1. Which of the following PDEs can be solved using the method of separation of variables? (Do not solve them.)

   a) $\frac{\partial^2 u}{\partial t^2} + (t^2 + x^2)\frac{\partial^2 u}{\partial x^2} = 0$
   b) $t^2 \frac{\partial^2 u}{\partial t^2} + x^2 \frac{\partial^2 u}{\partial x^2} = 0$
   c) $x^2 \frac{\partial^2 u}{\partial t^2} + t^2 \frac{\partial^2 u}{\partial x^2} = 0$
   d) $\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} - (1 - x^2)u = 0$
   e) $\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + (t - x)u = 0$
   f) $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial t \partial x} + \frac{\partial^2 u}{\partial x^2} - u = 0$

2. Find all of the eigenvalues and eigenfunctions for each of the following problems (*Be sure that you consider all possible values for* λ.)

   (a) $X'' - \lambda X = 0$ with $X(0) = X(l) = 0$.
   (b) $Y'' - \lambda Y = 0$ with $Y'(0) = Y'(l) = 0$.
   (c) $Z'' - \lambda Z = 0$ with $Z'(0) = Z(l) = 0$.
   (d) $F'' - \lambda F = 0$ with $F(0) = F'(l) = 0$.

3. Given that the eigenfunctions are orthogonal on $x \in [0, l]$ in each of the following expansions, find all of the coefficients.

   a) $1 = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$ *b*) $x = \sum_{n=0}^{\infty} a_n \cos \frac{n\pi x}{l}$

   c) $\pi = \sum_{n=0}^{\infty} a_n \cos \frac{(n+\frac{1}{2})\pi x}{l}$ *d*) $x = \sum_{n=0}^{\infty} b_n \sin \frac{(n+\frac{1}{2})\pi x}{l}$

4. The temperature $u(x, t)$ in a thin bar of length a units satisfies the partial differential equation $u_t = u_{xx}$ and is subject to the following homogeneous boundary conditions:

   $$u_x(0, t) = 0 \quad \text{and} \quad u(a, t) = 0.$$

   If the temperature also satisfies the initial condition $u(x, 0) = a$ for all $0 < x < a$ find the exact solution for $u(x, t)$ throughout the "strip" $x \in [0, a]$, $t \in [0, \infty)$, in the form of a Fourier expansion.