

**Note: Most of this sheet should be revision.**

1. Find the magnitude of the following vector functions:

$$a) y\mathbf{i} + x\mathbf{j}, \quad b) x\mathbf{j}, \quad c) \frac{(\mathbf{i} + \mathbf{j})}{\sqrt{2}}, \quad d) \mathbf{i} + y\mathbf{j}.$$

(And sketch them if you are feeling adventurous.)

2. a) Write a formula for a vector function in two dimensions which is in the positive radial direction and whose magnitude is 1.  
 b) Write a formula for a vector function in two dimensions whose direction makes an angle of  $45^\circ$  with the  $x$ -axis and whose magnitude at any point is  $(x + y)^2$ .  
 c) Write a formula for a vector function in three dimensions which is in the positive radial direction and whose magnitude is 1.

3. Prove that any given vector function  $\mathbf{F} = F_x(x, y, z)\mathbf{i} + F_y(x, y, z)\mathbf{j} + F_z(x, y, z)\mathbf{k}$  satisfies the identity:  $\nabla \cdot (\nabla \times \mathbf{F}) = 0$ .

4. Prove that for any scalar function  $f(x, y, z)$  we have  $\nabla \times (\nabla f) = 0$ .

5. Calculate the divergence of each of the following vector fields:

$$a) x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}, \quad b) yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}, \quad c) x\mathbf{i} + y\mathbf{j} + z\mathbf{k}, \quad d) \frac{-y\mathbf{i} + x\mathbf{j}}{\sqrt{x^2 + y^2}}.$$

6. Calculate the curl of each of the following vector fields:

$$a) z^2\mathbf{i} + x^2\mathbf{j} - y^2\mathbf{k}, \quad b) yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}, \quad c) xy\mathbf{i} + y^2\mathbf{j} + yz\mathbf{k}, \quad d) \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{\frac{3}{2}}}.$$

7. (Revision) Evaluate the following integrals and sketch the regions in the  $x - y$  plane that are determined by the limits of integration:

$$a) \int_0^1 \int_0^{x^3} 3 \, dy \, dx \quad b) \int_0^2 \int_0^{y^2} y \, dx \, dy.$$

8. (Revision) For the following double integrals, sketch the region of integration, reverse the order of integration and evaluate both integrals:

$$a) \int_0^1 \int_0^x 2 - x - y \, dy \, dx, \quad b) \int_0^3 \int_1^{e^x} 2 \, dy \, dx, \quad c) \int_0^{\frac{\pi}{2}} \int_0^{\cos x} \sin x \, dy \, dx.$$

9. Let

$$V = \{(x, y, z) \mid -2 \leq x \leq 3, 0 \leq y \leq 1, 0 \leq z \leq 5\}.$$

Evaluate the integral:

$$\int \int \int_V x^2 e^y + xyz \, dV.$$

10. Evaluate the following triple integrals

$$a) \int_{-1}^1 \int_0^2 \int_1^3 xyz \, dz \, dy \, dx, \quad b) \int_0^1 \int_0^2 \int_0^3 x^2 + y^2 + z^2 \, dz \, dy \, dx, \quad c) \int_{-1}^2 \int_1^{z^2} \int_0^{y+z} 3yz^2 \, dx \, dy \, dz.$$

11. Set up and evaluate a volume integral to find the volume of a cylinder of radius  $a$  units, with base on the  $x$ - $y$  plane and with height  $b$  units.