Human–structure interaction in vertical vibrations

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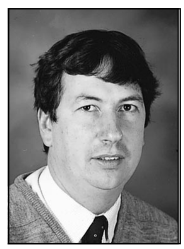

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This paper deals with the subject of human–structure interaction in vertical vibrations. It presents some results recorded on a cantilevered grandstand at Twickenham which showed that, although the empty structure had a clearly defined fundamental mode, two modes of vibration were apparent when the spectators were seated. The frequency of the empty stand was between the two frequencies of the occupied stand and a significant increase in damping was noted when the crowd was present. This suggests that the crowd should be modelled as a damped sprung mass system rather than as a simple added mass which is sometimes used in analysis. A study of an undamped system with two degrees-of-freedom is then presented which leads to three relationships which are consistent with the observations at Twickenham. The results of tests on a simple beam for a range of activities are given to provide further information on human–structure interaction and demonstrate that when a person is stationary on the beam the two interact, but when a person is moving he simply acts as a load. Results from tests on low- and high-frequency structures, when both empty and full of spectators, are given which show quite different vibrational characteristics but which are consistent with the basic two-degrees-of-freedom model. The implications of these findings are then considered.

Notation

A_h	displacement of the human modal mass (m)
A_s	displacement of the structure modal mass (m)
$\ddot{A}_h$	acceleration of the human modal mass (m/s^2)
$\ddot{A}_s$	acceleration of the structure modal mass (m/s^2)
K_h	modal stiffness of a human whole body (N/m)
K_s	modal stiffness of a structure (N/m)
M_h	modal mass of a human whole body (kg)
M_s	modal mass of a structure (kg)
α	the modal mass ratio of the human whole body to the structure
β	the frequency ratio of the human whole body to the structure
ω_h	fundamental circular frequency of a human whole body (radians/s)
ω_s	fundamental circular frequency of a test structure (radians/s)
ω_1,ω_2	frequencies of the combined human–structure system (radians/s)

Introduction

The response of structures to dynamic loads induced by individuals or crowds is becoming increasingly important. Concern may range from serviceability aspects (e.g. people walking across a structure and producing noticeable vibrations) to safety considerations (e.g. crowds of people jumping on floors or grandstands).[1] Besides generating loads people will interact with a structure, and this may be significant if the mass of the people is reasonably large in comparison with the mass of the structure. In this case the interaction between people and structure may effectively change the system characteristics, and this is important if the response of the system is to be evaluated. It has occasionally been assumed that the effect of the people is simply that of an added mass on the system, however, both site and laboratory tests demonstrate that human bodies do not act solely as mass on the structure and show that the problem is somewhat more complex. The purpose of this paper is to clarify this situation, to interpret what is happening at a range of events and to show how the interactions can be modelled.

2. Currently the subject of human–structure interactions is poorly covered in literature and no general descriptions of the problem have been published or are known to the authors. As this is important for structures such as grandstands which carry large crowds, it needs to be fully explained. This paper provides an explanation for the phenomena for vertical vibrations, using both laboratory and full-scale tests, together with the mathematical background which is based on a model with two degrees-of-freedom. Two examples are given where modelling human–structure interaction will be significant, and its importance is discussed.

Observations from a cantilevered grandstand at Twickenham

3. The Building Research Establishment (BRE) was asked by Mott MacDonald Ltd to

measure the structural characteristics of part of the North Stand at the Rugby Football Union ground at Twickenham, before the new East Stand was constructed. Part of MacDonald's interests was to understand how the cantilevered tiers of the grandstand would respond to dynamic loads induced by spectators, and, as this particular aspect was directly linked to work being undertaken by BRE, arrangements were made to extend the original study to include measurements of the stand's response to spectator loads. The measurements were taken for the 1991 Varsity match which attracted a capacity crowd.

4. A photograph of the stand in 1991 is shown in Fig. 1. The middle tier consists of 19 trusses (numbered 3 to 21), the majority of which are 7 m apart, with their front section cantilevered over the hospitality boxes. Truss 3 is at the west end of the stand, truss 21 at the east end. Initially each truss in the middle tier was subjected to an impact test which essentially monitored how the structure responded to a local impact produced by a single jump. The tests did excite several modes, and these were identified as being related to each truss (i.e. local modes). This is not surprising as the horizontal precast beams which carry the seating are not positively connected to adjacent beams. If the horizontal beams had been rigidly connected to all adjacent beams then a monolithic behaviour of the middle tier would have been expected.

5. Three trusses, namely 5, 9 and 11, were selected for the measurements. They were chosen on the basis that their fundamental modes of vibration, evaluated from the impact tests, were clearly defined. Geophones, mounted in metal boxes, were glued under the front concrete beam immediately adjacent to the truss. The geophones were oriented to monitor velocity in the vertical direction. The geophone output was passed through a 20 Hz low-pass filter before being recorded. The signals were recorded on a digital tape recorder and later transferred to a personal computer. The data were then processed to reduce the effective sampling rate to 40 Hz and split into contiguous files each containing 8192 (8K) data points per channel, hence 204·8 s of recording per file. The recordings were for $2\frac{1}{4}$ hours which included the match and yielded 40 files for analysis.

