Tame or wild Toeplitz shifts

The Ellis semigroup $E(X,T)$ of a topological dynamical system is defined to be the compactification of the action T in the topology of pointwise convergence on the space of all functions X^X. *Tameness* is a concept whose roots date back to Rosenthal's ℓ^1 embedding theorem, which says that if a sequence in ℓ^1 does not have a weakly Cauchy subsequence, then it must be the sequence of unit vectors in ℓ^1. Köhler linked the concept of tameness to the Ellis semigroup. A system is *tame* if its Ellis semigroup has size at most the continuum. Non-tame systems are very far from tame, as they must contain a copy of $\beta\mathbb{N}$, the Stone-Čech compactification of $\mathbb{N}$.

Starting with Köhler's work, the dynamics community has investigated the question of which systems are tame. In this talk I will give a brief exposition of these results, and talk about my recent work with Gabriel Fuhrmann and Johannes Kellendonk, where we study tameness, or otherwise, of Toeplitz shifts.