# §5 Triangulations and Euler characteristic

**Problem 1.** Using suitable triangulations find the Euler characteristics for the following topological spaces:
**(a)** $I = [0, 1]$ *Answer:* 1
**(b)** $I^2$ *Answer:* 1
**(c)** $S^1$ *Answer:* 0
**(d)** $S^2$ *Answer:* 2
**(e)** $S^1 \times I$ *Answer:* 0
**(f)** closed Möbius strip $M$ *Answer:* 0

**Problem 2.** Consider two different triangulations for each of the given topological spaces and verify that the Euler characteristic does not depend on a choice of triangulation: **(a)** $I^2$; **(b)** $S^1$; **(c)** $S^2$.

**Problem 3.** **(a)** For a square check that the number of vertices (4) minus the number of edges (4) plus the number of faces (1) coincides with the Euler characteristic (1).
**(b)** Generalize to show that the Euler characteristic of a surface can be calculated from any "tiling" by (homeomorphic images of) squares, each two of them either not meeting or meeting by a single common edge.
Calculate the Euler characteristic of $S^2$ considering it as the boundary of a cube and using tiling by squares.

**Problem 4.** Triangulate the following spaces and find the Euler characteristic:
**(a)** Klein bottle $K$ *Answer:* 0
**(b)** $\mathbb{R}P^2$ *Answer:* 1
**(c)** $T^2$ *Answer:* 0
*Hint:* triangulate the square using the subdivision into 9 smaller squares by lines parallel to the sides. Triangulating surfaces, mark vertices carefully, keeping track of all identifications.

**Problem 5.** Using the excision formula or triangulations show that the Euler characteristic of the sphere with $k$ holes equals $2 - k$.

**Problem 6.** **(a)** Use the excision formula to prove for the standard surface $H^2_g$ (the sphere with $g$ handles) that $\chi(H^2_g) = 2 - 2g$.
**(b)** Use the excision formula to prove for the standard non-orientable surface $M^2_\nu$ (the sphere with $\nu$ Möbius strips) that $\chi(M^2_\nu) = 2 - \nu$.
*Hint:* use the results of Problem 1 and Problem 5.

**Problem 7.** A surface $X^2_{g,\nu}$ is obtained from the sphere by attaching $g$ handles and gluing in $\nu$ Möbius strips. Find its Euler characteristic.

**Problem 8.** Prove that the surface $X_{g,\nu}$ defined in Problem 7 is homeomorphic to the standard surface $M^2_{2g+\nu}$. *Hint:* apply the classification theorem for closed surfaces and the result of the previous problem.