6. The objective of the analysis was to show the frequency content of the record, and this was accomplished by transforming the data using a fast Fourier transform (FFT) procedure and then calculating the autospectrum (or PSD) for each file. Individual spectra appeared to be very ragged due to the large number of spectral lines in the FFT, so it was decided to average eight spectra and apply a smoothing algorithm to the resulting spectrum. As there were 40 consecutive spectra, nine averaged spectra were obtained for each truss by averaging the first eight spectra, then the spectra from files 5–12 and so on.

7. Results for truss 9 are given in Fig. 2. Fig. 2(a) shows a clearly defined fundamental mode of vibration for the empty structure.

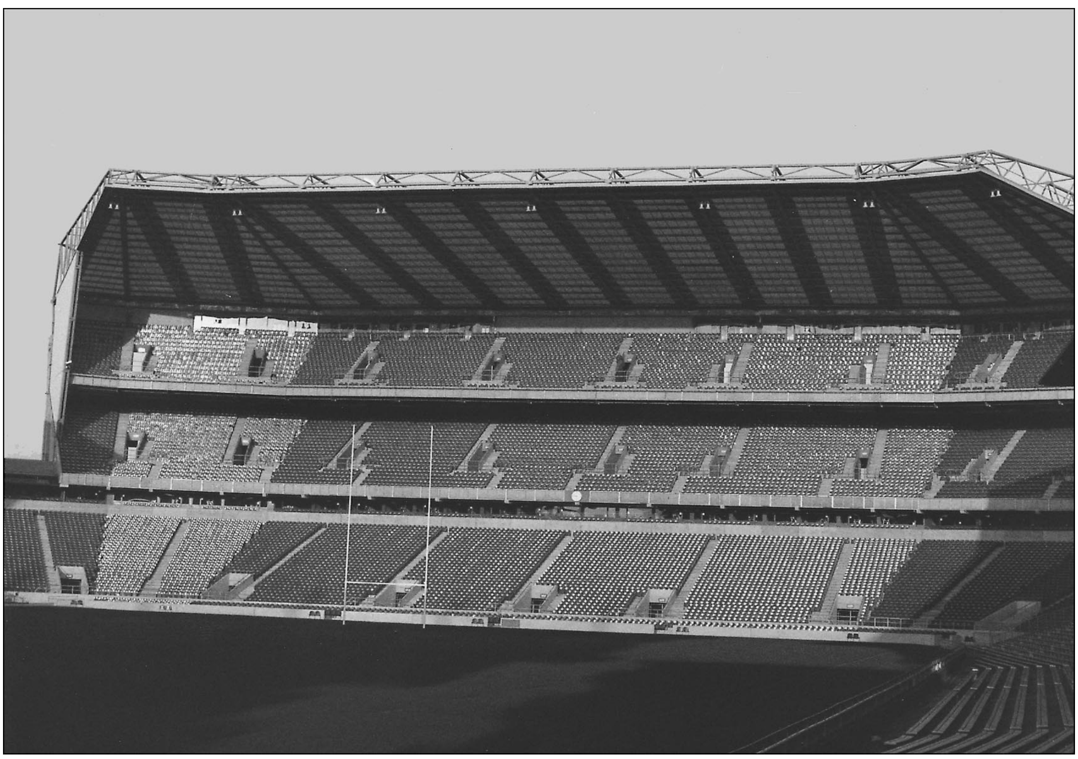

Fig. 1. The North Stand, Twickenham, 1991

Instead of the expected gradual reduction in frequency of the mode as the crowd assembled, the presence of the spectators appeared to result in the single mode changing into a two-mode system (Fig. 2(b)), which means that rather than the spectators being effectively an added mass, they acted as a damped sprung mass. Fig. 2 shows that the characteristics changed significantly when the crowd was involved and that the structure and crowd interact, thereby suggesting that the two should be treated as an integrated system with two degrees of freedom (d.o.f.). Examination of all of the spectra showed that this two-d.o.f. system was gradually changing as the stand filled with people and also during the half-time interval when a large number of people went to seek the nearest convenience. This pattern was also noted for trusses 5 and 11. It can be seen that the single fundamental mode is replaced by two modes, the frequencies of which straddle the original frequency, and the damping increases significantly.

8. Best-fit viscoelastic curves were obtained for the spectra, using one-d.o.f. and two-d.o.f. models respectively. The frequencies which were obtained from the best-fit curves for the empty and full (seated) conditions are given in Table 1.

9. From these observations three significant phenomena are apparent. First, an additional frequency is observed in the occupied stand. Second, the frequency of the empty stand is between the two frequencies of the occupied stand. Third, the damping increases significantly when people are involved.

Some relationships for an undamped system with two degrees-of-freedom

10. The measurements at Twickenham indicated that the system was behaving similarly to a two-d.o.f. system when people were involved but as a one-d.o.f. system when the stand was empty. This suggests that, in this situation, the crowd acts as a one-d.o.f. system. Hence it is useful to examine the mechanics of such a system to see whether it can be used to describe the human–structure system. In order to identify the modes of vibration a two-d.o.f. undamped system is considered, which can be thought to represent the fundamental modes of vibration of both the structure and the crowd.

