1. Find all the first and second partial derivatives of the function $u(x, y) = x^2y^4 + e^{xy}$.

2. Show that the function $u(x, y) = \ln \sqrt{x^2 + y^2}$ is a solution to Laplace's equation written in Cartesian co-ordinates. Convert $u(x, y)$ into a function of polar co-ordinates r, θ and show that it satisfies Laplace's equation in polar co-ordinates.

3. Verify that the function:
$$u(x, t) = \frac{e^{-\frac{x^2}{4t}}}{\sqrt{t}},$$
is a solution of the one-dimensional heat equation: $u_t = u_{xx}$ for values $0 \leq x < \infty$ and $t > 0$. How does this function behave at $x = 0$ and $x = \infty$ and at time $t = 0$? Hence, write down boundary conditions and an initial condition satisfied by this solution and describe, in words, the physical situation being modelled.

4. Prove that the one-dimensional wave equation and Laplace's equation, in two space dimensions, are linear PDEs.

5. For each of the following PDEs, state the order of the equation and whether or not the equation is linear. For those that are linear, state whether or not they are homogeneous.

   i) $2u_x - 5uu_y = 0$,

   ii) $u_{xx} = xu_x$,

   iii) $y^2u_x + 3x^3u_y = (x + y)u$,

   iv) $\nabla^2 u = 4u + 2xy$,

   v) $\nabla^2 \left(\nabla^2 u\right) = \lambda^2 u$,

   vi) $\nabla^2 u = \sinh u$.

6. Classify the following PDEs as being either elliptic, parabolic or hyperbolic:

   i) $u_{tt} + au_t + ku - c^2u_{xx} = f(x, t)$

   ii) $u_x + u_{xx} + u_y + u_{yy} = \sin(xy)$,

   iii) $u_x + u_{xx} - u_y - u_{yy} = \cos(xyu)$,

   iv) $u_{tt} + xu_{xx} + u_t = h(x, t)$,

   v) $u_{xx} + 2yu_{xy} + xu_{yy} - u_x + u = 0$.

7. Set up the boundary and initial value problems associated with the following physical problems.

   i) Find the temperature in a metal wire of length π units, with uniform thermal conductivity constant $K = 3.2$ which is insulated at both ends and has initial temperature distribution $f(x) = \cos(2x)$ at time $t = 0$ seconds.

   ii) Find the displacement, measured from the horizontal axis, of a string of length 2 units, fixed at both ends, in the time interval, zero to two seconds, vibrating with speed 1.5 units and with initial displacement profile $\sin(3x)$ and initial velocity zero.

   iii) Find the steady-temperature in a square metal plate, of edge length 10 units, if the horizontal edges are insulated, the left hand vertical wall is heated constantly to a temperature of 5^oC and the right hand edge is dipped in an ice bath of temperature 0^oC.