1. For the following functions, sketch (i) $f(x)$ on the stated intervals, (ii) the periodic extension $\tilde{f}(x)$ and (iii) the Fourier series associated with the periodic extension. (You do not need to compute the Fourier coefficients.)

$$a)\ f(x) = x, \quad -1 \le x \le 1, \quad b)\ f(x) = e^x \quad -1 \le x \le 1$$

$$c)\ f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ x & 0 < x \le \pi \end{cases}, \quad d)\ f(x) = x^2, \quad -1 \le x \le 1.$$

2. Compute the Fourier series of the periodic extension $\tilde{f}(x)$ of each of the following functions defined on the interval $[-L, L]$,

$$a)\ f(x) = \sin\left(\frac{\pi x}{L}\right), \qquad b)\ f(x) = x, \qquad c)\ f(x) = \begin{cases} 1 & -L \le x \le 0 \\ 2 & 0 < x \le L \end{cases}.$$

3. Determine which of the following functions are even and which are odd:

$$a)\ f(x) = |x| \qquad b)\ f(x) = x^{175} \qquad c)\ f(x) = x^{-1}, \qquad d)\ f(x) = \cos\left(\frac{4\pi x}{3}\right).$$

4. Sketch the odd extensions $f_{odd}(x)$ of the following functions defined on $[0, 1]$, to the symmetric interval $[-1, 1]$.

$$a)\ f(x) = 10 \qquad b)\ f(x) = x \qquad c)\ f(x) = e^x$$

   Next, consider the periodic extension $\tilde{f}_{odd}(x)$ of the odd extension, and sketch the associated Fourier sine series. Compute the Fourier coefficients for cases (a) and (b).

5. Starting from the definition of the standard Fourier series, use standard properties of even and odd functions to derive the Fourier cosine series of an even function $g(x)$ that is piecewise smooth on $[-L, L]$ and periodic, with period $2L$.

6. Sketch the even extensions $f_{even}(x)$ of the following functions defined on $[0, 1]$, to the symmetric interval $[-1, 1]$.

$$a)\ f(x) = 10 \qquad b)\ f(x) = x \qquad c)\ f(x) = e^x.$$

   Next, consider the periodic extension $\tilde{f}_{even}(x)$ of the even extension, and sketch the associated Fourier cosine series. Compute the Fourier coefficients for cases (a) and (b).