11. The frequencies of the independent human body system ω_h (the crowd) and the structure system ω_s are respectively

$$\omega_h^2 = \frac{K_h}{M_h} \tag{1}$$

$$\omega_s^2 = \frac{K_s}{M_s} \tag{2}$$

where K_h, M_h, K_s and M_s represent the modal stiffness and modal mass of the human body system and structure. For the combined human–structure system the human body system is attached to the structural system. The frequencies of the combined system can be obtained by solving the corresponding eigenvalue problem of the following motion equations:

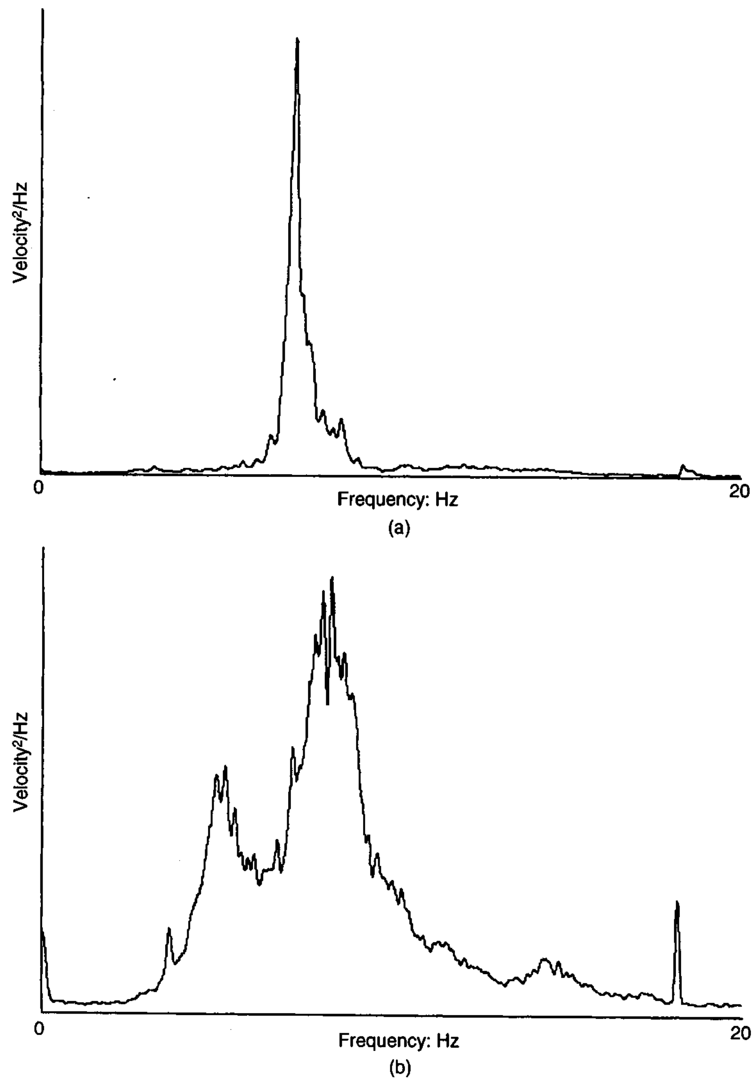

Fig. 2. Autospectra for vertical vibrations of truss 9 at Twickenham: (a) with no crowd, for files 1–8; (b) with full crowd, for files 17–24

Table 1. Frequencies observed at Twickenham

Truss	Frequency when empty: Hz	Frequencies when full: Hz	
5	8·55	5·44	8·72
9	7·32	5·41	7·91
11	7·24	5·13	7·89

$$\begin{bmatrix} M_s & \\ & M_h \end{bmatrix} \begin{bmatrix} \ddot{A}_s \\ \ddot{A}_h \end{bmatrix} + \begin{bmatrix} K_s + K_h & -K_h \\ -K_h & K_h \end{bmatrix} \begin{bmatrix} A_s \\ A_h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (3)$$

For convenience the following terms are defined:

$$\alpha = M_h/M_s \qquad \beta = \omega_h/\omega_s$$

where α and β are the mass ratio and the frequency ratio of the simplified human body system and simplified structure system respectively, and are positive values. The frequencies ω_1 and ω_2 of the two-d.o.f. system can be represented using ω_h and ω_s:

$$\omega_1^2 = \frac{1}{2}\left[\omega_s^2 + \alpha\omega_h^2 + \omega_h^2 - \sqrt{(\omega_s^2 + \alpha\omega_h^2 + \omega_h^2)^2 - 4\omega_s^2\omega_h^2}\right] \quad (4)$$

$$\omega_2^2 = \frac{1}{2}\left[\omega_s^2 + \alpha\omega_h^2 + \omega_h^2 + \sqrt{(\omega_s^2 + \alpha\omega_h^2 + \omega_h^2)^2 - 4\omega_s^2\omega_h^2}\right] \quad (5)$$

Thus three frequency relationships between the combined human–structure system (ω_1 and ω_2) and the independent human and structure systems (ω_s and ω_h) can be found. First,

$$\omega_1^2 + \omega_2^2 = \omega_s^2 + (1 + \alpha)\omega_h^2 > \omega_s^2 + \omega_h^2 \quad (6)$$

