group of research workers in Canada[6-11]. It is informative to compare their findings with the results given here.

Dynamic load factors (Fourier coefficients)

An experimental study on dynamic load factors for pedestrian movements (walking and jogging) and rhythmic exercises (jumping, stride jumps and running-on-the-spot) was conducted by Pernica[9]. Allen[6] also contributed by determining the second and the third dynamic load factors based on the first one which came from testing[9]. The dynamic load factors suitable for design were also suggested for floors subject to the above-mentioned activities.

Table 1 lists the suggested load factors[6,3,9] and the Fourier coefficients

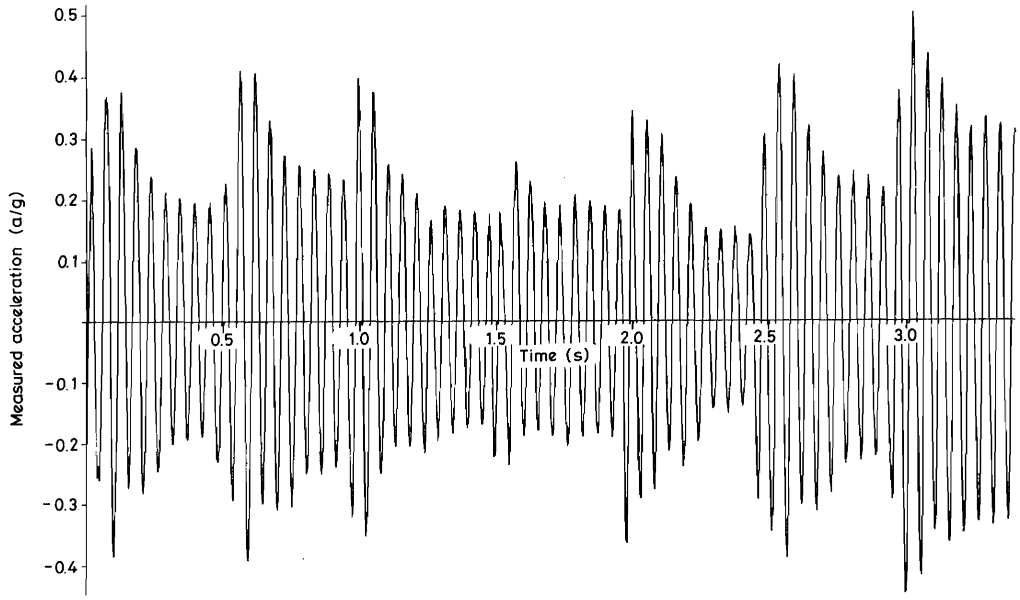

Fig 8. Measured acceleration on the beam showing resonance induced by the ninth Fourier component of the load

derived in ref. 1. It can be seen that the suggested values based on measurements and the proposed analytical values are close, except for the case when $\alpha = 1$ and $n = 1$, and this suggests a 30 % difference for non-jumping dances. The load model (eqns. (6) and (7) in ref. 1) is actually for modelling jump dancing loads where $\alpha < 1$ and where several Fourier terms are required; hence some error for $\alpha = 1$ and $n = 1$ is to be expected.

Another item to compare is how many Fourier terms should be adopted in the analysis and design of floor vibration induced by human activities. It has been suggested[7] that the first three Fourier terms should be included for jumping and aerobics since resonance occurring at three times the jumping frequency has been observed in practice. In ref. 1 it is suggested that the number of Fourier terms to be considered depends on the ratio of the structure and load frequencies. Considering only the first three Fourier terms is reasonable for calculating displacements, but may not be sufficient for calculating accelerations. This fact has been demonstrated numerically and experimentally in the last section. It is likely that safety evaluation will be concerned with displacements and that serviceability, regarding human acceptance of vibration, will relate to accelerations. The following observations can be made:

(1) Based on a comparison of dynamic load factors, the contact ratio α can be determined for different activities as listed in Table 1.
(2) The dynamic loads used in analysis, corresponding to different activities, can now be described using eqns. (6) and (7) in ref. 1, which includes the phase information.

Design criteria and acceleration

A design criterion for floor structures subject to rhythmic activities without jumping is given in refs. 7, 8 and 2:

$$f_{11} \geq f_p \sqrt{1.0 + 1.3 \frac{r_1 g m_l}{a_0 m}} \quad \text{....(5)}$$

This criterion was derived from the following formula by assuming $\zeta = 0$

$$\frac{a}{g} = 1.3 \frac{r_1 m_l / m}{\sqrt{\left[\left(\frac{f_{11}}{f_p}\right)^2 - 1\right]^2 + \left(2\zeta \frac{f_{11}}{f_p}\right)^2}} \quad \text{....(6)}$$

where

a/g is the acceleration as a fraction of the acceleration due to gravity
r_1 is the first dynamic load factor
a_0 is the limiting value of acceleration

Eqn. (6) is used to calculate acceleration for vibration due to rhythmic activities[8,2]. This equation was derived using beam theory and considering the first Fourier component of load. When just the first Fourier term is considered, a similar formula can be arrived at from eqn. (3) in ref. 1 as follows:

$$\frac{a}{g} = B_{11} \frac{r_1 m_l / m}{\sqrt{\left[\left(\frac{f_{11}}{f_p}\right)^2 - 1\right]^2 + \left(2\zeta \frac{f_{11}}{f_p}\right)^2}} \quad \text{....(7)}$$

The structural factor B_{11} can be found in Table 3 in ref. 1. The coefficient 1.3 used in eqn. (6) corresponds to the exact structural factor $4/\pi$ for simply supported beams, though eqn. (6) was derived in different manner[8]. For a floor with a given fundamental frequency, the acceleration calculated using the beam model is at least 20 % less than it would be using the plate model.

It is therefore apparent that selecting an appropriate model (beam or plate) for the floor under consideration is important. The use of the beam model (eqn. (6)) does not necessarily produce conservative or safe answers. Also, if, in eqn. (5) $r_1 g m_l / a_0 m \geq 4$, which is quite likely, there is over 9 % difference in the simple design criterion between a simply supported beam model ($B_{11} = 1.3$) and a simply supported plate model ($B_{11} = 1.62$) for a given frequency f_{11}.

It is also important to note, particularly for lightweight floors under dance-type loads, that m in eqns. (5) and (6) is the mass of the floor rather than the mass of people and the floor.

Final comments

This paper presents an experimental and numerical verification of the analytical method of calculating the response of floors to dance-type loads which is presented in ref. 1. The main points drawn from this study are as follows:

(1) It has been identified experimentally that people jumping act only as a load and not as an additional mass on the floor. This observation is adopted in ref. 1.
(2) The loads calculated using the analytical model are in close agreement with those measured experimentally.
(3) The derived Fourier coefficients correspond reasonably well with the dynamic load factors that have been determined experimentally elsewhere.
(4) The floor response calculated using the analytical method, using the response contributed from the fundamental mode, compares favourably with both experimental and numerical investigation.
(5) A potential resonance, due to higher harmonic components ($n > 3$), is numerically and experimentally verified. It is confirmed that significant accelerations could occur on a relatively stiff dance floor ($f_{11} > 10$ Hz). This may cause serviceability problems.
(6) It is not safe to apply design criterion based on simply supported beam for all types of floor. For many dance floors, the structural coefficient could be 1.62, rather than 1.3.

Acknowledgement

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