§4 Manifolds and surfaces

Problem 1. Consider the unit circle with center at the origin in $\mathbb{R}^2$. Introduce two charts covering the circle using angles as coordinates (for one chart you may take the circle without the point $(1,0)$ and count angles from the positive direction of the x-axis; for another chart you may take the circle without the point $(-1,0)$ and count angles from the negative direction of the x-axis). Find the transition functions between these two charts.

Problem 2. Introduce two charts on S^1 using stereographic projections and calculate the transition functions for them.

Problem 3. Find the transition functions for S^n for the charts obtained from stereographic projections with centers at the north pole and the south pole.

Problem 4. Find the transition functions for $\mathbb{R}P^1$ for the "canonical" charts: $(x^1 : x^2) = (y : 1)$ where $x^2 \neq 0$, and $(x^1 : x^2) = (1 : y')$ where $x^1 \neq 0$.

Problem 5. Do the same for $\mathbb{C}P^1$. Write the answer in the complex and the real forms.

Problem 6. Do the same for $\mathbb{R}P^n$ and $\mathbb{C}P^n$. (You have to introduce a notation for charts and coordinates. It is suggested that the coordinates in the j-th chart will be denoted $y^i_{(j)}$, with $i = 1, \ldots, j-1, j+1, \ldots, n+1$, and with a "fake coordinate" $y^j_{(j)} = 1$ (constant).)

Problem 7. Use coordinates to establish homeomorphisms between S^1 and $\mathbb{R}P^1$, and between S^2 and $\mathbb{C}P^1$.