This formula indicates that *the sum of square of the frequencies of the combined human–structure system is larger than that of the corresponding human and structure systems*; it is obtained by adding equations (4) and (5). Second,

$$\omega_1\omega_2 = \omega_s\omega_h \quad (7)$$

This formula indicates that *the product of frequencies of the human–structure system equals that of the corresponding human and structure systems*; it is derived by multiplying equations (4) and (5). Finally,

$$\omega_1 < (\omega_s, \omega_h) < \omega_2 \quad (8)$$

This formula indicates that *the frequencies of the human and structure systems are always between those of the human–structure system*; it can be proved as follows.

$$\omega_2/\omega_s = \sqrt{\frac{1 + \alpha\beta^2 + \beta^2 + \sqrt{(1 + \alpha\beta^2 + \beta^2)^2 - 4\beta^2}}{2}}$$

$$> \sqrt{\frac{1 + \alpha\beta^2 + \beta^2 + \sqrt{(1 + \beta^2)^2 - 4\beta^2}}{2}}$$

$$= \sqrt{\frac{1 + \alpha\beta^2 + \beta^2 + (1 - \beta^2)}{2}}$$

$$= \sqrt{1 + (\alpha\beta^2/2)} > 1$$

Similarly $\omega_2/\omega_h > 1$ can be proved. Equation (8) is then obtained from equation (7). These three relationships are valid for any system that can be represented as a two-d.o.f. system.

12. Examining the measured data in Table 1 and the three frequency relationships, the following observations can be made:

(*a*) Inequality (8) holds for the Twickenham data, i.e. the frequency of the bare stand ω_s is between the two frequencies of the human occupied stand ω_1 and ω_2.

(*b*) If the crowd acts as a sprung mass the vertical frequency of the crowd on the stand would be between 5·4 Hz and 7·9 Hz according to equation (8).

(*c*) The unknown crowd frequency can be calculated using equation (7), and this gives a range of 5·5–5·8 Hz, which is within the range of human body frequencies suggested for sitting positions.[2]

(*d*) The mass ratio can then be calculated using relationship (6), and this gives values which correspond to the observed distribution of the crowd on the structure.

Laboratory tests

13. Following the observations from Twickenham and the basic theoretical explanation, further tests were required for three purposes: to verify the phenomena observed at Twickenham; to identify the range of conditions over which the phenomena exist; and to check the validity of the mathematical relationships based on a two-d.o.f. system. In order to obtain such information a number of simple laboratory experiments were devised. The essential requirements for the test structure to be used in the experiments were: simple and well-defined structural characteristics; simple means of supporting people in various positions; and similar order of structural mass to mass of supported people. The selected test structure was a precast reinforced concrete beam, with dimensions 3·0 × 0·4 × 0·083 m, and it was simply supported 0·1 m from either end.

14. Measurements of frequency were made using impact tests for experiments where people were stationary and recording time histories when people were moving. The impact test involved hitting the beam with a single blow from a soft-headed hammer to cause the system to vibrate. The response of the beam was monitored using accelerometers mounted under the centre of the beam. The output from these accelerometers was filtered before digitizing and recorded on a computer. For each test a sampling frequency of 1000 Hz was used

and 8192 data points were recorded per channel. Information on the frequency content of any particular signal was obtained by calculating an autospectral density function (autospectrum) from a Fourier transform of the recorded data. The actual autospectrum is composed of discrete points, with a step of 1000/8192 Hz (approx. 1/8 Hz) between each point. This gives the basic frequency resolution of the procedure.

15. A large number of tests were conducted using this beam, but for this paper only a small number need be given, and for illustration purposes measurements made using one of the authors will be used. The results of frequency measurements made on the beam are given in Table 2. Spectra obtained for the bare beam and the beam with the standing person are shown in Fig. 3. The table contains a large amount of information, the most significant items being the following:

(*a*) The fundamental frequency of the beam is 18·68 Hz, which is much higher than the frequencies examined at Twickenham.
(*b*) The dead weights which were placed centrally on the beam for two tests served to reduce the frequency as expected.
(*c*) The experiment with the person standing showed an increase in the measured frequency, but, unlike Twickenham, only one frequency was noted. This is consistent with the mathematics in the previous section, and suggests that the body frequency is 9·96 Hz,[3] and that the 'other' mode of the system is primarily the person vibrating, which would be at 9·29 Hz; this however, cannot be observed directly from measurements on the beam.
(*d*) The measured frequency for the vibrations where the person sat on the stool on the beam was also higher than that of the bare beam, and this is consistent with relationship (8).
(*e*) Significant damping contributions from the human body were observed for both sitting and standing positions, as can be appreciated from Fig. 3.
(f) Jumping and walking provided interesting results in that they did not affect the measured system characteristics, either frequency or (as far as can be resolved) damping, although the forced response could be seen clearly.[4] The unchanged system characteristics would appear to be because the human body is not vibrating with the beam and hence for these situations it acts solely as a load.

