MATH20411 Problem sheet 7: Finite Difference Methods A

---

1. Using the definition of the centered finite difference of $u(x)$ at a point $x = x_j$, derive the second centered finite difference and hence give an approximation to the second derivative $\frac{d^2u}{dx^2}$ at a point $x = x_j$.

2. Consider the model reaction-diffusion problem: find $u(x)$ satisfying

$$-\frac{d^2u}{dx^2} + r(x)u = f(x), \quad 0 < x < 1 \quad \text{with} \quad u(0) = 0,\, u(1) = 0.$$

   a) Let $r(x) = 0$ and $f(x) = 1$. Choose 5 equally spaced intervals on the x-axis and generate an approximation to the solution at the grid points using the centered finite difference scheme. Compare your answer with the exact solutions at the grid points. What do you observe?

   b) Repeat your calculations with $r(x) = 16$ and $f(x) = 1$. Compare your answer with the exact solutions at the grid points. What do you observe?

   You should derive the systems of equations to be solved by hand. To solve the tridiagonal systems easily - download the MATLAB program file `trisolve.m` from the course webpage. Save it to your P-drive. Open it in MATLAB and follow the instructions. You may also find the code `reac_diff_1d.m` useful to check your answers and plot the results.

3. Write down the second centered difference approximation to $\frac{d^2u}{dx^2}$ at $x = x_j$. By expanding the terms $u(x_j + h)$ and $u(x_j - h)$ about the point x_j with a Taylor series, determine that the truncation error, T_j in the approximation satisfies:

$$T_j = -\frac{h^2}{12}\frac{d^4u(x_j)}{dx^4} + \text{terms with higher powers of}\, h.$$

   That is, the method is second-order accurate. How does this explain the observations made in question 2?