Further full-scale experiments

16. From the previous sections it can be seen that the simple two-d.o.f. model can be useful for explaining the observed behaviour. However, further full-scale measurements are important. At Twickenham it appears that the structural frequencies were of a similar order to the crowd frequency, which was somewhat fortuitous. However, there are two other situations which may be more common, and these are where the structural frequency is either significantly lower or higher than the crowd frequency. If the basic two-d.o.f. model is valid, these two situations will exhibit different characteristics. For the low-frequency structure the

Table 2. Frequencies observed on the beam

Description of experiments	Measured frequency: Hz
Bare beam (Fig. 3(a))	18·68
Beam plus dead mass of 100 lb	15·75
Beam plus dead mass of 200 lb	13·92
Beam with T. Ji standing (Fig. 3(b))	20·02
Beam with T. Ji sitting on a high stool	19·04
Beam with T. Ji jumping on spot	18·68
Beam with T. Ji walking on spot	18·68

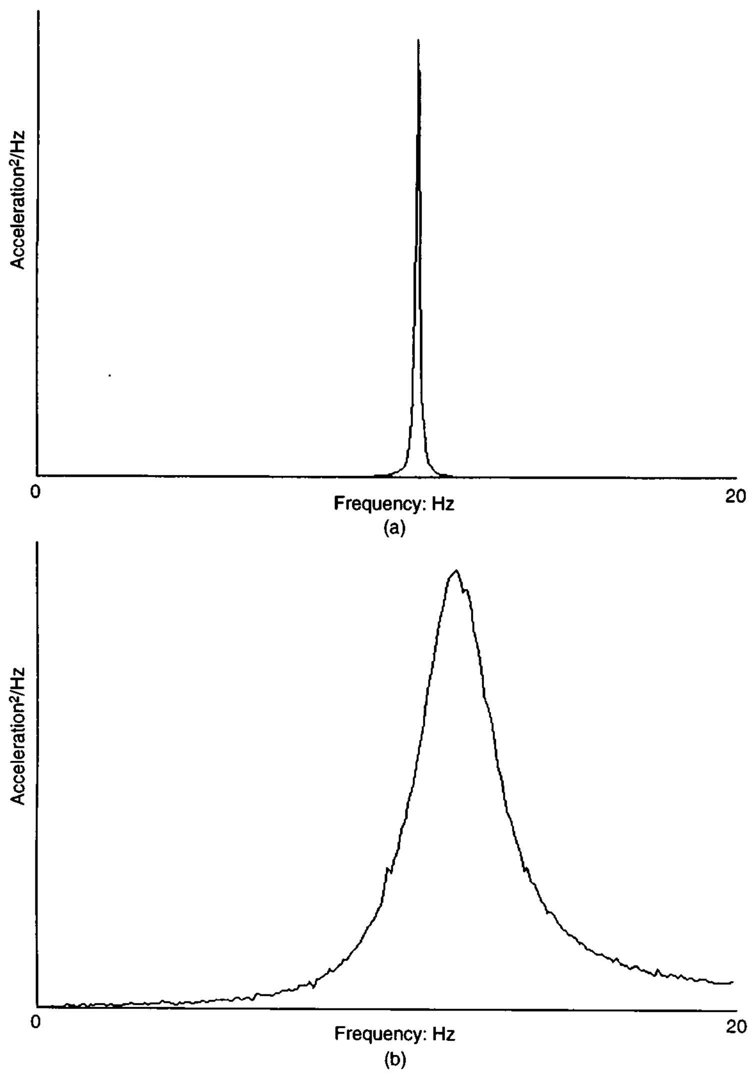

Fig. 3. Autospectrum for vertical vibrations of: (a) the bare beam, frequency 18·68 Hz; (b) the beam supporting a standing person, frequency 20·02 Hz

fundamental frequency will gradually reduce as the crowd occupies the structure and a higher-frequency mode will appear. For the high-frequency structure the original mode will increase in frequency as the crowd increases with a new lower-frequency mode appearing. In both cases the 'new' modes will be mainly motion of the people and will not necessarily be seen from the structural response.

17. Consequently, in this section results from two further structures are given. The actual locations of the structures are not important and they have been selected to show low- and high-frequency systems. The monitoring of both structures was similar to that at Twickenham, with the vertical response being recorded on a digital tape recorder. In each case the monitoring started well before the event, continuing through the event when the structure was full of spectators, and was finally terminated when the event finished and the structure was again empty. The continuous records were then split into a number of contiguous files which were analysed to identify the predominant frequencies of the monitored response. In both cases the changes in frequencies between empty and full conditions will be examined and compared with both the 'added mass' and 'sprung mass' models of the crowd.

A cantilevered grandstand: a low-frequency structure

18. The vertical vibrations of a large cantilevered grandstand were monitored, but unlike Twickenham the stand behaved as one monolithic structure. The analysis of the response of the stand yielded the frequencies of the first three modes of vibration; these were examined for both empty and full conditions and are given in Table 3.

19. Concentrating on the fundamental mode, the changes which were observed as the crowd assembled were a gradual decrease in the measured frequency, and this aligns with both the added mass and sprung mass models of a crowd. Given the frequency measured for the empty structure, the measured mode shape and the number of people on the structure, it is possible to calculate what the frequency would be for a full structure.

20. Assuming an added mass model for the crowd the frequency of the occupied stand would be 2·45 Hz. If it assumed that the crowd acts as a sprung mass system with a fundamental frequency of 5·5 Hz (similar to that derived at Twickenham) then the calculated frequency of the occupied stand would be 2·41 Hz. Thus it can be seen that the calculations using either the added mass model or the sprung mass system are equally close to

Table 3. Frequencies observed on a cantilevered grandstand

	Fundamental mode: Hz	Second mode: Hz	Third mode: Hz
Empty	2·67	2·98	3·40
Full	2·43	2·62	2·87

the measured value of 2·43 Hz. If the above observations had been made before the studies presented above, it would have been easy to conclude that the added mass model was correct.

A temporary grandstand: a high-frequency structure

21. A further set of results was obtained for the vertical vibrations of a temporary grandstand which had a higher fundamental frequency than the two cantilevered structures previously reported. Fig. 4 presents spectra obtained for both empty and full situations. The spectra have been obtained by averaging

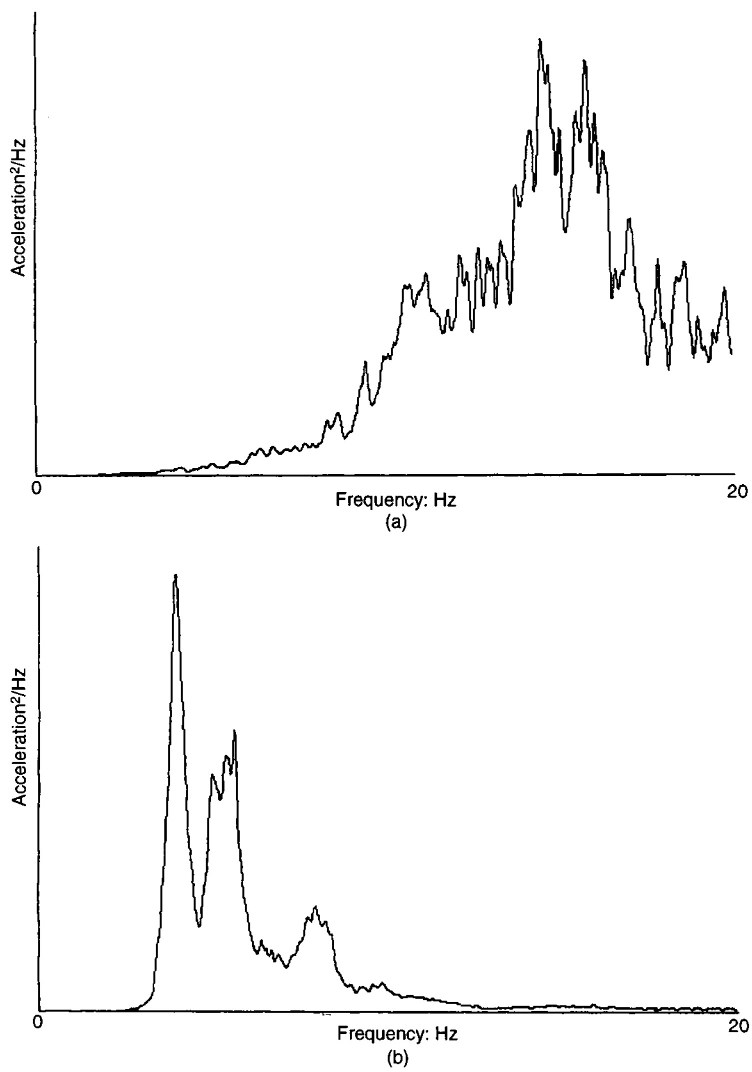

Fig. 4. Autospectra for vertical vibrations of a temporary grandstand: (a) with no crowd, for files 1–20; (b) with full crowd, for files 90–109

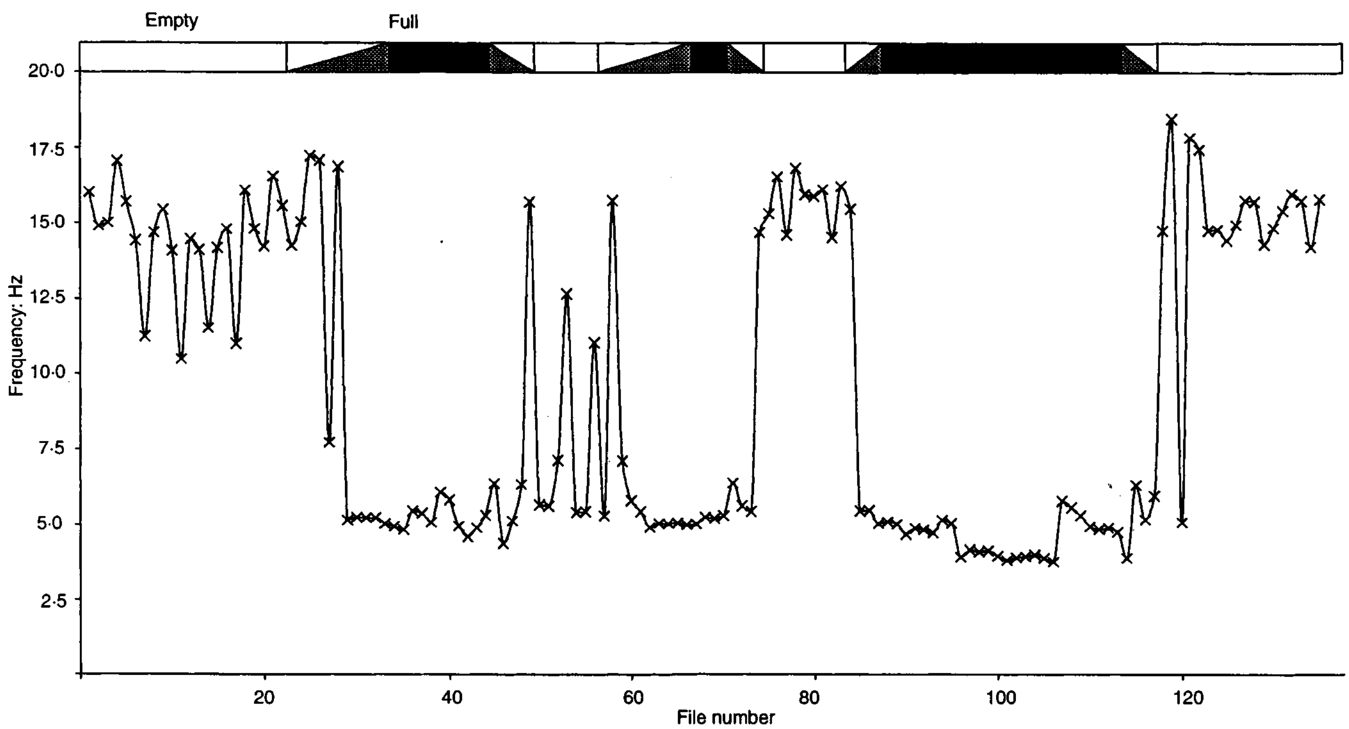

Fig. 5. Changes in frequency of the maximum vertical spectral response of a temporary grandstand during a tennis match

spectra obtained for 20 contiguous files, each of 204·8 s duration, with a smoothing algorithm being applied to the resulting spectrum. It can be seen that there is a significant difference between the empty and the full conditions. Furthermore, the frequency did not gradually reduce as the crowd occupied the structure, as for the previous example, rather a new lower-frequency mode appeared. This can be seen by examining the dominant frequency for the spectra from each file throughout the event as the stand filled and emptied several times. The dominant frequency was extracted from each of the 136 files, and these values, which are shown by crosses, are plotted against the file number (hence time) in Fig. 5. As can be appreciated from Fig. 4, extracting one dominant frequency value from a relatively ragged spectrum can result in a range of frequencies from spectra obtained for the same conditions, but Fig. 5 does serve to show a step change in characteristics between the empty and full conditions rather than a gradual change.

22. If the crowd could be modelled as an added mass, then a gradual reduction in the fundamental frequency would occur, which does not align with the observations. Also a calculation adopting this model gives a structural mass less than one-ninth of the human mass which is obviously far too small. Hence this model is not valid.

23. If the sprung mass model of the crowd is considered, with a frequency of 5·5 Hz (similar to that at Twickenham) then the situation which occurred can be explained as follows. First the empty structure exhibits quite a high frequency, say 16 Hz (see Fig. 5). When the first person takes his seat a two-d.o.f. system is obtained, with frequencies of 16 and 5·5 Hz. However, as there is only one person in a large stand the 5·5 Hz mode is insignificant in comparison with the 16 Hz mode. As the crowd increases, the 16 Hz frequency should increase slightly (like the observations made in the laboratory using the beam) and the 5·5 Hz mode should decrease in frequency. As the crowd increases the lower-frequency mode becomes more significant, until at one stage it dominates the response, i.e. the predominant response switches from the high frequency to the lower frequency, with no transitional zone. The sprung mass model for the crowd therefore serves to explain the observed phenomena at least for the fundamental modes.

Implications

24. The observations made in the previous sections show that a crowd can be represented as a sprung mass system in some situations, which is contrary to the model which considers only the added mass of the crowd. This has important implications for three situations: interpreting measurements of structural response involving crowds; evaluating structural response where people are involved; and evaluating human response to vibration. The first situation relates to experimental observation, for example monitoring grandstand vibrations, when it is important to understand any

interaction to help interpret the changes in frequency and the large range of structural responses which may be encountered. The other two situations relate to safety and serviceability problems respectively, and these will be discussed next. Although the basic interaction effects have been described, further information is required in order to calculate both human and structural response, and this is outlined at the end of this section.

Crowds and safety

25. When structures are built to accommodate crowds of people, such as grandstands and dance floors, there are two different conditions to consider in design and analysis. First, a stationary crowd will interact with a structure, alter the structural frequencies and increase the damping significantly. In this situation, the crowd acts as a mass–spring–damper system and the human–structure model should be adopted. Second, a moving crowd will act as a load only, hence any calculations can be made using the 'empty' structural characteristics. Crowds of people can generate significant loads, and the dynamic load may be several times the static load. In particular, if the crowd jumps in a coordinated manner, usually in response to a musical beat, and the jumping frequency coincides with a structural frequency (or a half or a third thereof), resonance will occur, and this is the main safety concern. In this situation the structural response can be calculated using a method proposed by the authors.[4]

26. A single example will suffice to illustrate these points. On the cantilevered stand mentioned in the previous section, one person jumping on the empty stand at its fundamental frequency produced a response of 0·0145 g whereas the peak response during a game with a crowd of several thousand on the stand was 0·094 g. Thus the load produced by one person jumping induced roughly 15% of the maximum response of a full crowd during a football match. The crowd was mainly seated, hence effectively absorbing vibrational energy. When the crowd moved it was not coordinated, even when the home team nearly scored and the crowd stood and clapped; but not all the crowd stood at exactly the same time and many remained seated. However, if all of those present had started to jump in a coordinated manner at the resonance frequency of the stand a totally different situation would have arisen. But this latter situation is highly unlikely to be encountered at a football match, although it would have to be considered if the structure was used for events like pop concerts.

Individuals and serviceability

27. The serviceability problem of people feeling vibrations is becoming increasingly important, especially for lightweight structures. For example, people sitting in an office may be disturbed by people walking in the room. When this problem is being investigated, the practice is generally to place a transducer on the structure to monitor the vibration levels, which can then be compared with prescribed guidance levels, e.g. BS 6472.[5] This is actually a different situation from the real situation where a person feels the vibration. For a person to feel the vibration he must be stationary on that structure. This is why someone walking over a road bridge will not feel any vibrations until he stands still. If a person is stationary on a structure then he will interact with it, absorbing some of the vibrational energy, and change the system. Also the person will sense his body vibrations, which will not be the same as the structural vibrations. Hence the suggested human–structure model provides a means to evaluate human response to structural vibrations.

28. A somewhat similar problem is that of testing people to establish vibration limits such as those given in BS 6472. It is well known that such tests produce quite a wide spread of results, and if measurements are actually made on the test structure, which is likely, they may not describe the human body vibrations well. Indeed, two different structures may interact quite differently with people, so that the same response measured on the two structures might relate to widely varying human responses, and this should really be considered when interpreting the data.

Further information required

29. The basis of the human–structure interaction model has been observed on site, examined theoretically and verified in the laboratory. However, the basic parameters for the human body model are unknown, although Griffin[2] gives some useful information. The following items are required for a comprehensive understanding of the subject:

(*a*) the nature of the human body model for lateral vibrations (sway)
(*b*) the measurements of the fundamental frequency for the human body for a range of conditions[3]
(*c*) the measurement of damping of the human body
(*d*) evaluation of the modal mass of the human body[6]
(*e*) calculation of human–structure interaction.

Conclusions

30. In this paper a new subject of human–structure interaction has been examined. The fundamental vertical vibrations of a human–structure system can be described by a two-d.o.f. system in which the human body, or crowd, is modelled as a one-d.o.f. system and the fundamental mode of a structure is represented as another one-d.o.f. system.

31. The main points are presented in this paper as follows:

32. Observations made on a cantilevered grandstand at Twickenham, which had a clearly defined fundamental mode when empty, showed that:

- an additional frequency was observed when the crowd occupied the stand
- the frequency of the empty stand was between the two frequencies of the occupied stand
- the damping increased significantly when people were involved.

These observations suggest that the crowd acts as a mass–spring–damper system on the structure and the crowd and the structure form a two-d.o.f. system.

33. Examination of a two-d.o.f. undamped model provides three frequency relationships between the combined human–structure system and the independent human and structure systems:

- the sum of the square of the frequencies of the combined human–structure system is larger than that of the corresponding human and structure systems
- the product of frequencies of the human–structure system equals that of the corresponding human and structure systems
- and the frequencies of the human and structure systems are always between those of the human–structure system.

These relationships are consistent with the observations made at Twickenham and for two other structures which had lower and higher frequencies than the crowd frequency.

34. Experiments using a simple beam and a variety of human actions produced by one person provide further data to support the model:

- when the person was stationary on the beam, he acted as a spring–mass–damper on the structure
- when the person was moving on the beam, he acted solely as a load and the structural characteristics were those of the empty beam.

35. The human–structure interaction which has been examined has important implications for three situations

- interpreting measurements of structural response involving crowds
- evaluating structural response where people are involved
- evaluating human response to vibration

Therefore it is important that an appropriate model of human–structure interaction is adopted in these situations.

Acknowledgements

36. The measured data on the grandstands were obtained by various members of the Structural Performance Division at BRE. The authors would particularly like to acknowledge the contribution of Mr M. Beak who was involved with most of the measurements, and Dr J. Littler who was responsible for the measurements on the temporary grandstand.

37. The grandstand at Twickenham was designed by Mott MacDonald Ltd for the Rugby Football Union, and the authors would like to acknowledge the help given by Mr D. A. Webster of Mott MacDonald in arranging the tests at Twickenham and also for support in publishing the results.

References

1. Ellis B. R., Ji T. and Littler J. D. The response of grandstands to dynamic loads induced by crowds. *Proceedings of the Australasian Structural Engineering Conference, Sydney*, 1994, 457–462.
2. Griffin J. *Handbook of human vibration*. Academic Press, 1990.
3. Ji T. and Ellis B. R. A method for the indirect measurement of the human whole-body frequency. Submitted to the Journal of Sound and Vibration.
4. Ji T. and Ellis B. R. Floor vibration induced by dance type loads – theory. *Struct. Engr*, 1994, **72**, No. 3, 37–44.
5. British Standards Institution. *Guide to the evaluation of human exposure to vibration in buildings (1 Hz to 80 Hz)*. BSI, London, 1992, BS 6472.
6. Ji T. A continuous model for the vertical vibration of the human body in a standing position. *Proc. of the UK Informal Group Meeting on Human Response to Vibration, Sept. 1995, Silsoe, UK.*